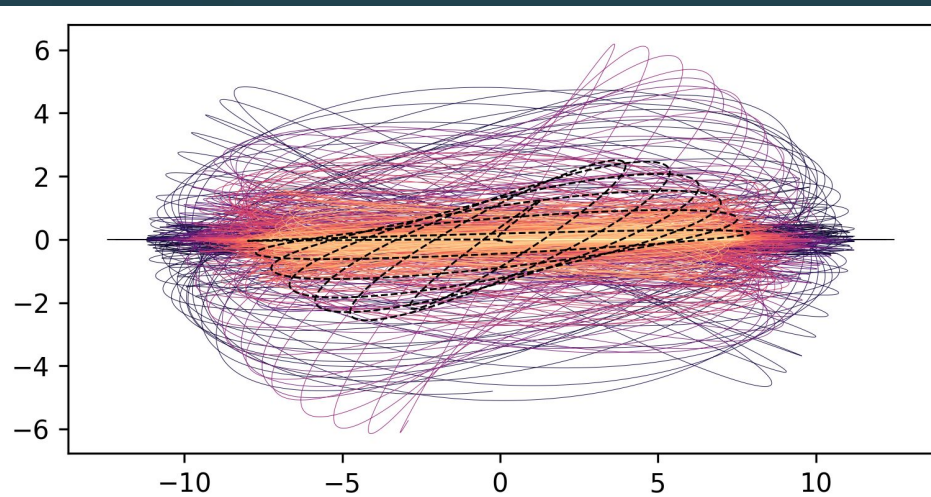
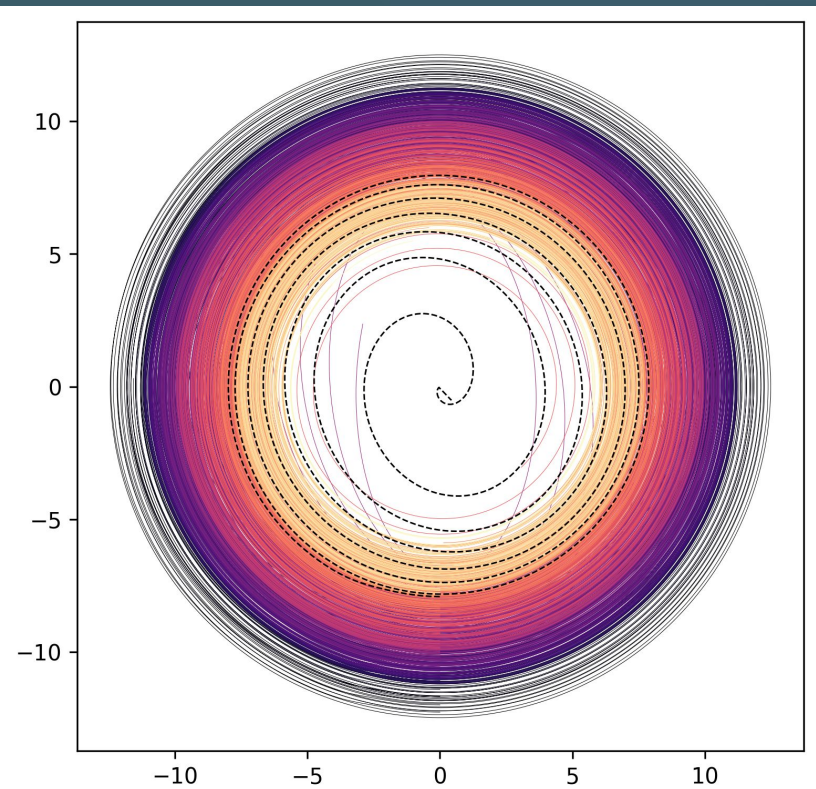


BlackHoles@Home Update



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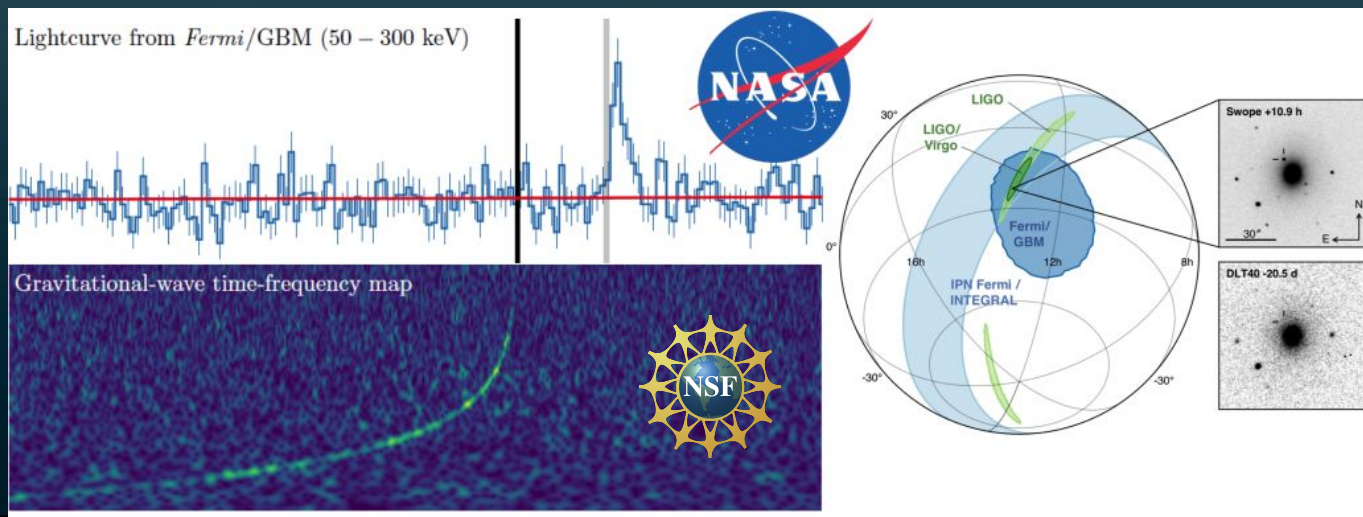
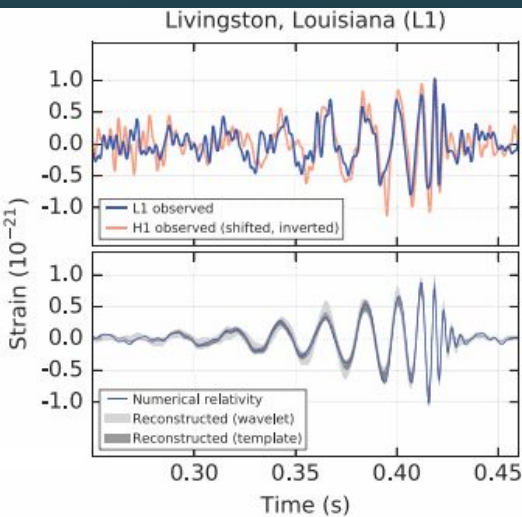
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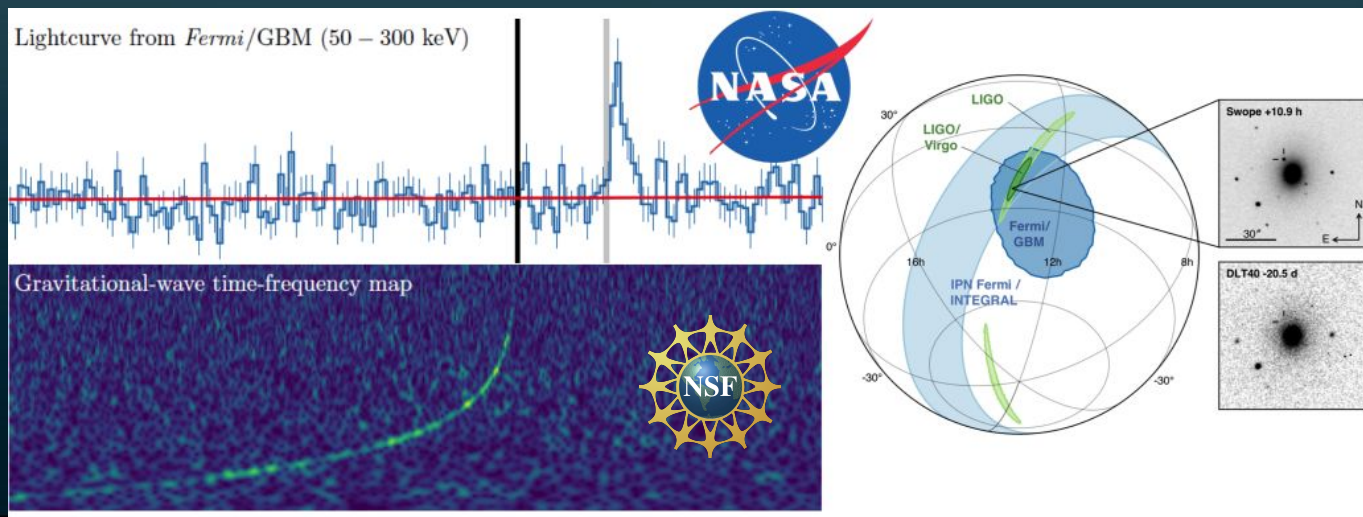
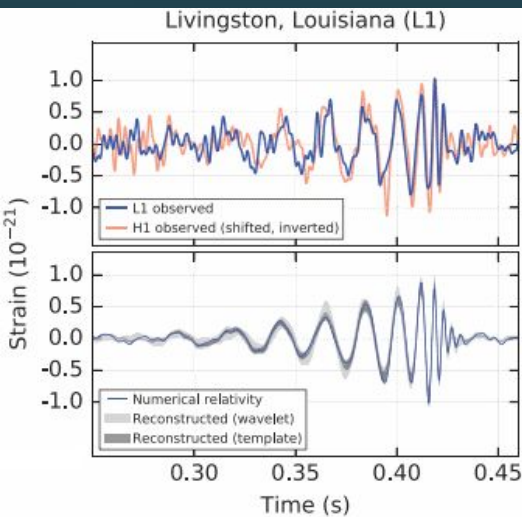
Importance of modeling gravitational wave and multimessenger sources

- Example: LIGO detects a gravitational wave from a black hole or neutron star binary



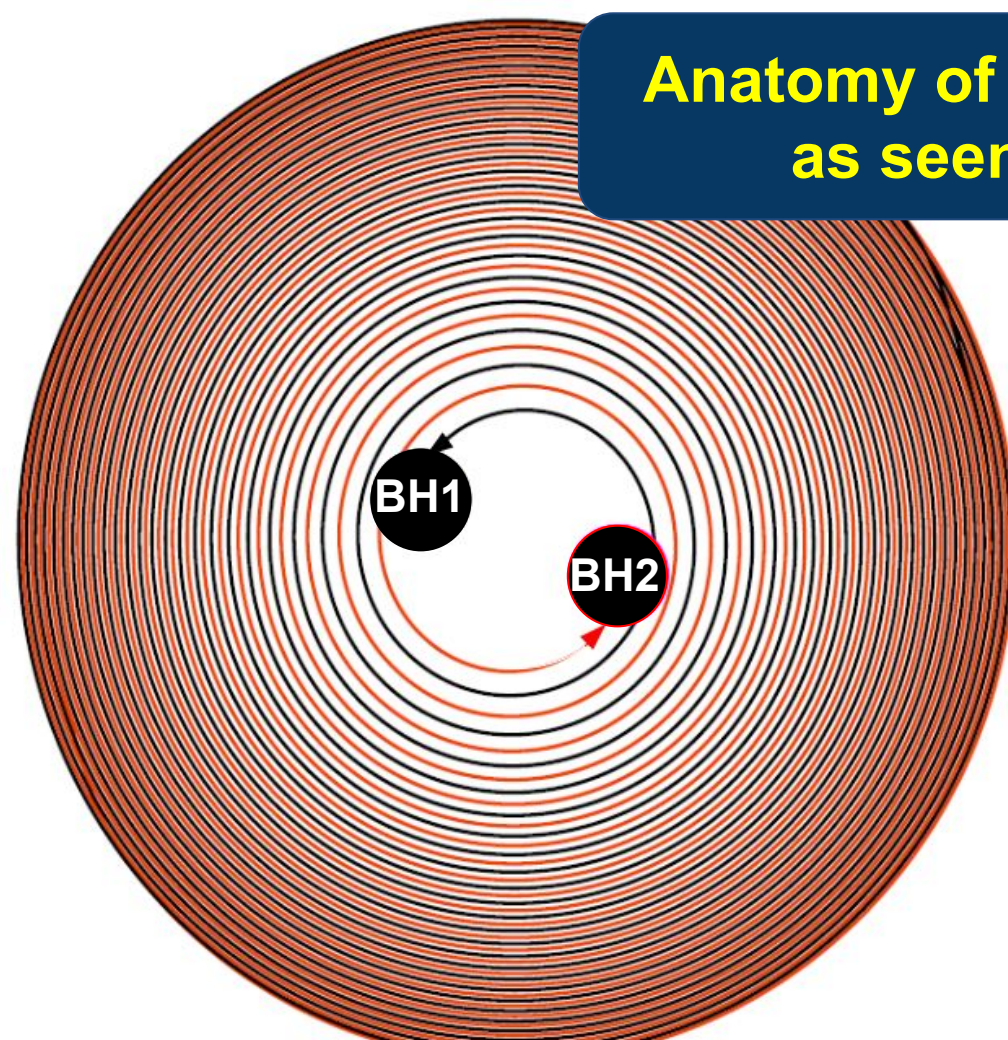
Importance of modeling gravitational wave and multimessenger sources

- \$1B+ Question: What *exactly* caused this and *how*?
 - Answer → insights into extreme gravity and matter, pushing theor. limits.
 - To advance science, must compare observations with theoretical predictions
 - Theoretical predictions must span observ. & theor. uncertainties



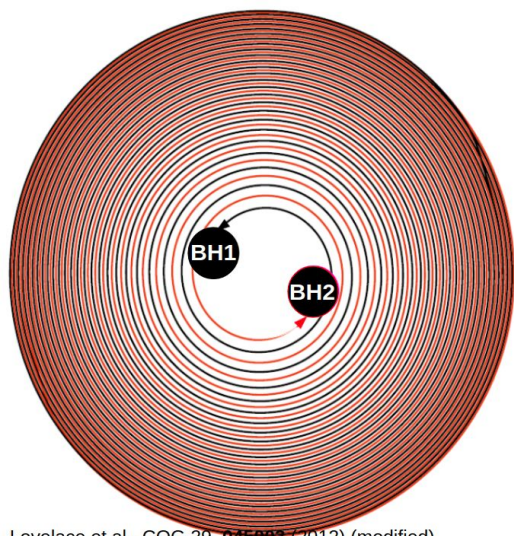
Anatomy of a Binary Black Hole Merger, as seen in gravitational waves

- Gravitational-wave driven
“Relativistic death spiral”

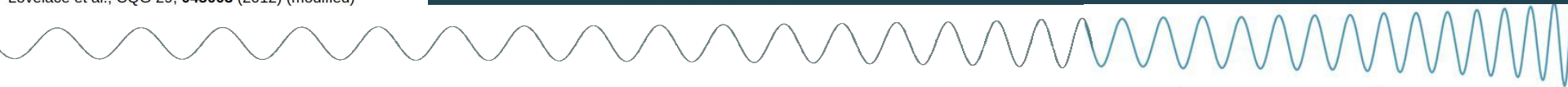


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Lovelace et al., CQG 29, 045003 (2012) (modified)



Time axis \Rightarrow

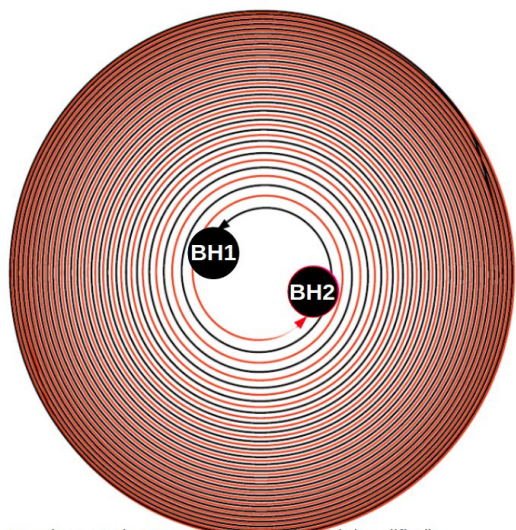
(spans $\sim 200\text{ms}$)

Wave amplitude \Uparrow

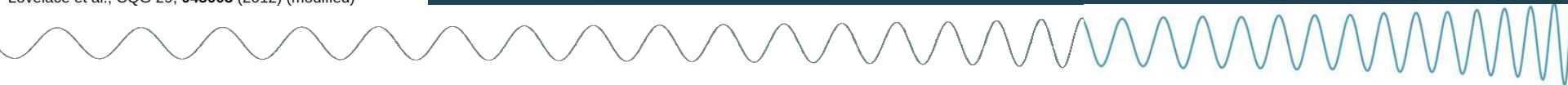
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These waves encode info
about masses, spins, and
eccentricity of orbiting
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Time axis \Rightarrow

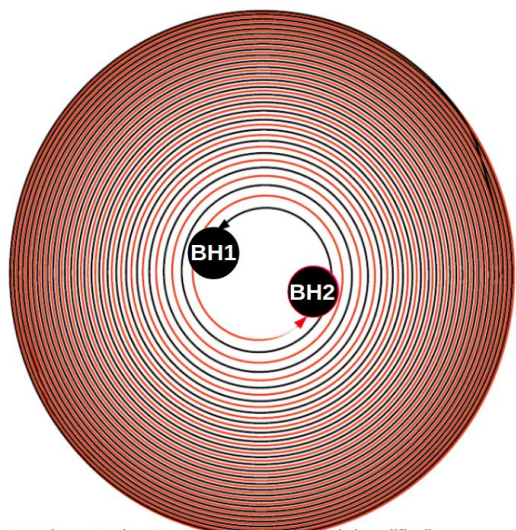
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Perturbative solutions
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Time axis ⇨

(spans ~200ms)

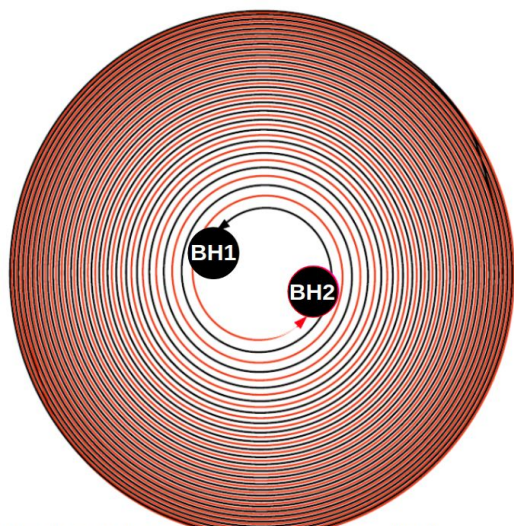
Wave amplitude ⇧

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Late inspiral: Perturb.
theory breaks down;
Only full GR solutions



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$$\begin{aligned}\partial_t \bar{\gamma}_{ij} &= [\beta^k \partial_k \bar{\gamma}_{ij} + \partial_i \beta^k \bar{\gamma}_{kj} + \partial_j \beta^k \bar{\gamma}_{ik}] + \frac{2}{3} \bar{\gamma}_{ij} (\alpha \bar{A}_k^k - \bar{D}_k \beta^k) - 2\alpha \bar{A}_{ij} , \\ \partial_t \bar{A}_{ij} &= [\beta^k \partial_k \bar{A}_{ij} + \partial_i \beta^k \bar{A}_{kj} + \partial_j \beta^k \bar{A}_{ik}] - \frac{2}{3} \bar{A}_{ij} \bar{D}_k \beta^k - 2\alpha \bar{A}_{ik} \bar{A}_j^k + \alpha \bar{A}_{ij} K \\ &\quad + e^{-4\phi} \{ -2\alpha \bar{D}_i \bar{D}_j \phi + 4\alpha \bar{D}_i \phi \bar{D}_j \phi + 4\bar{D}_{(i} \alpha \bar{D}_{j)} \phi - \bar{D}_i \bar{D}_j \alpha + \alpha \bar{R}_{ij} \}^{\text{TF}} , \\ \partial_t \phi &= [\beta^k \partial_k \phi] + \frac{1}{6} (\bar{D}_k \beta^k - \alpha K) , \\ \partial_t K &= [\beta^k \partial_k K] + \frac{1}{3} \alpha K^2 + \alpha \bar{A}_{ij} \bar{A}^{ij} - e^{-4\phi} (\bar{D}_i \bar{D}^i \alpha + 2\bar{D}^i \alpha \bar{D}_i \phi) , \\ \partial_t \bar{\Lambda}^i &= [\beta^k \partial_k \bar{\Lambda}^i - \partial_k \beta^i \bar{\Lambda}^k] + \bar{\gamma}^{jk} \hat{D}_j \hat{D}_k \beta^i + \frac{2}{3} \Delta^i \bar{D}_j \beta^j + \frac{1}{3} \bar{D}^i \bar{D}_j \beta^j \\ &\quad - 2\bar{A}^{ij} (\partial_j \alpha - 6\partial_j \phi) + 2\alpha \bar{A}^{jk} \Delta_{jk}^i - \frac{4}{3} \alpha \bar{\gamma}^{ij} \partial_j K \\ \partial_t \alpha &= [\beta^i \partial_i \alpha] - 2\alpha K \\ \partial_t \beta^i &= [\beta^j \partial_j \beta^i] + B^i \\ \partial_t B^i &= [\beta^j \partial_j B^i] + \frac{3}{4} \partial_0 \bar{\Lambda}^i - \eta B^i\end{aligned}$$

Most popular formulation

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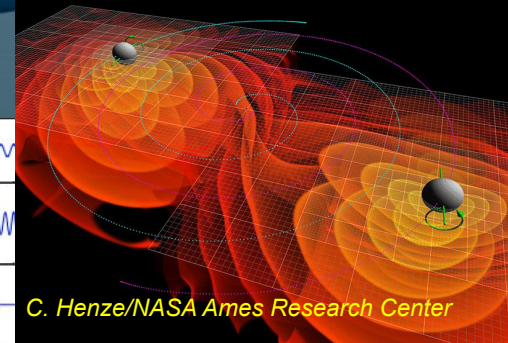
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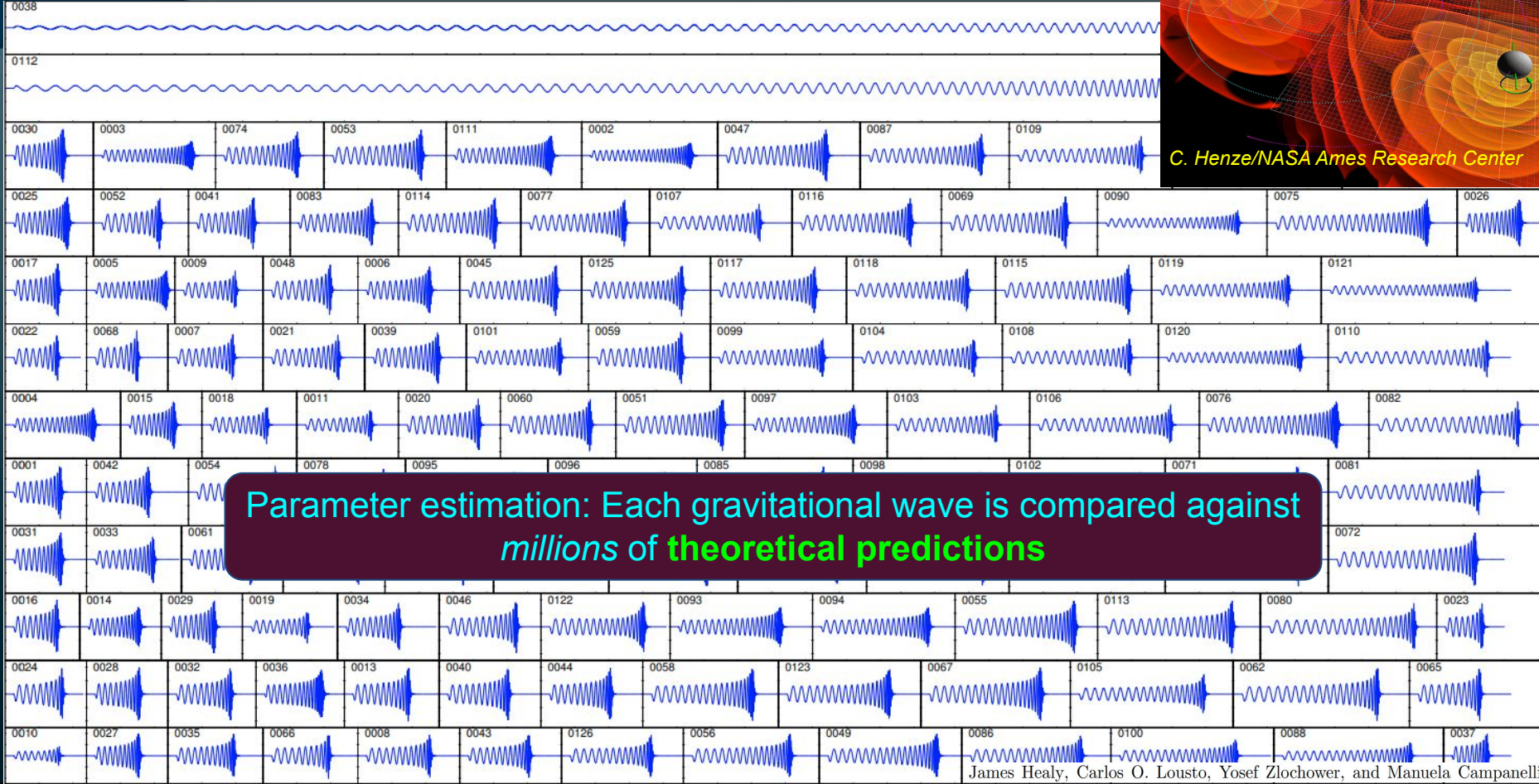
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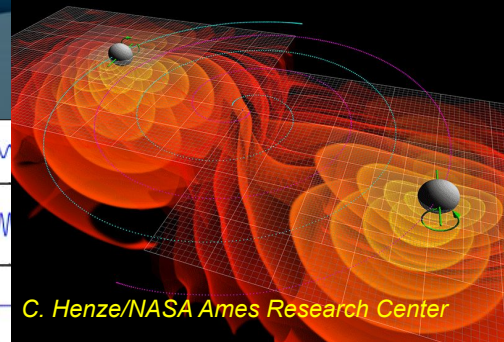


C. Henze/NASA Ames Research Center



Parameter estimation: Each gravitational wave is compared against
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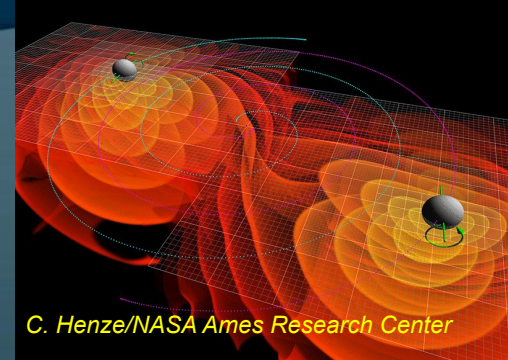


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Theoretical predictions are built upon
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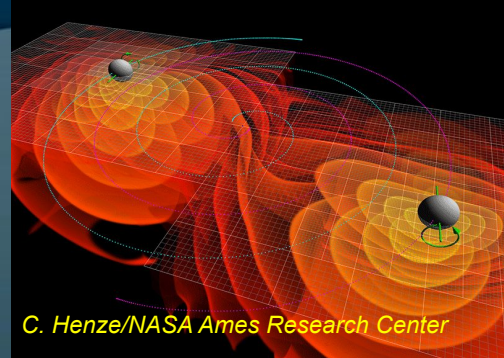
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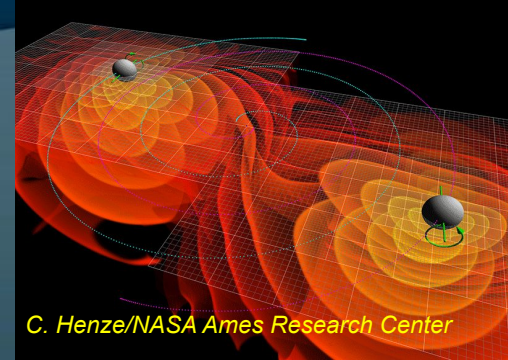
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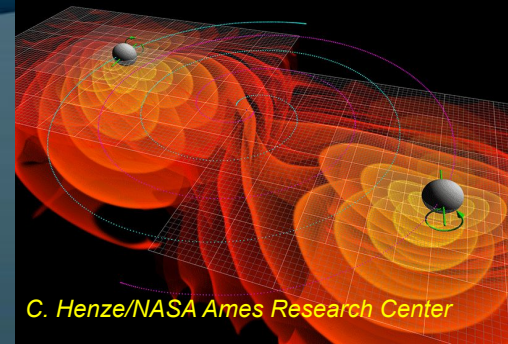
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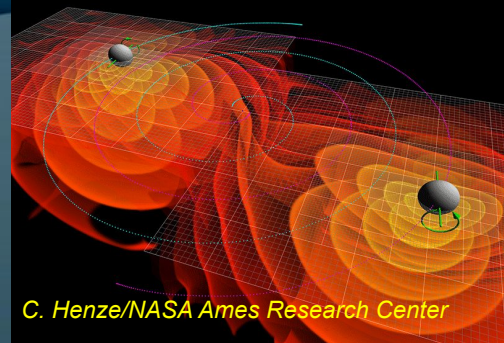
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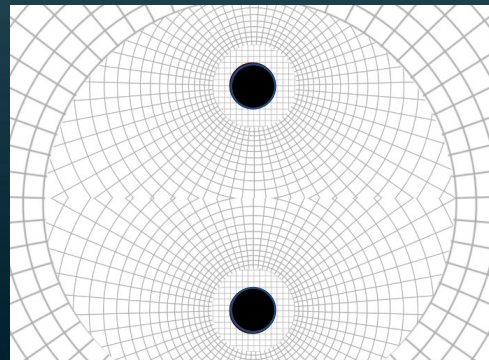
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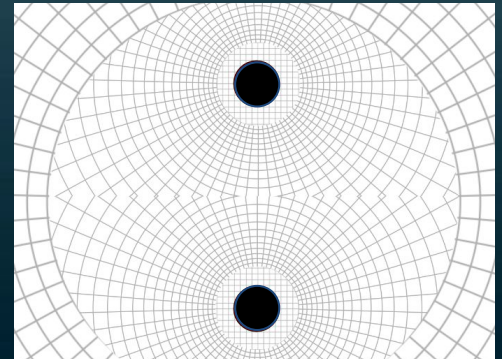
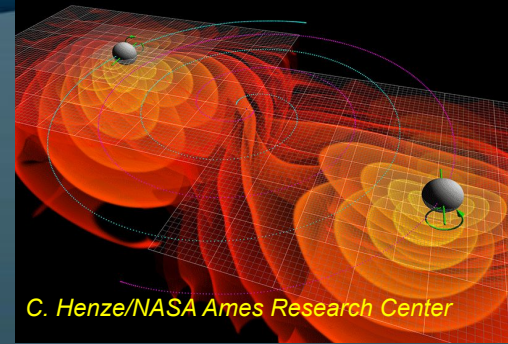
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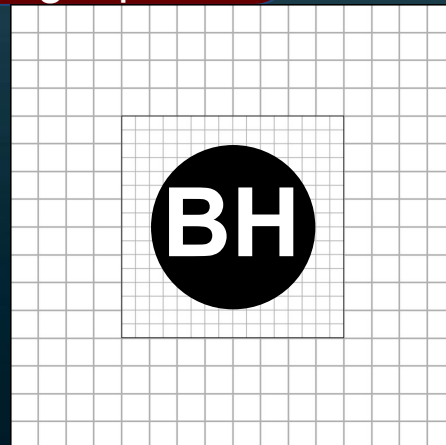
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How to address?³⁷

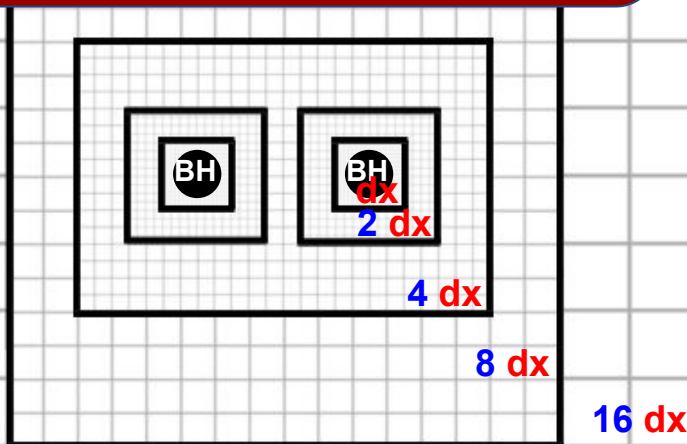
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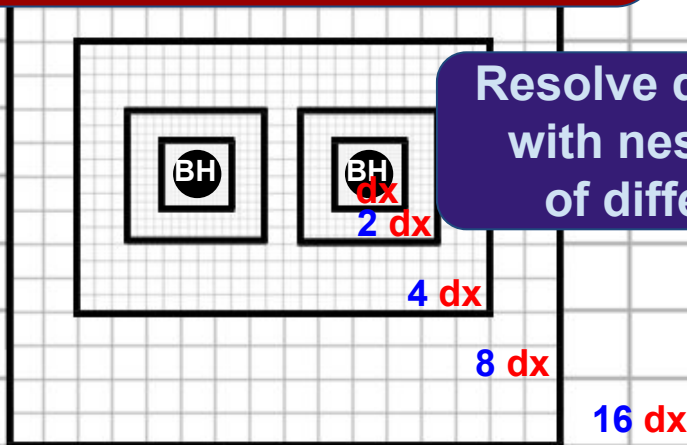
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Resolve disparate lengthscales
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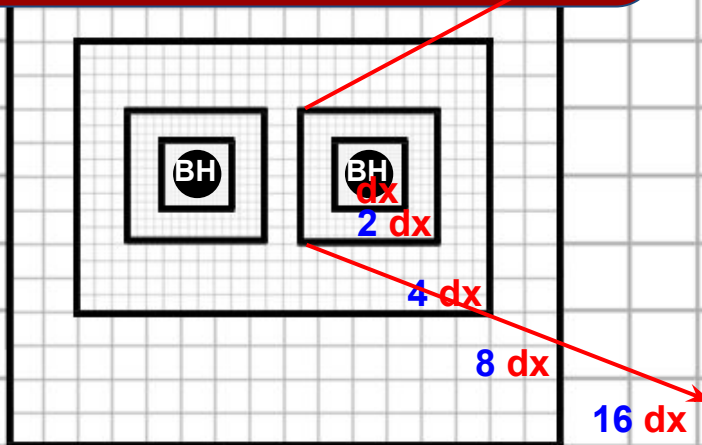
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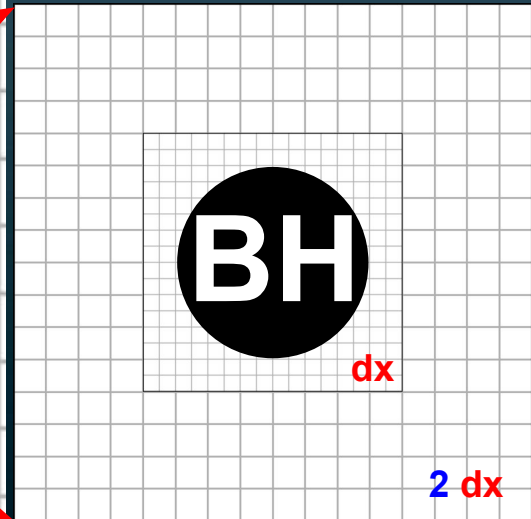
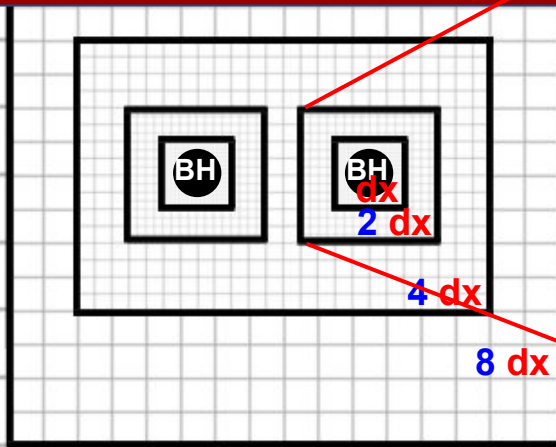
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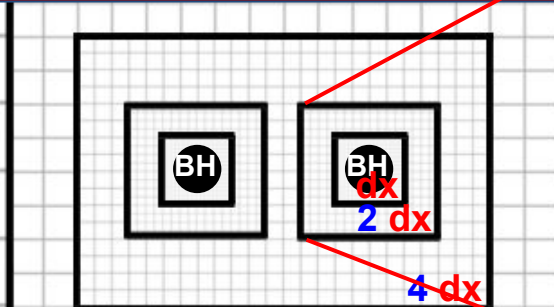
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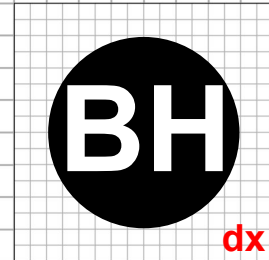
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Resolution highest where fields are sharpest -- near BHs for example



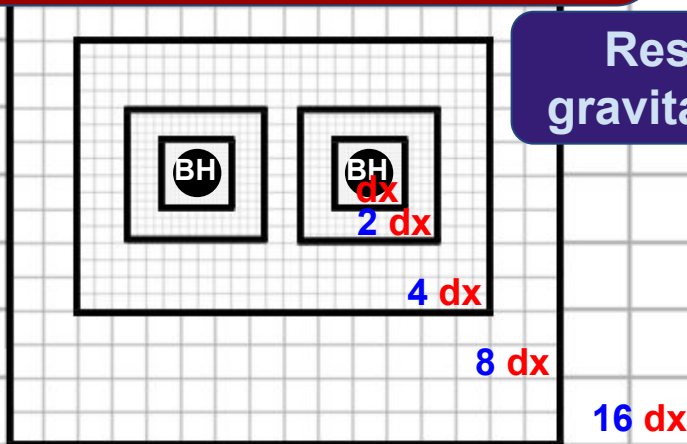
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Resolution lower where
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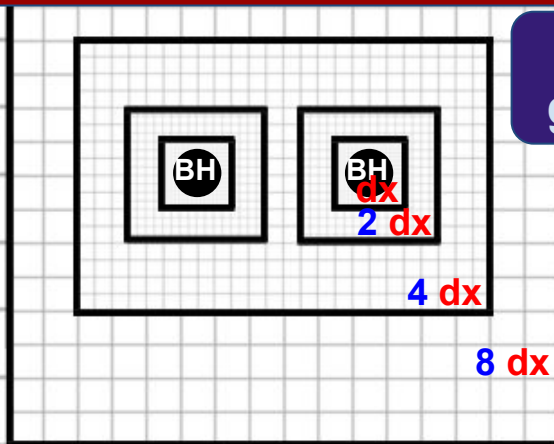
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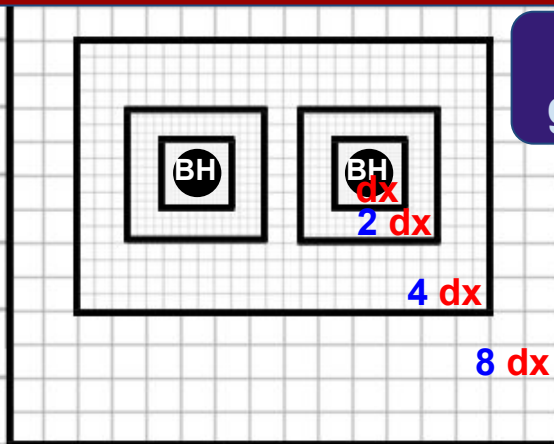
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(Most Popular Method in NR)*



Resolution lower where
gravitational waves modeled

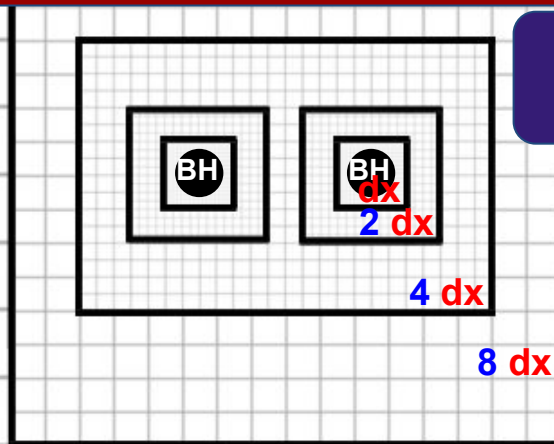
Modeling Challenges

Address (~5 orders of mag) disparity in physical scales

- ✓ 1. Resolve **sharp**, rapidly changing grav fields near BHs and NSs
- ✓ 2. Model **long-wavelength** gravitational waves far away
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AMR

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Add coarser grids to push
outer boundary far away

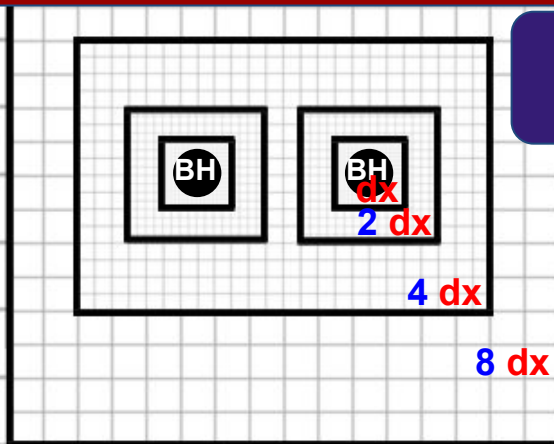
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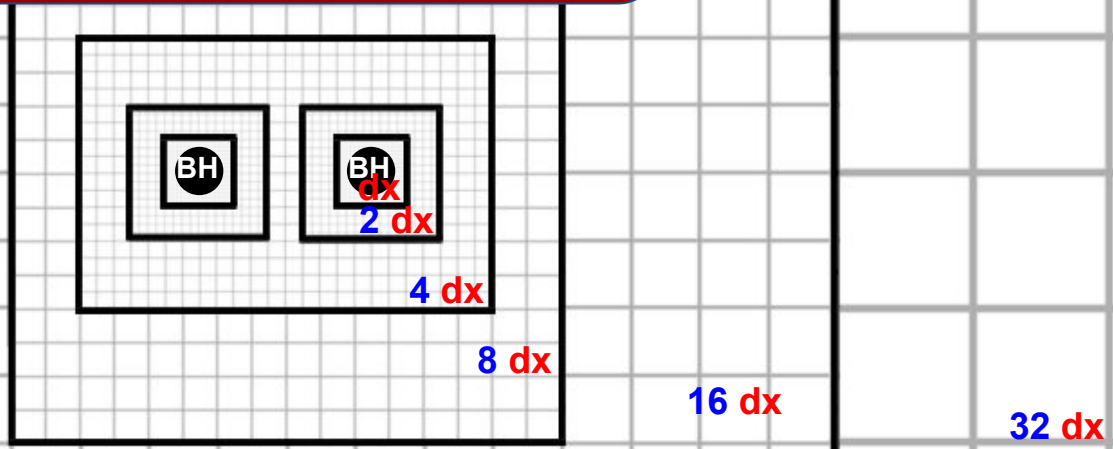
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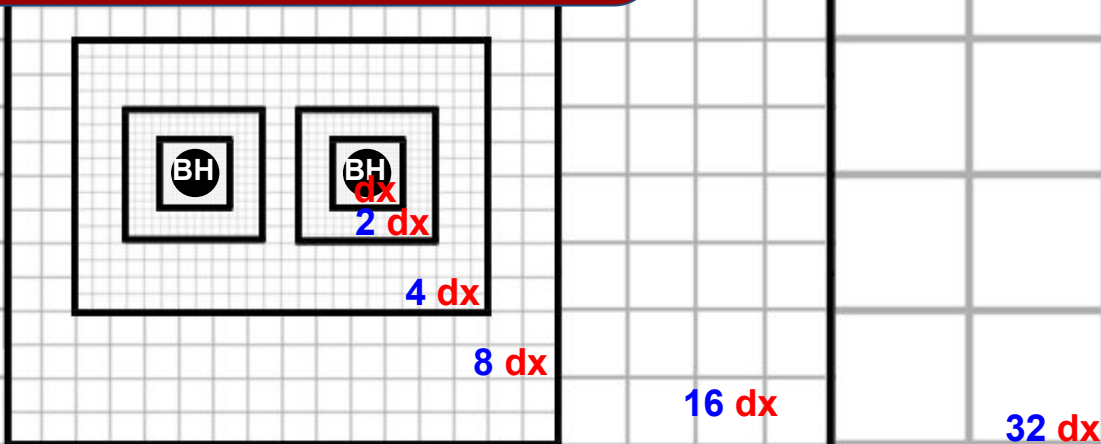
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AMR

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Modeling Challenges

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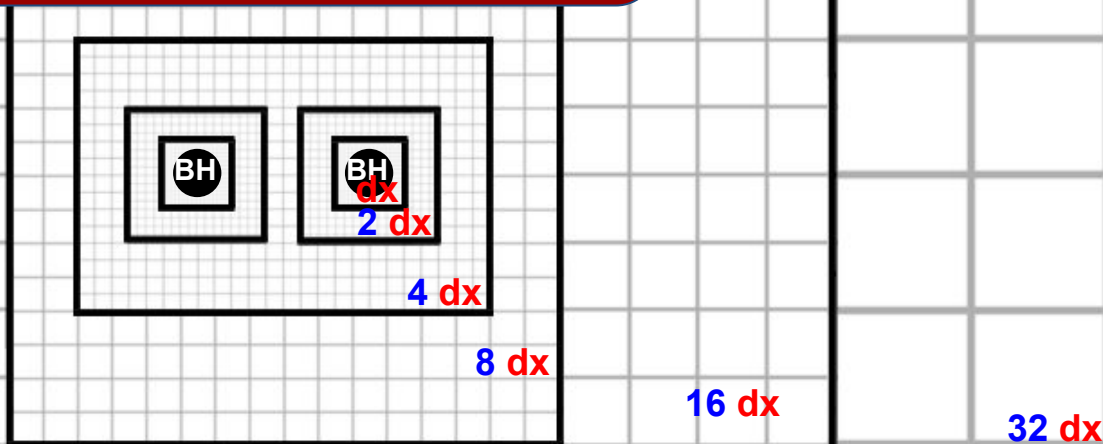
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What's the problem?

A: Inefficient!
⇒ greater comp. cost

AMR

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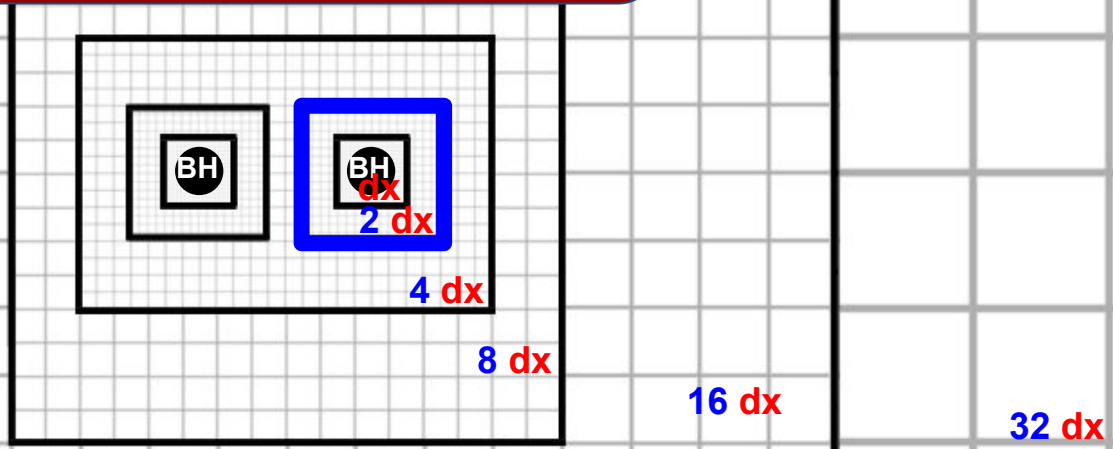
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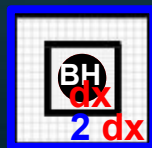
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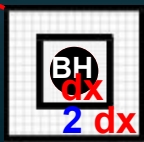
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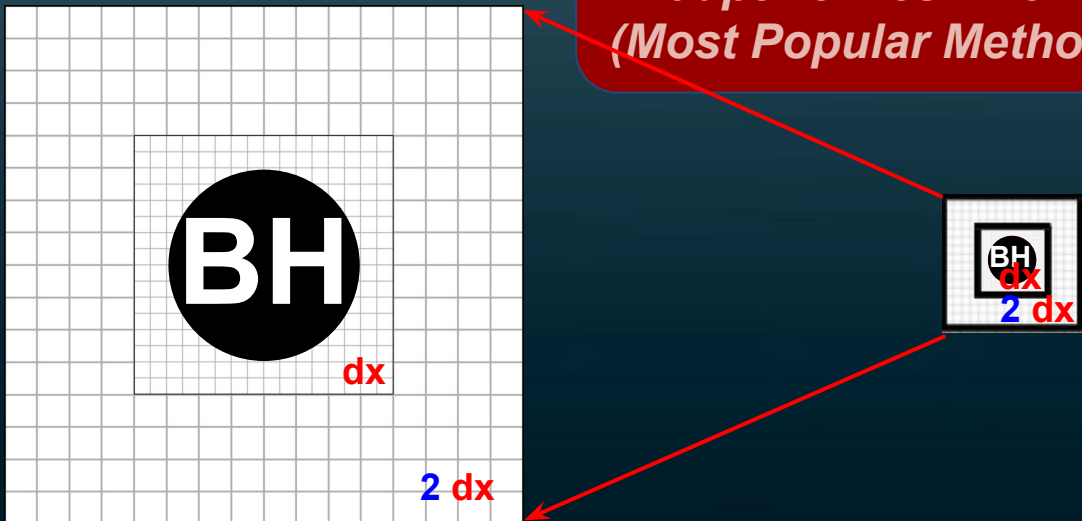
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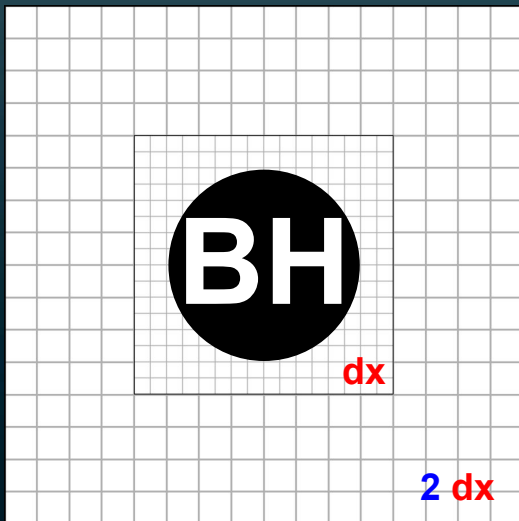
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nearly spherical/axisymmetric
 - \Rightarrow grav/matter fields drop off strongly in *radial* direction
 - \Rightarrow need highest sampling in r direction



Modeling Challenges

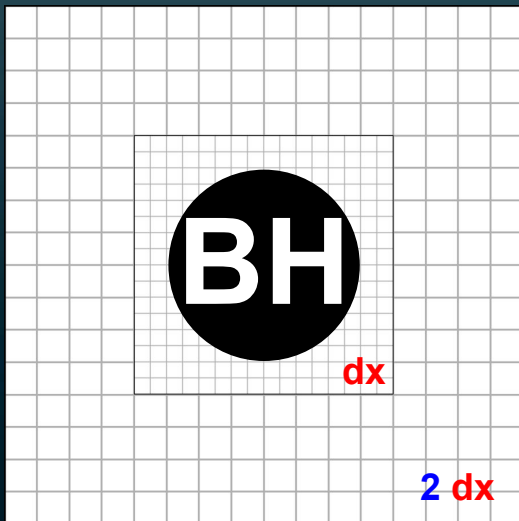
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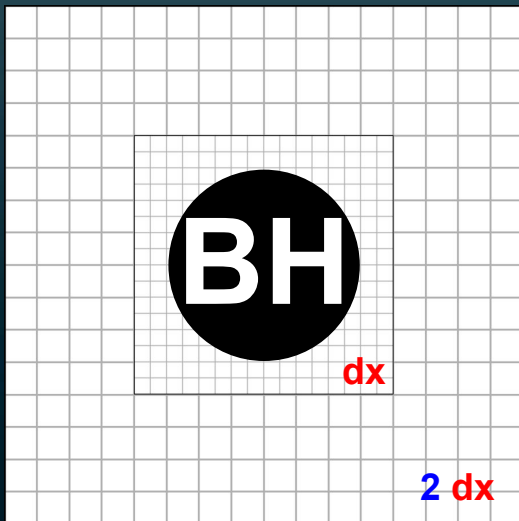
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 - Cartesian **AMR** grids: *x*, *y*, & *z* directions are all radial!
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 - **Spherical** grids: **~5x more efficient**;
 - need high sampling only in *r* direction

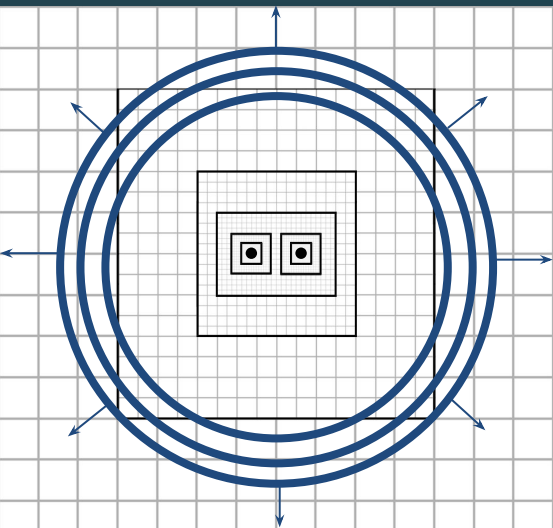
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AMR Inefficiencies

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2. Gravitational waves far away nearly spherical
 - \Rightarrow grav. waves vary most strongly in *radial* direction
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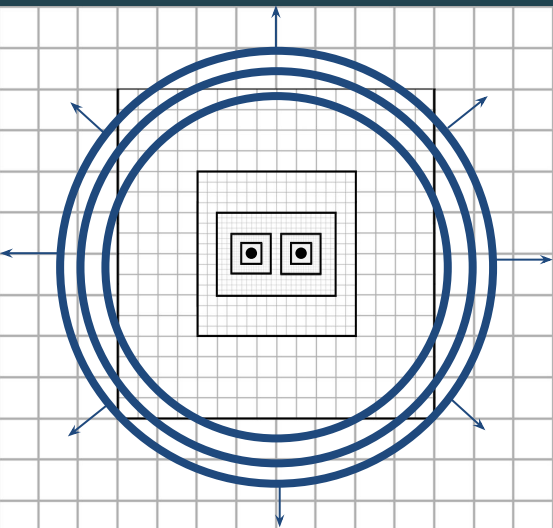
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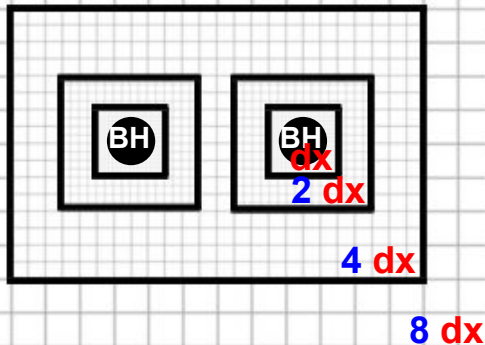
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AMR Inefficiencies

*Adaptive Mesh Refinement
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3. Grav & matter fields are mostly smooth

- Cartesian **AMR** grids:
 - 2x jumps in resolution between boxes
 - Boxes have sharp corners
- **Bi-spherical-like** grids: **another ~4x efficiency boost**
 - Smooth, logarithmic r coordinate from NSs
 - Uniform angular coordinates

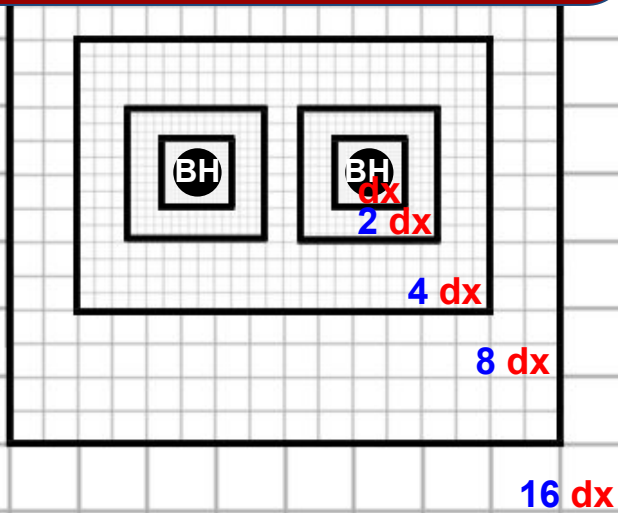


BlackHoles@Home: BBHs on the Desktop

Development of Super-Efficient Grid Structures

AMR Grids

*Adaptive Mesh Refinement
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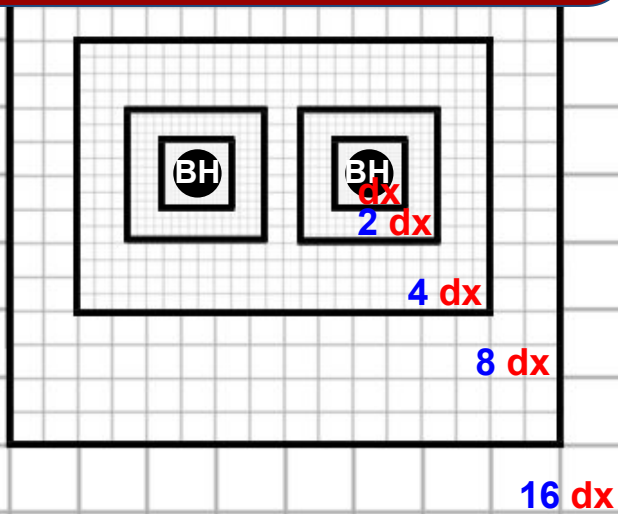


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More efficient grids would...

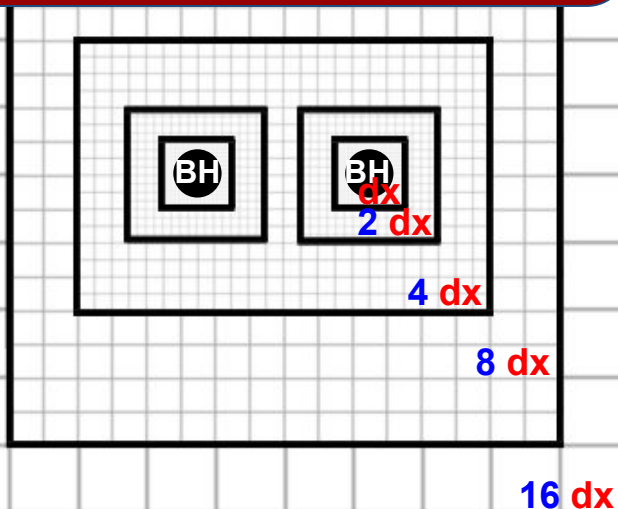
- Employ smoother transitions in resolution
- Exploit near-symmetries
 - Singular coords!

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Development of Super-Efficient Grid Structures

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More efficient grids would...

- Employ smoother transitions in resolution
- Exploit near-symmetries
 - Singular coords!

NR in Spherical coordinates

- Brown (PRD 79, 104029, **2009**)
- Baumgarte, Montero, Cordero-Carrión, Müller (PRD 87, 044026, **2012**)

NR in Spherical-like, Cyl-like, etc.

- Ruchlin, Etienne, Baumgarte (PRD 97, 064036, **2018**)

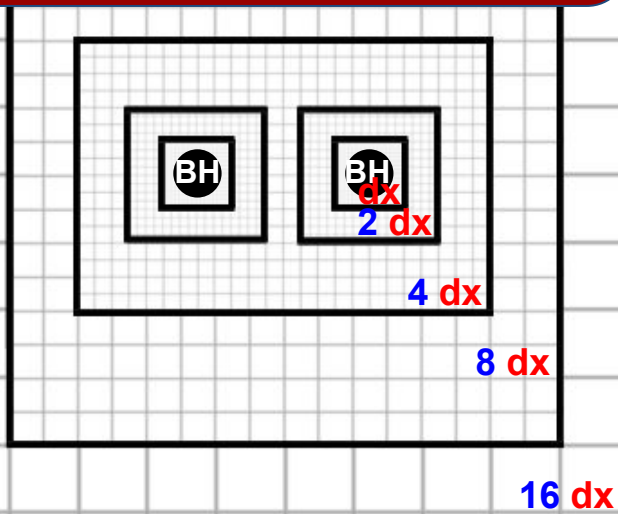


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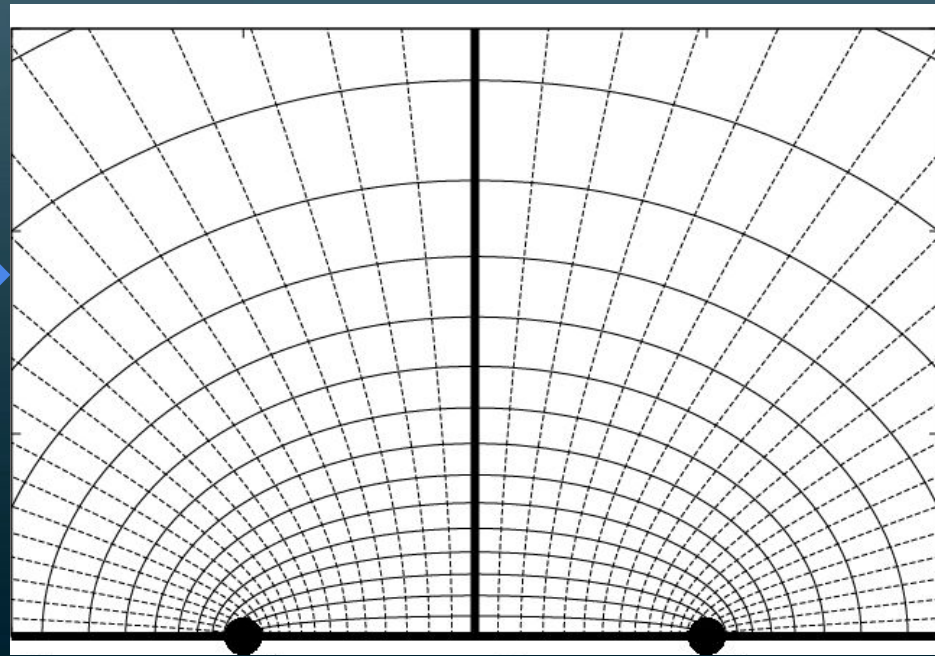
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AMR Grids

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TwoPuncture Grids

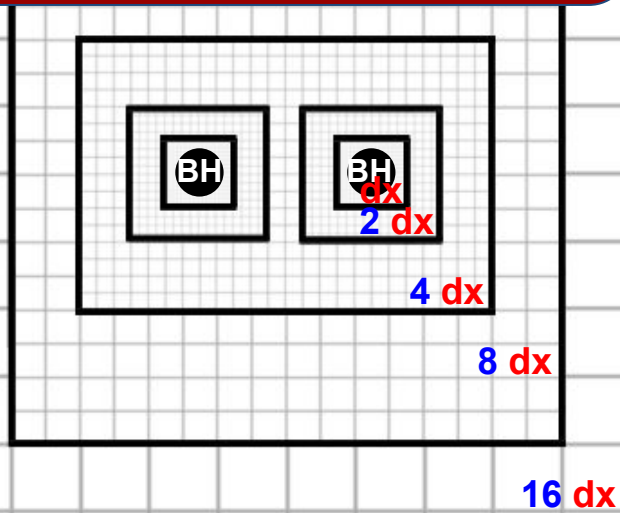


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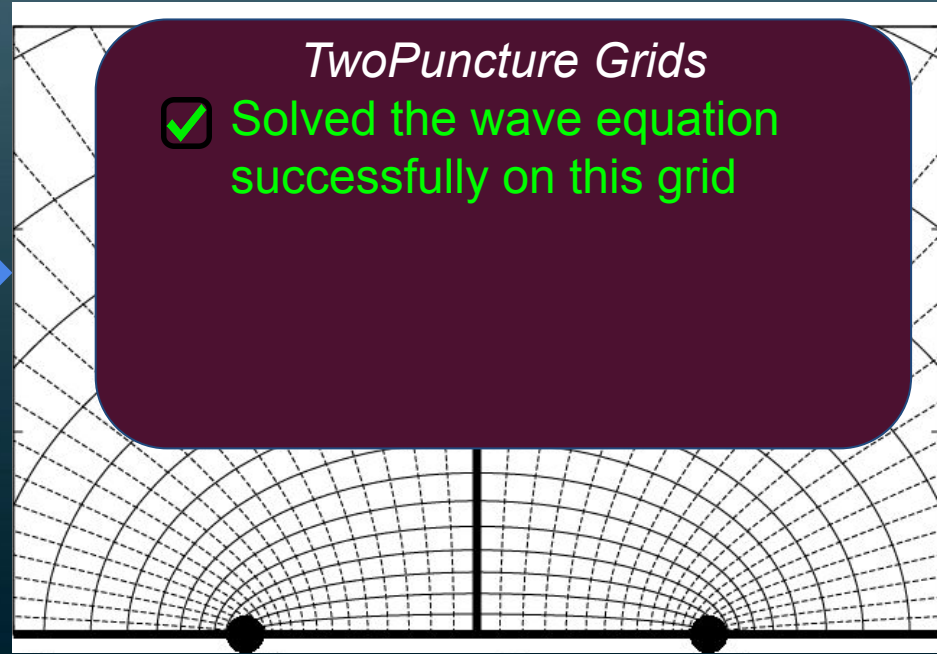
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TwoPuncture Grids

TwoPuncture Grids

- ✓ Solved the wave equation successfully on this grid

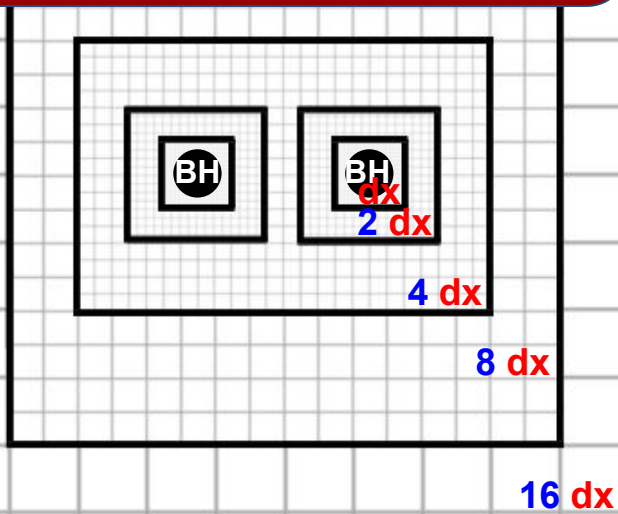


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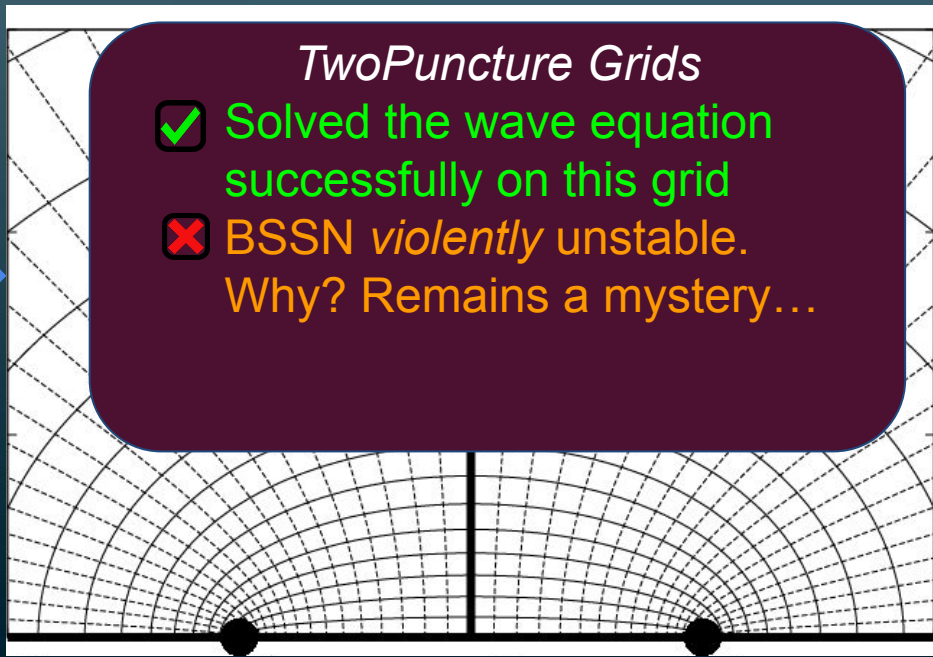
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- ✗ BSSN *violently* unstable. Why? Remains a mystery...

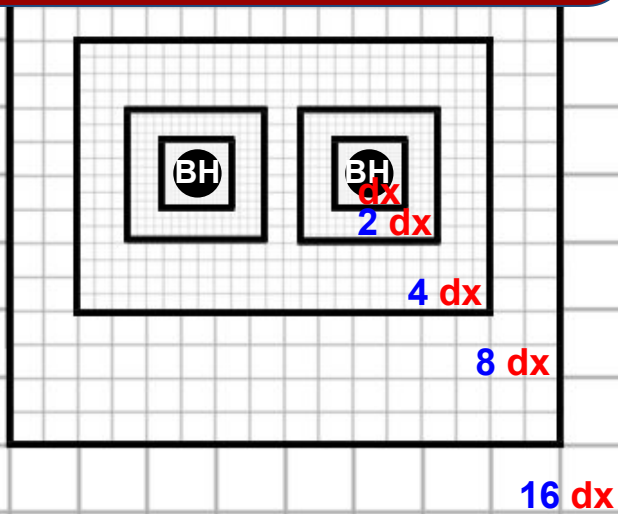


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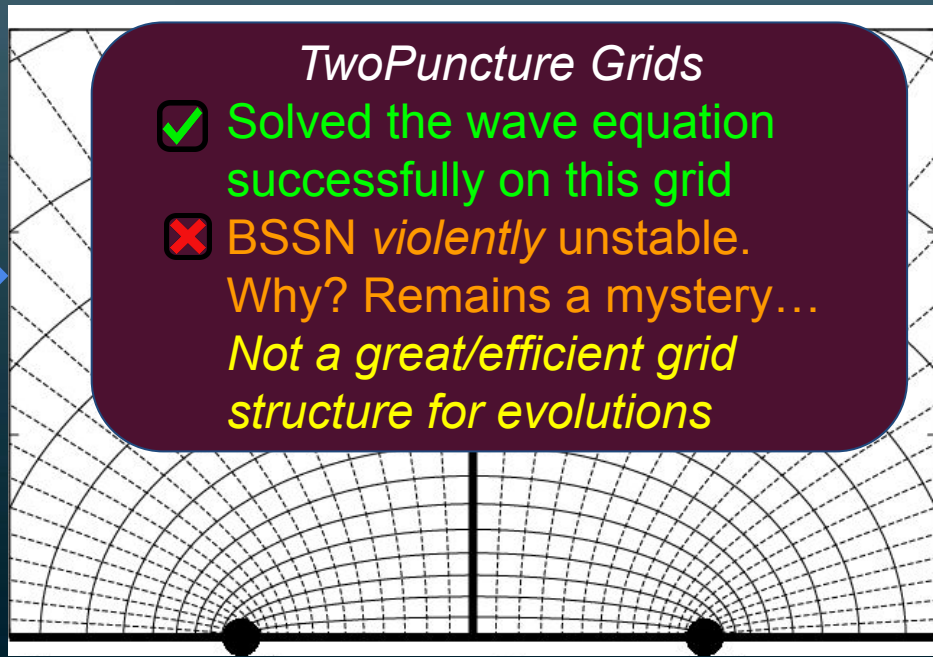
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Not a great/efficient grid structure for evolutions

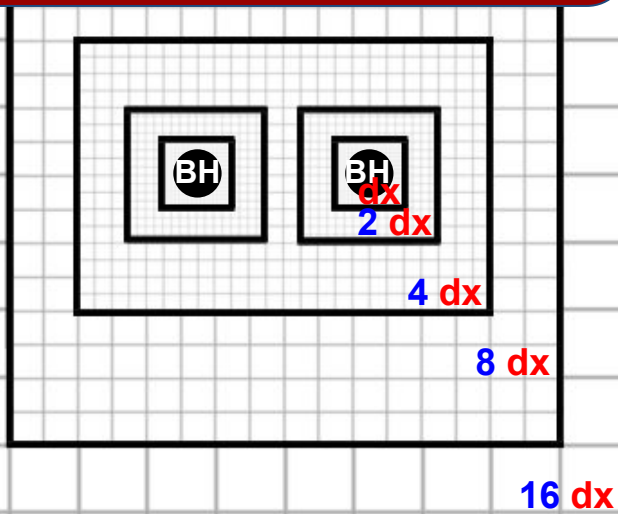


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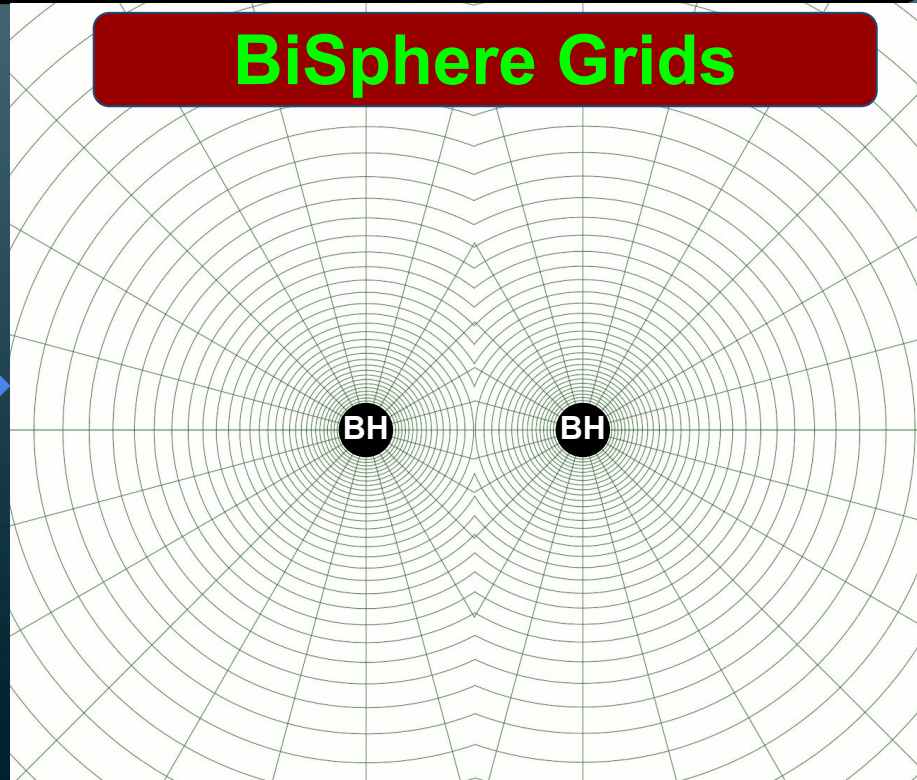
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BiSphere Grids

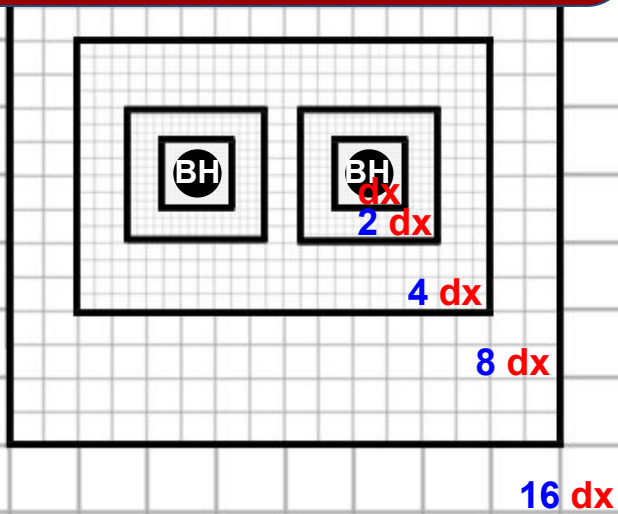


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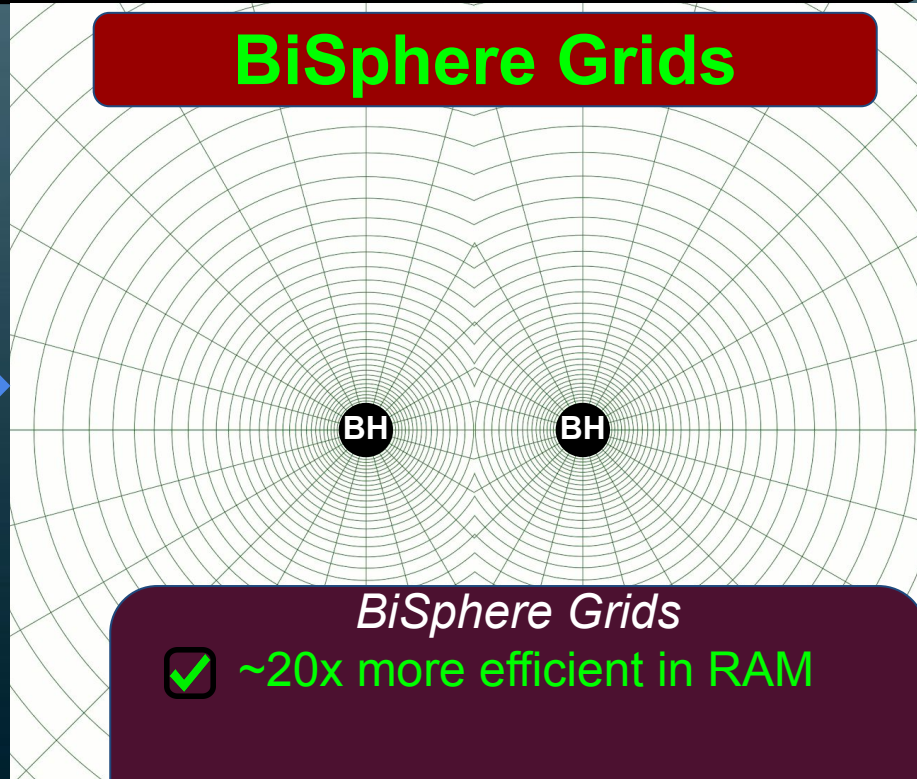
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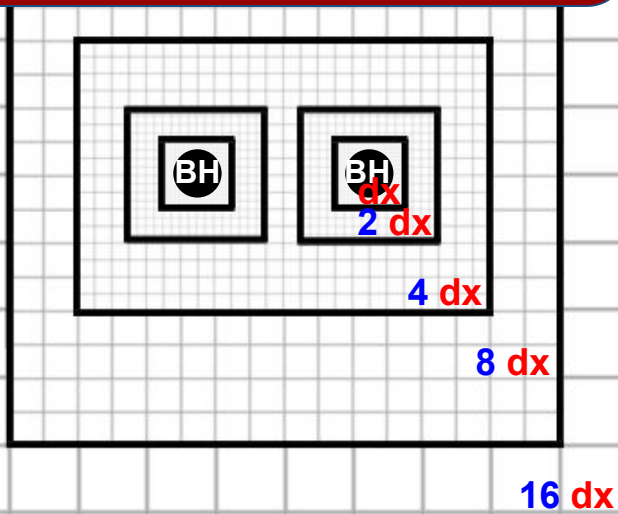
~20x more efficient in RAM

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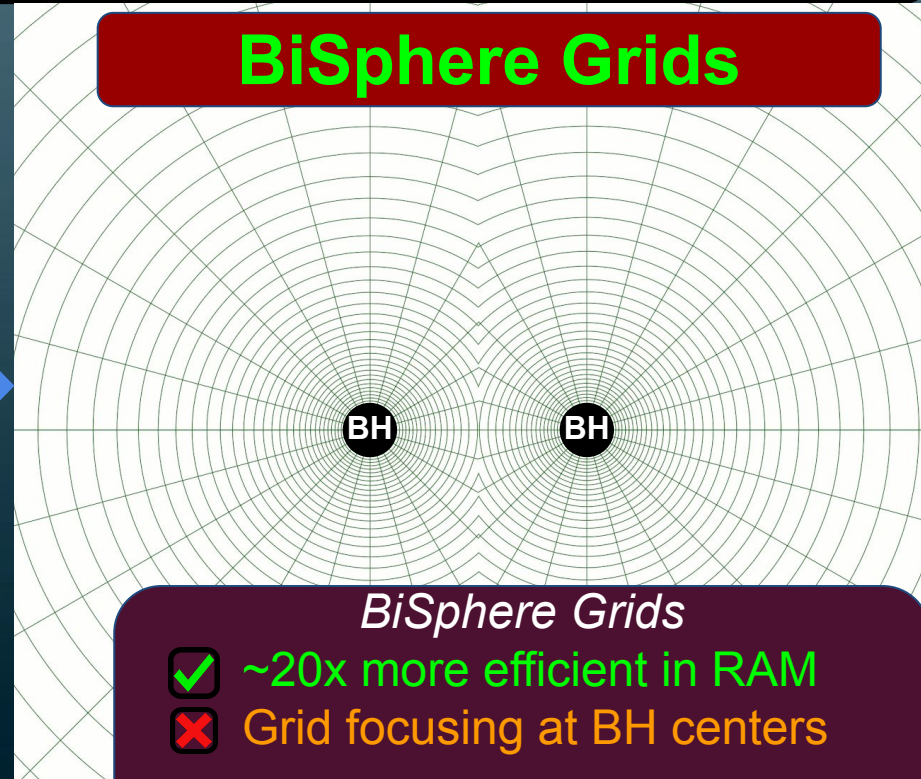
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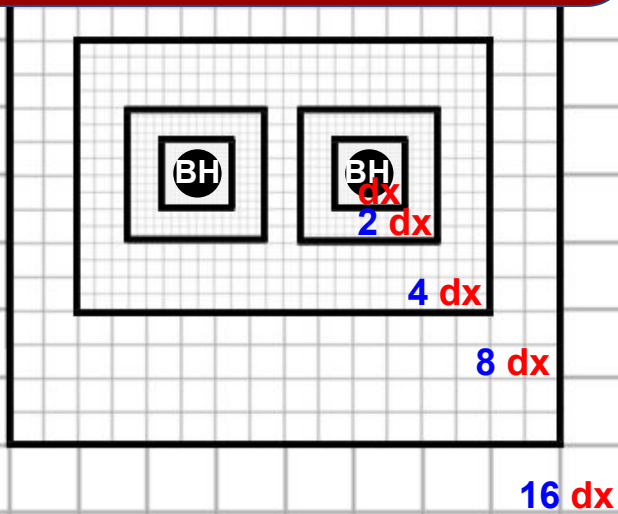
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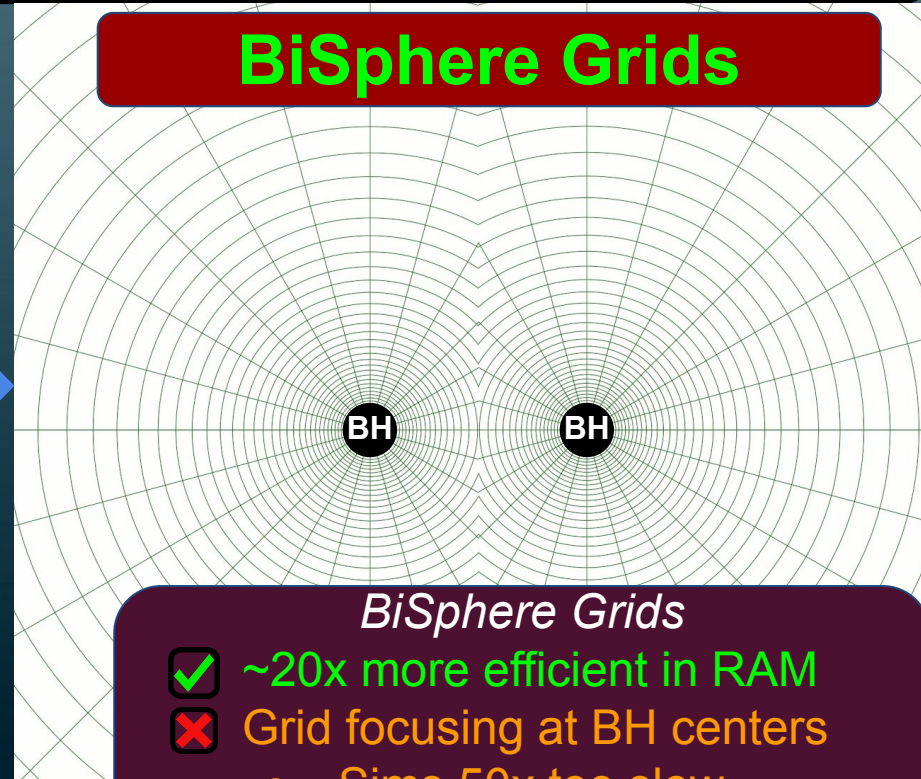
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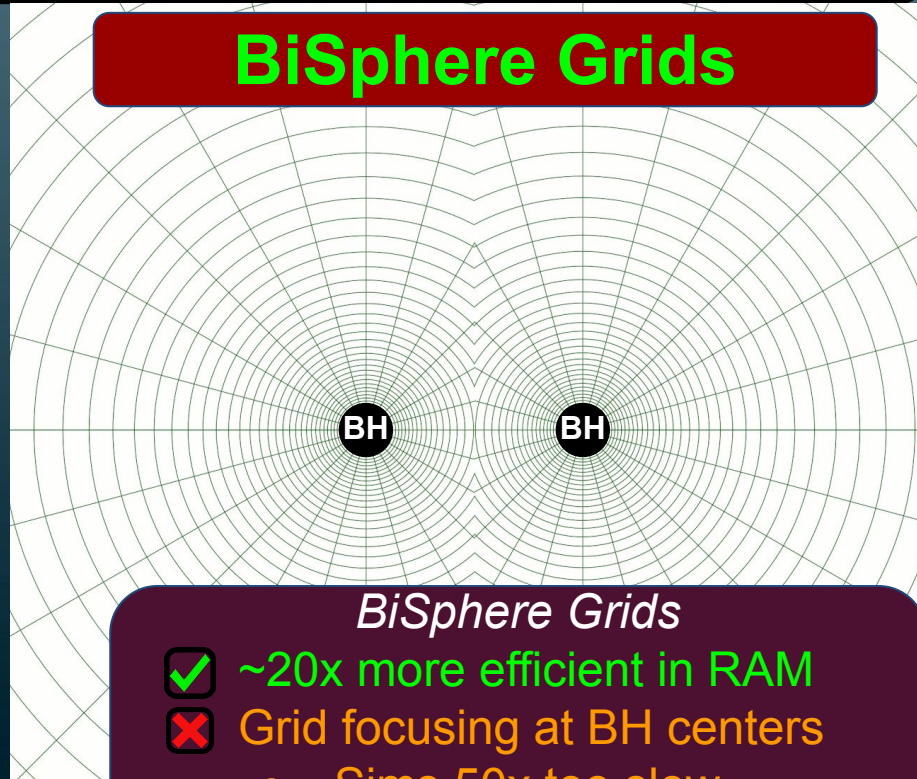
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Development of Super-Efficient Grid Structures

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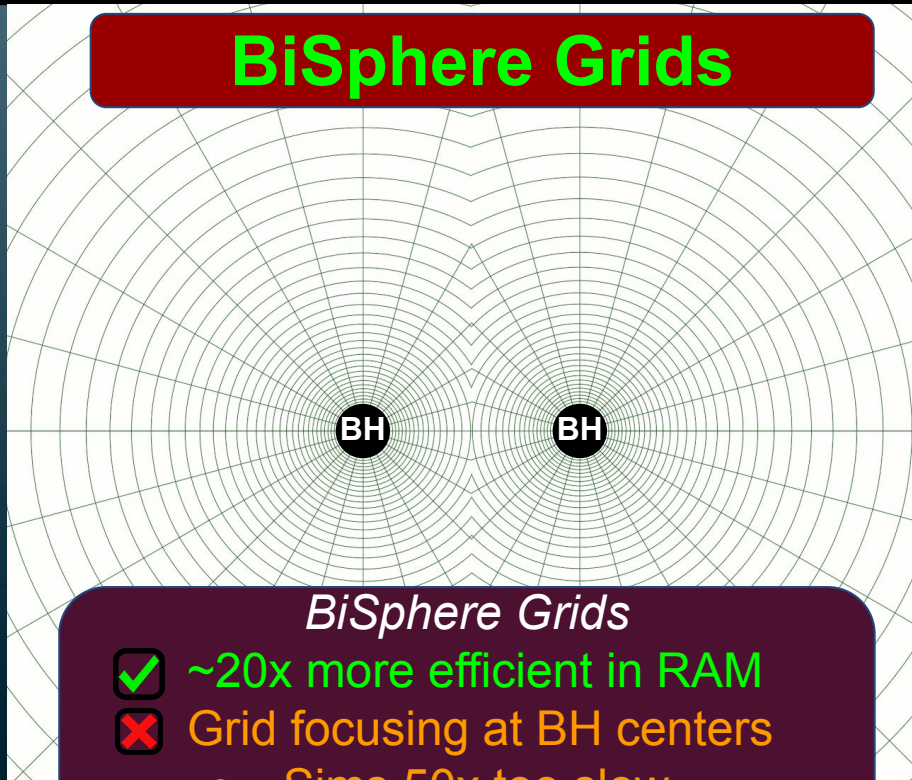
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Development of Super-Efficient Grid Structures

BiSphere Grids

BH

BH

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Hybrid Grids

schematic

BlackHoles@Home: BBHs on the Desktop

Development of Super-Efficient Grid Structures

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BH

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Hybrid Grids

●

●

Hybrid Grids

- ✓ ~100x larger dt!

BlackHoles@Home: BBHs on the Desktop

Development of Super-Efficient Grid Structures

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BH

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Hybrid Grids

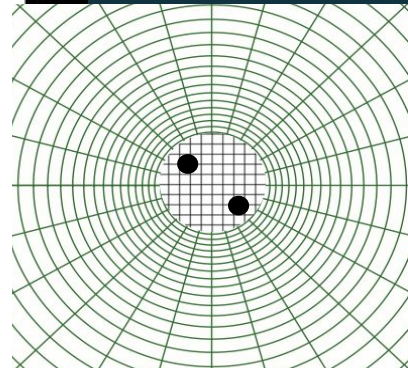
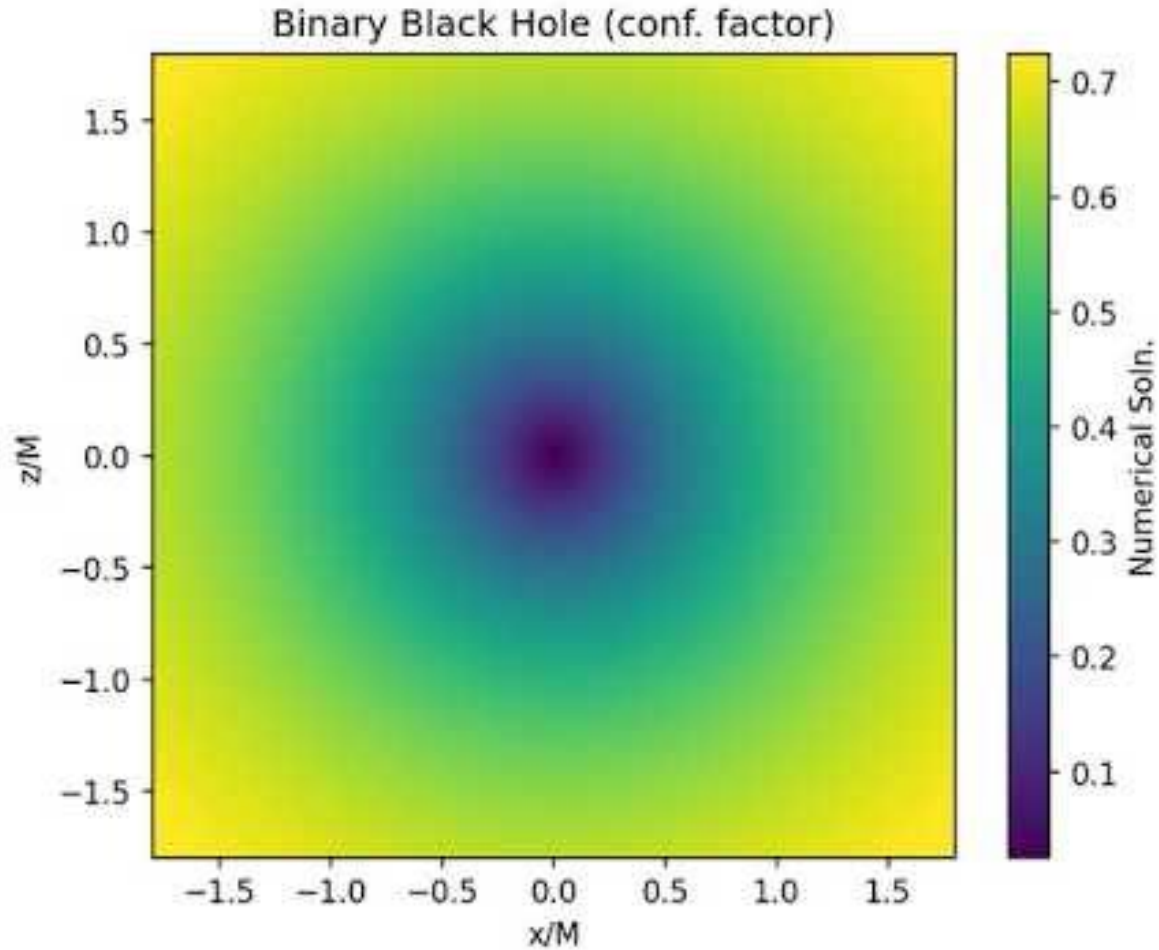
BH

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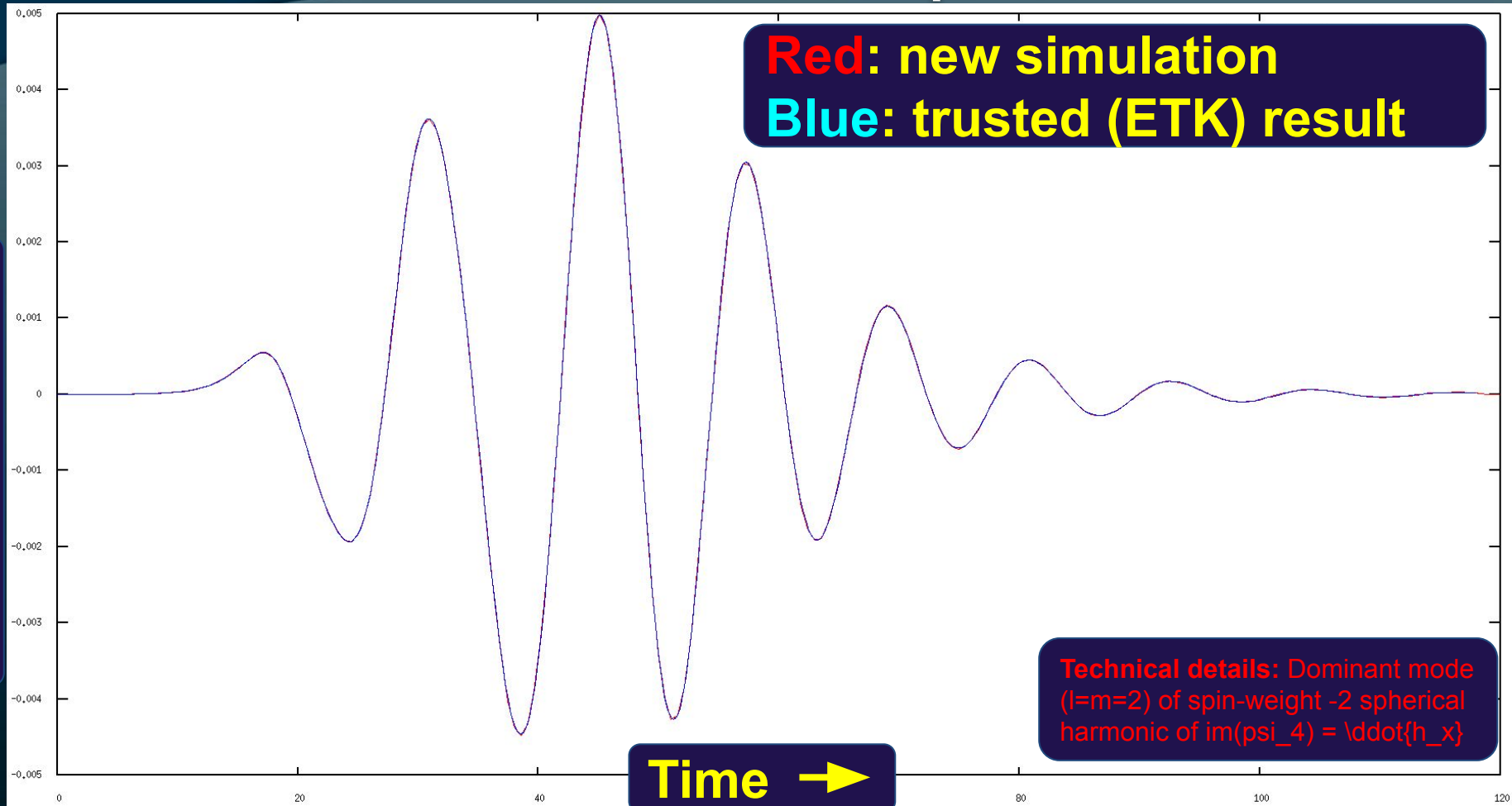
- ✓ ~100x larger dt!
- Memory usage?!
- Will it even work?!

Results from Hybrid Grids



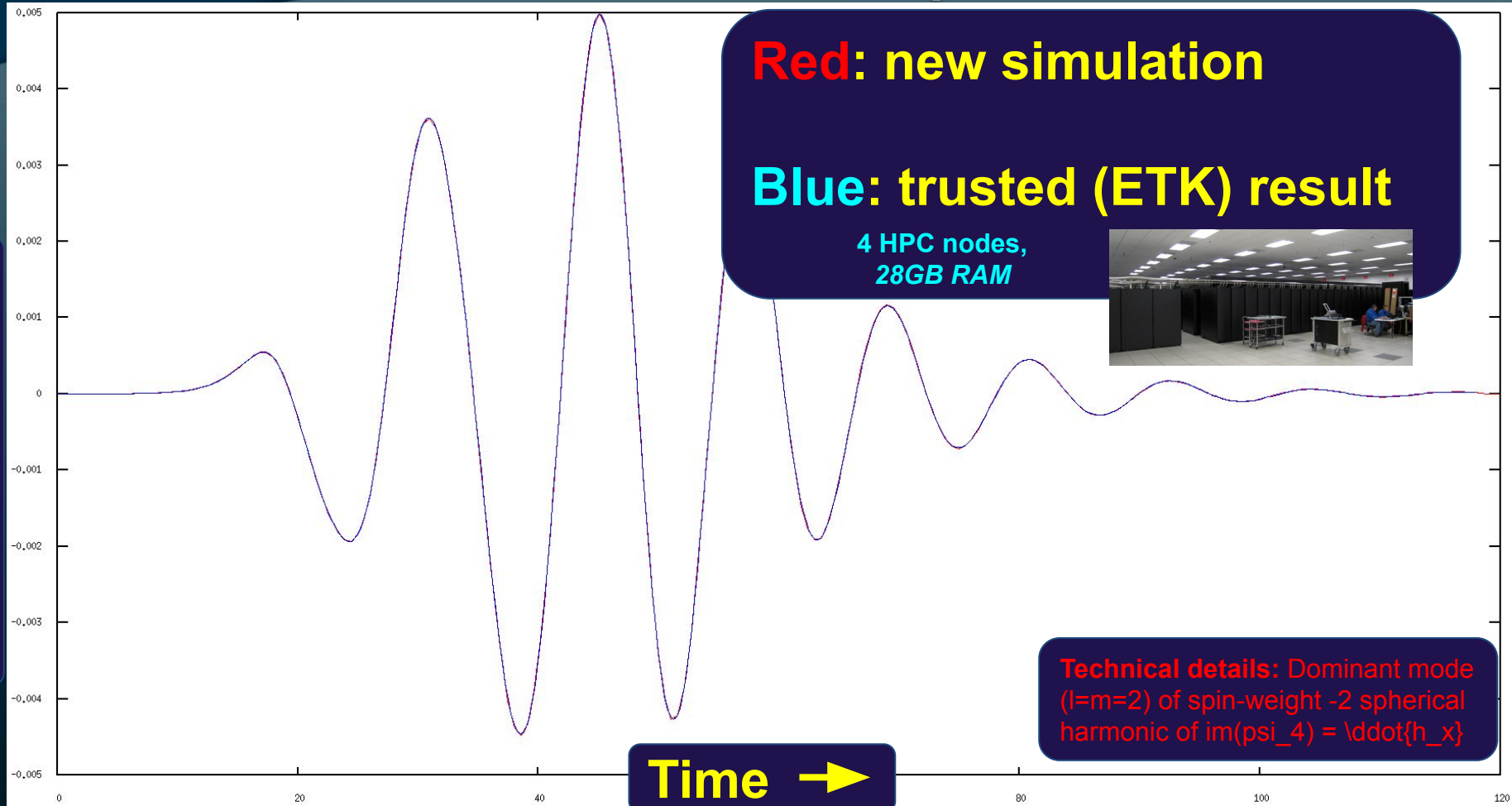
Gravitational Wave Comparison

Wave amplitude



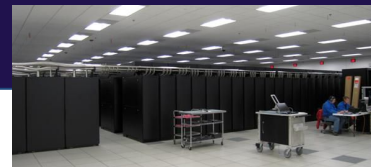
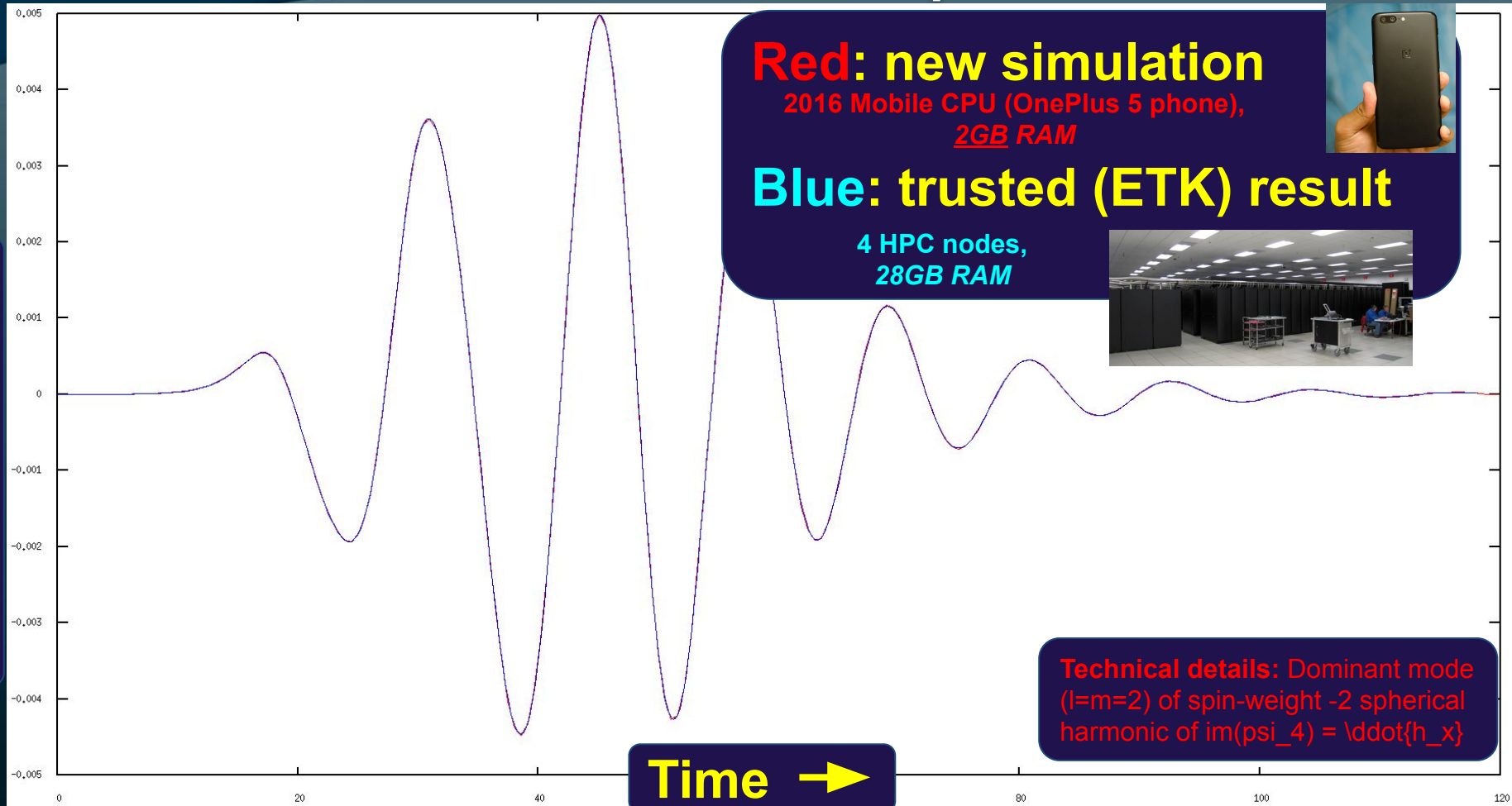
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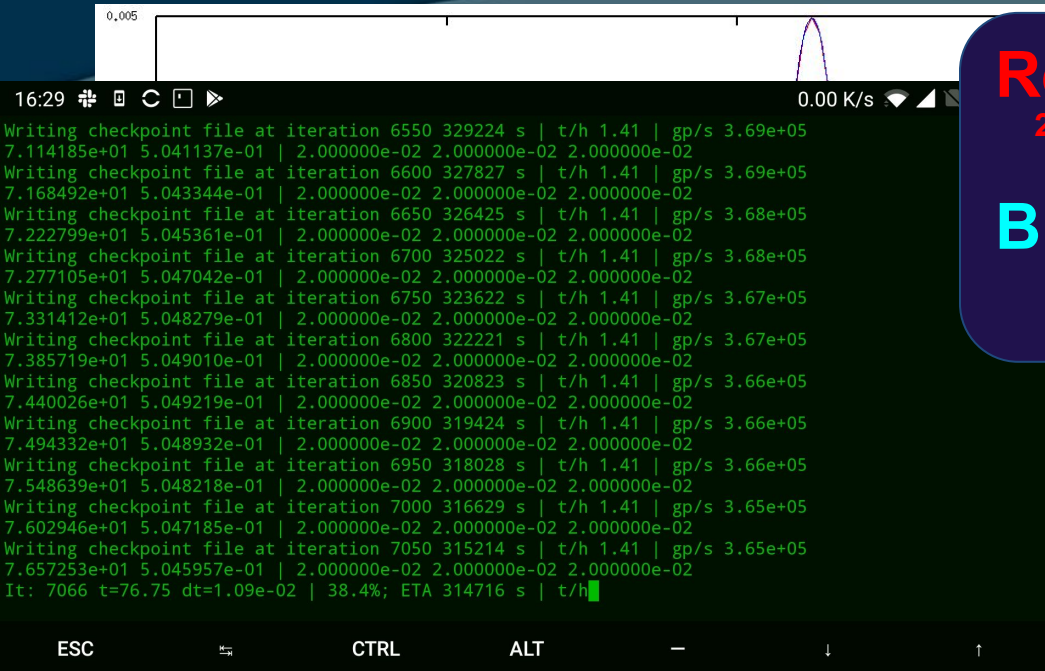


Gravitational Wave Comparison

Wave amplitude



Gravitational Wave Comparison



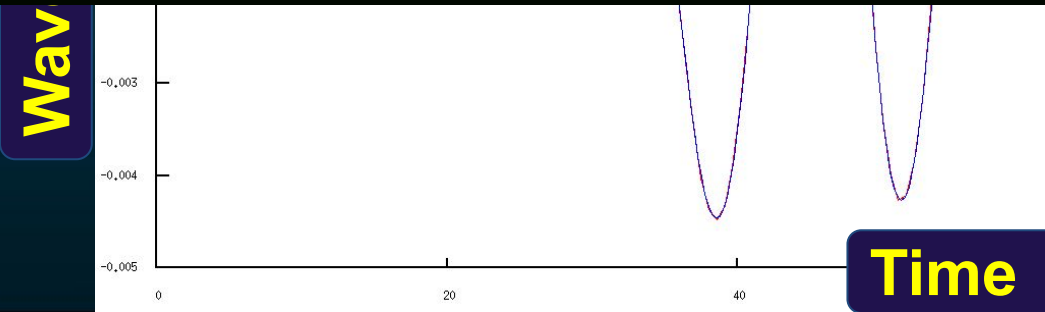
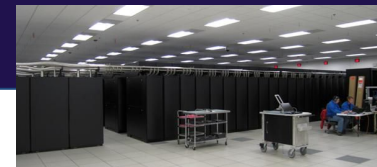
Red: new simulation

2016 Mobile CPU (OnePlus 5 phone),
2GB RAM



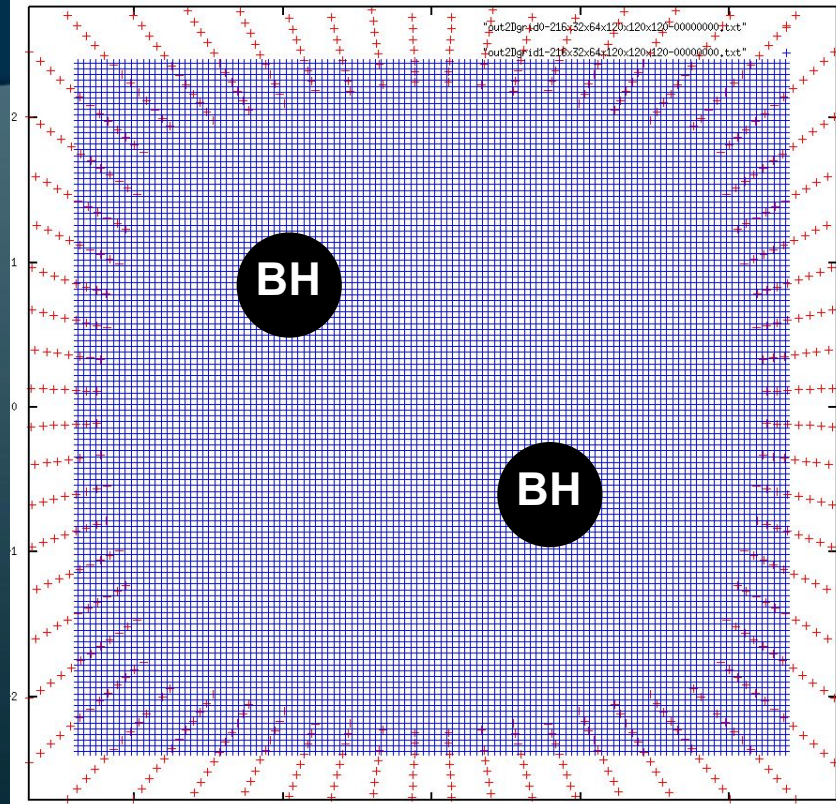
Blue: trusted (ETK) result

4 HPC nodes,
28GB RAM



Technical details: Dominant mode ($l=m=2$) of spin-weight -2 spherical harmonic of $\text{im}(\psi_4) = \ddot{h}_x$

Early 2020 Results



- Problem with this grid structure:
 - Only works well for two orbiting black holes **very close to merger**
 - Larger separations -- Cartesian grid too large -- too much memory!

- **What to do?!**

Latest:
7-grid Bispheres
 $q \sim 1$ config.

BH

BH

Latest:
7-grid Bispheres
 $q \sim 1$ config.

Benefits

1. No $r=0$ focusing on Spherical grids
2. Fully dynamical grids, comove & inspiral with BHs

BH

BH

**Latest:
7-grid Bispheres
 $q \sim 1$ config.**

**SinhCart
AMR**

BH

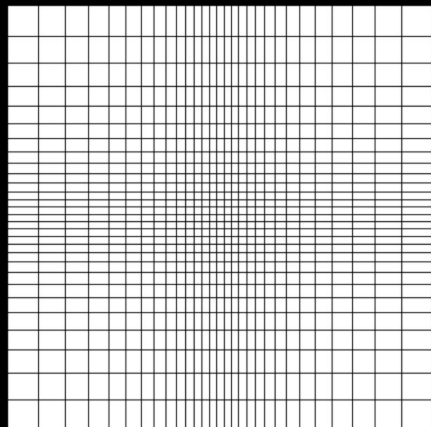
**Small
SinhSpherical**

**Small
SinhSpherical**

**SinhCart
AMR**

BH

**Large
SinhSpherical**

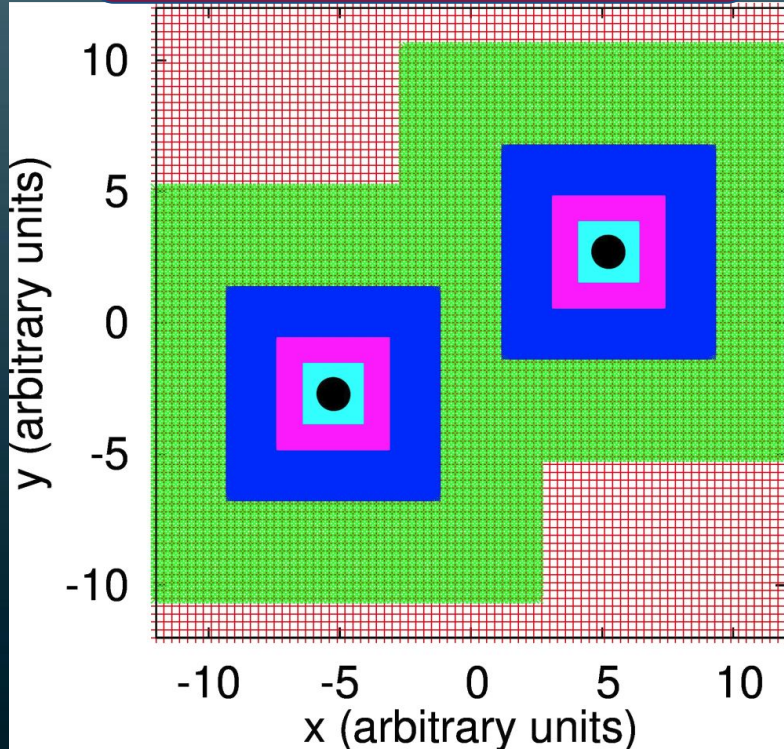


**SinhCartesian focuses
gridpoints along x,y,z axes.
Great for puncture BHs
AMR: $\sim 8x$ in 1 refinement**

Basic Idea:

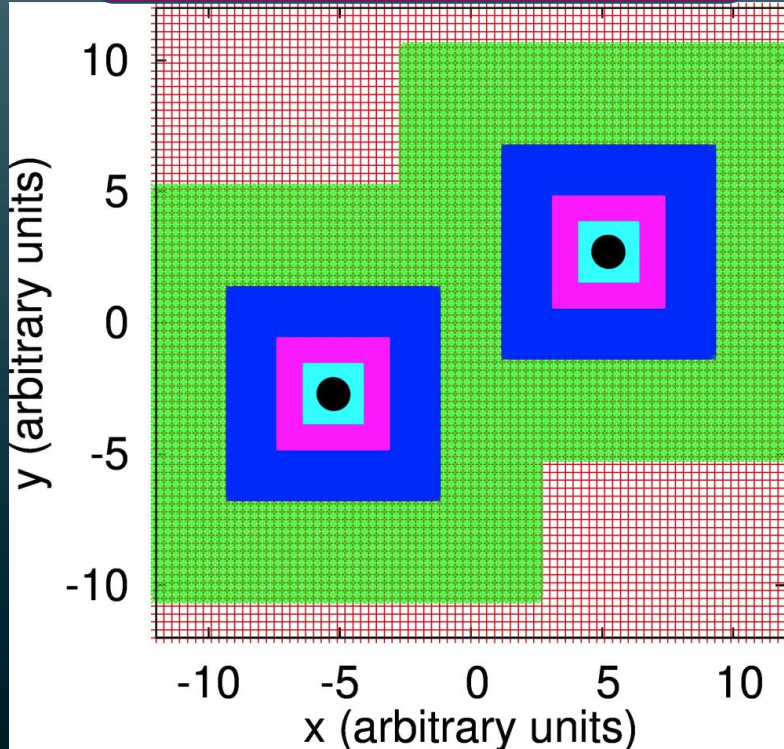
More efficient simulation grids

Most popular grid approach:
Needs ~100GB of RAM

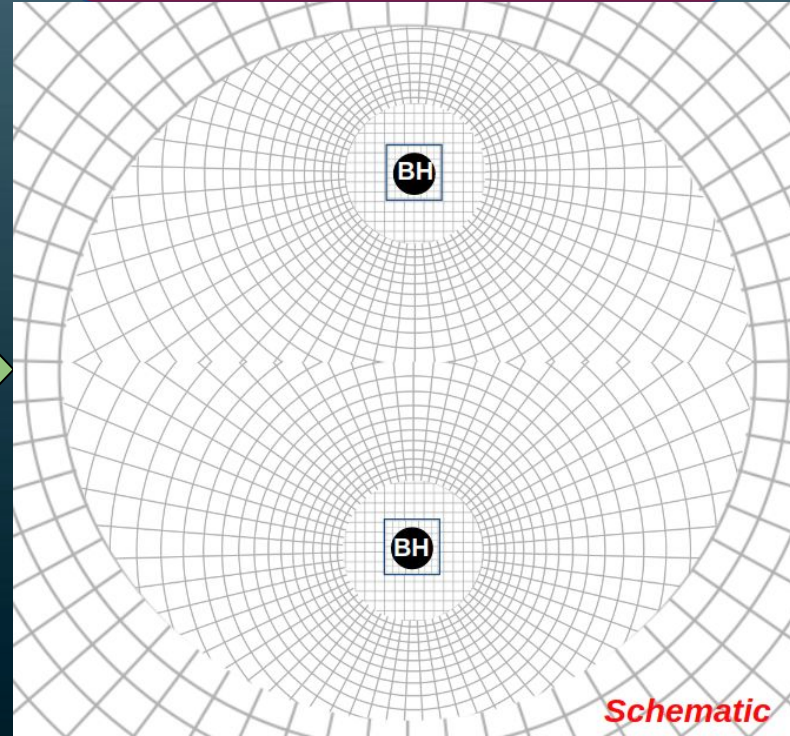


Basic Idea: More efficient simulation grids

Most popular grid approach:
Needs ~100GB of RAM



BlackHoles@Home grids:
Needs ~3GB of RAM



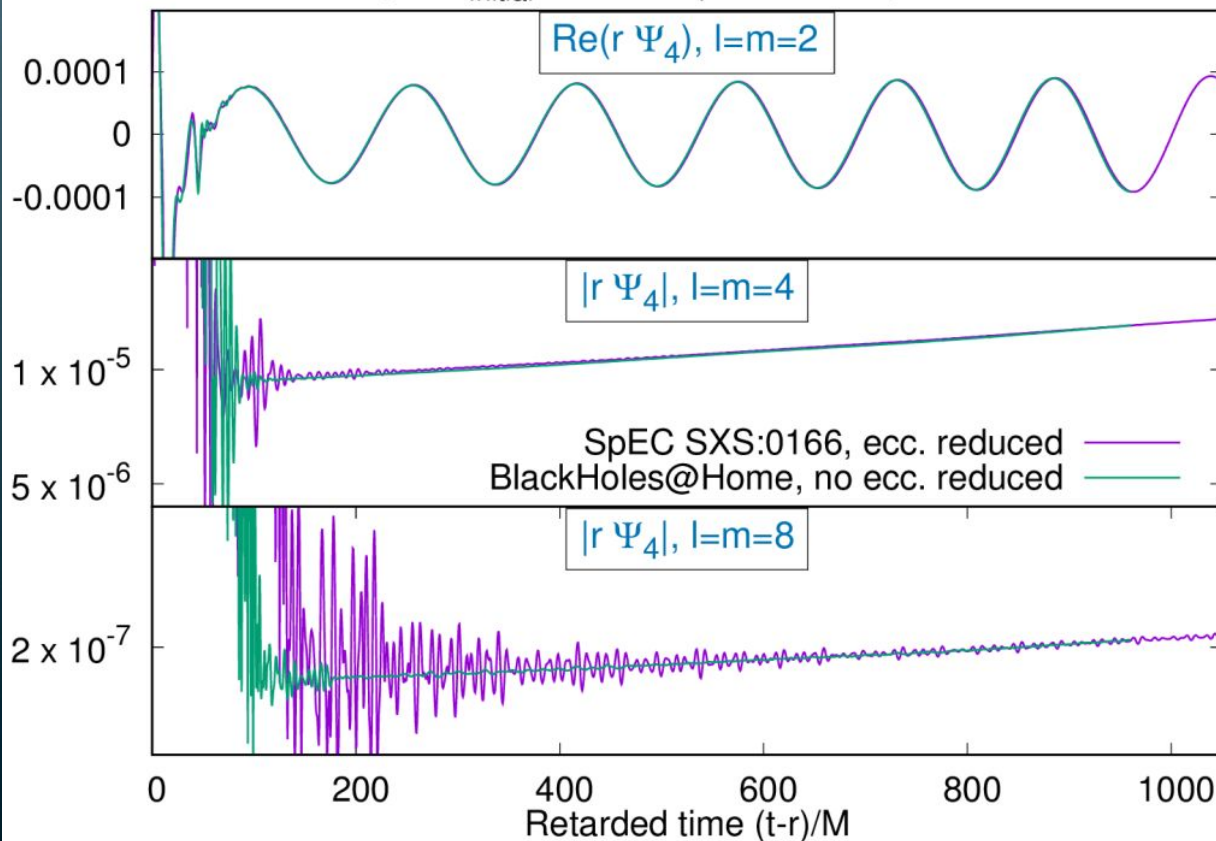


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Dev. Highlights
2020-2023

- 2020-2021: 7-grid BiSpheres grids; inspirals **look great vs SpEC**

$q=6$, $d_{\text{initial}}=13M$: $r \Psi_4$ Mode Comparison

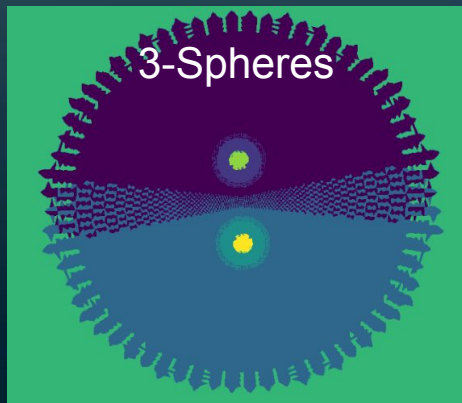




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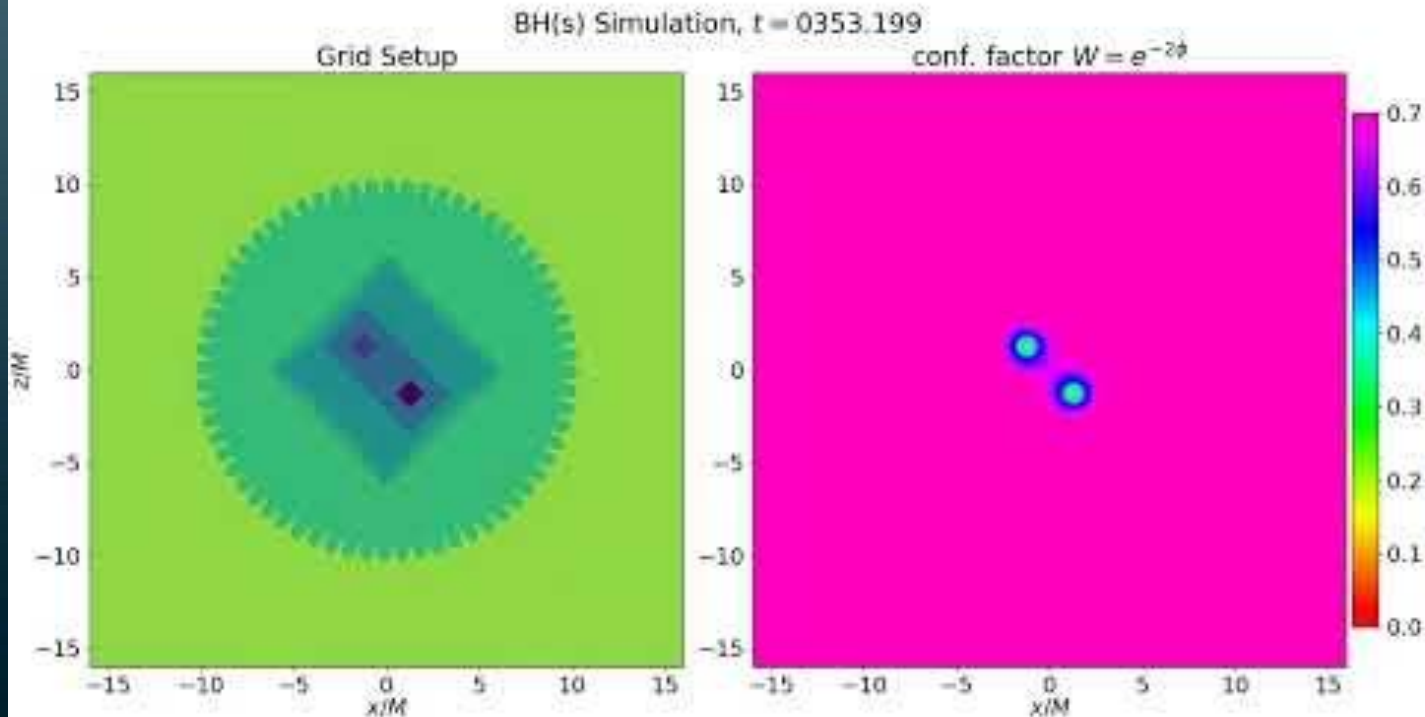
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- 2020-2021: 7-grid BiSpheres grids; inspirals **look great vs SpEC**
- 2022: Rewrote multipatch to handle *mergers*



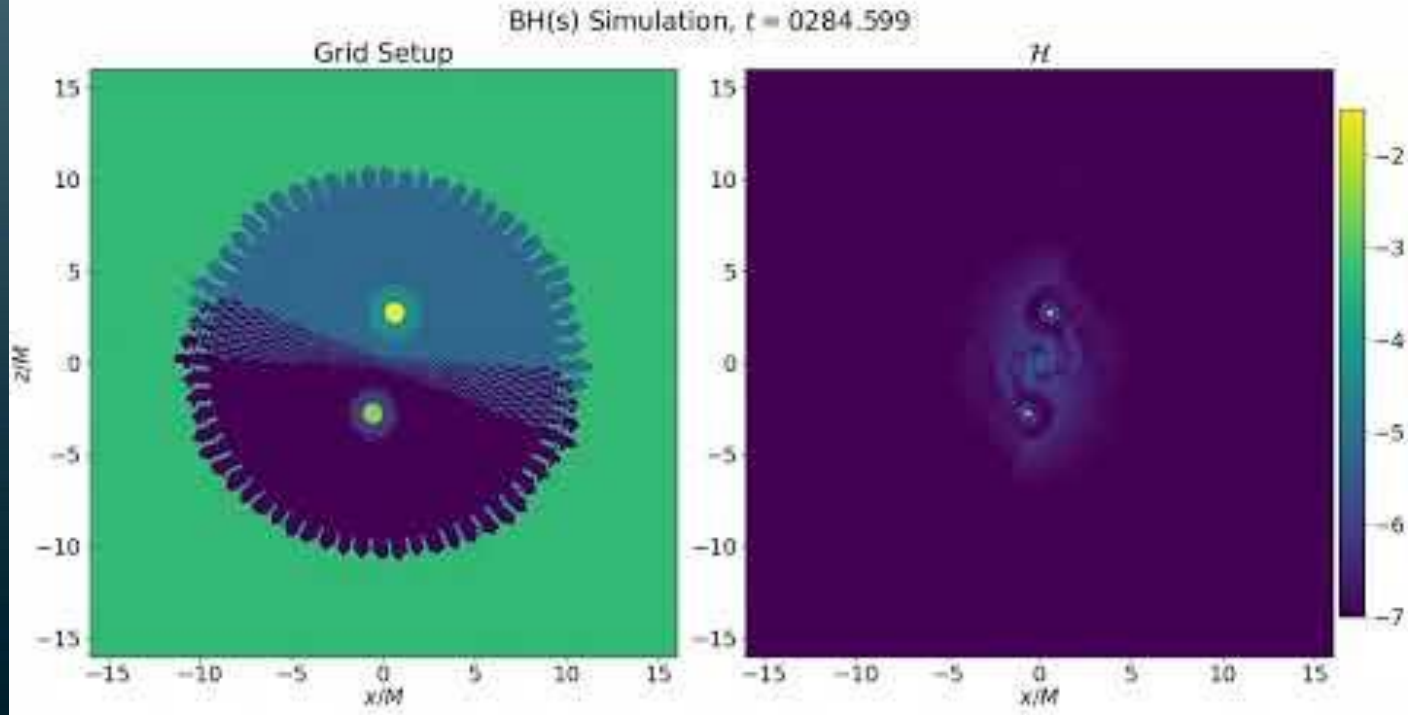
New grid structures example

$q=1$ nonspinning, initial sep= $8M$ BBH



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$q=1$ nonspinning, initial sep= $8M$ BBH

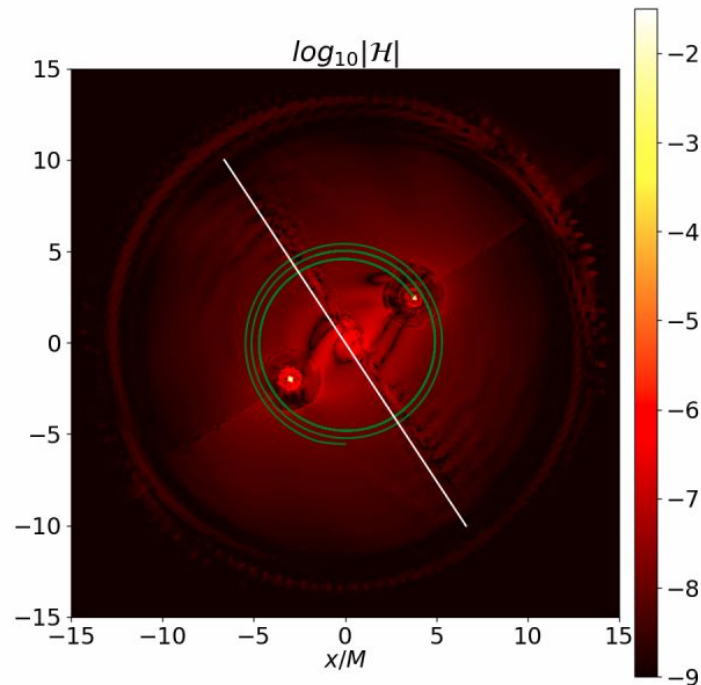
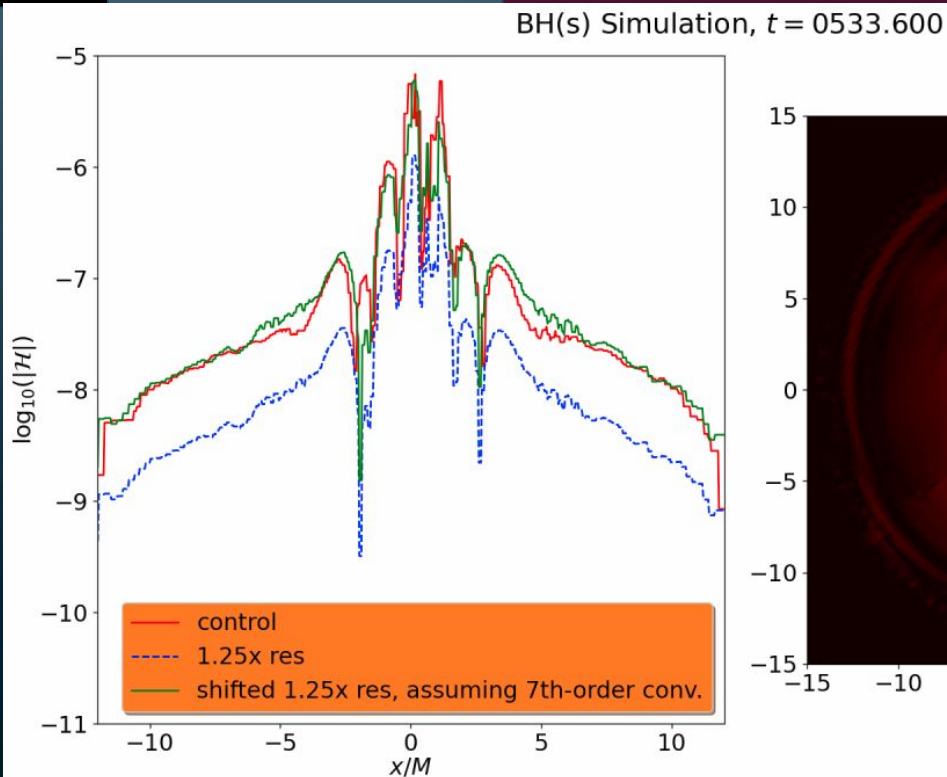




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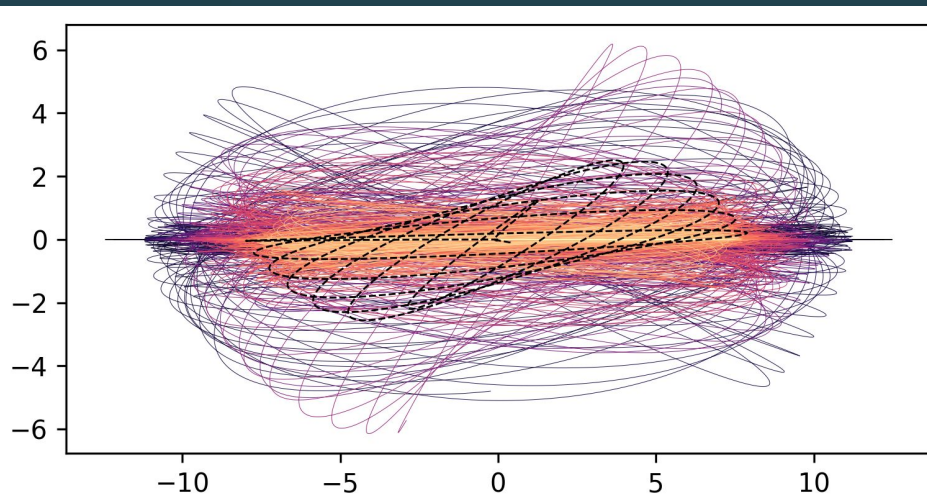
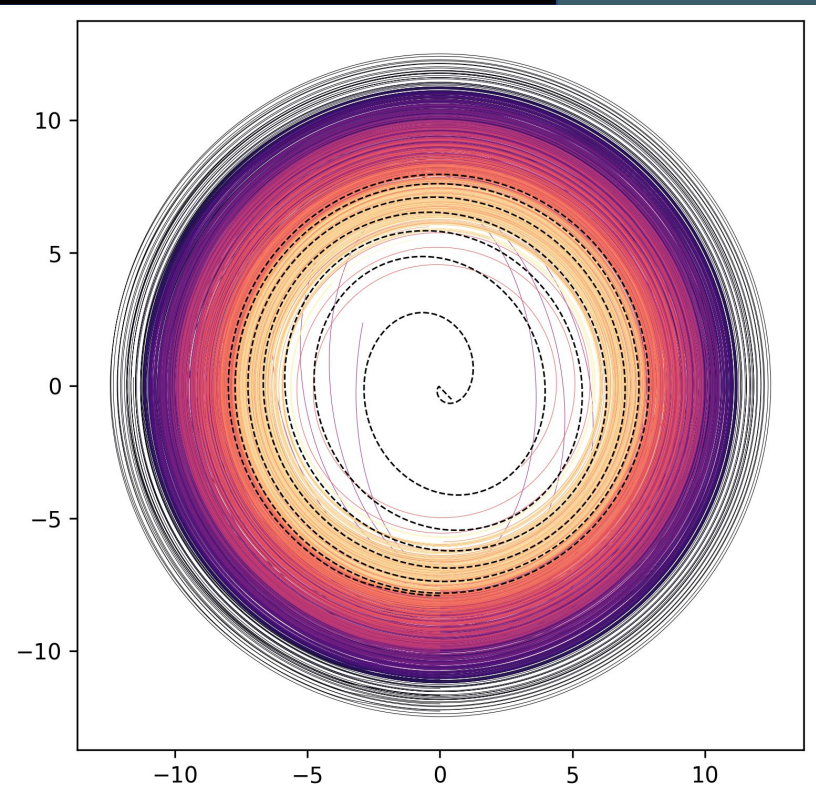




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- 2020-2021: 7-grid BiSpheres grids; inspirals **look great vs SpEC**
- 2022: Rewrote multipatch to handle **mergers**; 7th order convergence!
- 2023: Performed **hundreds** of BBH simulations, found instability in ~20%

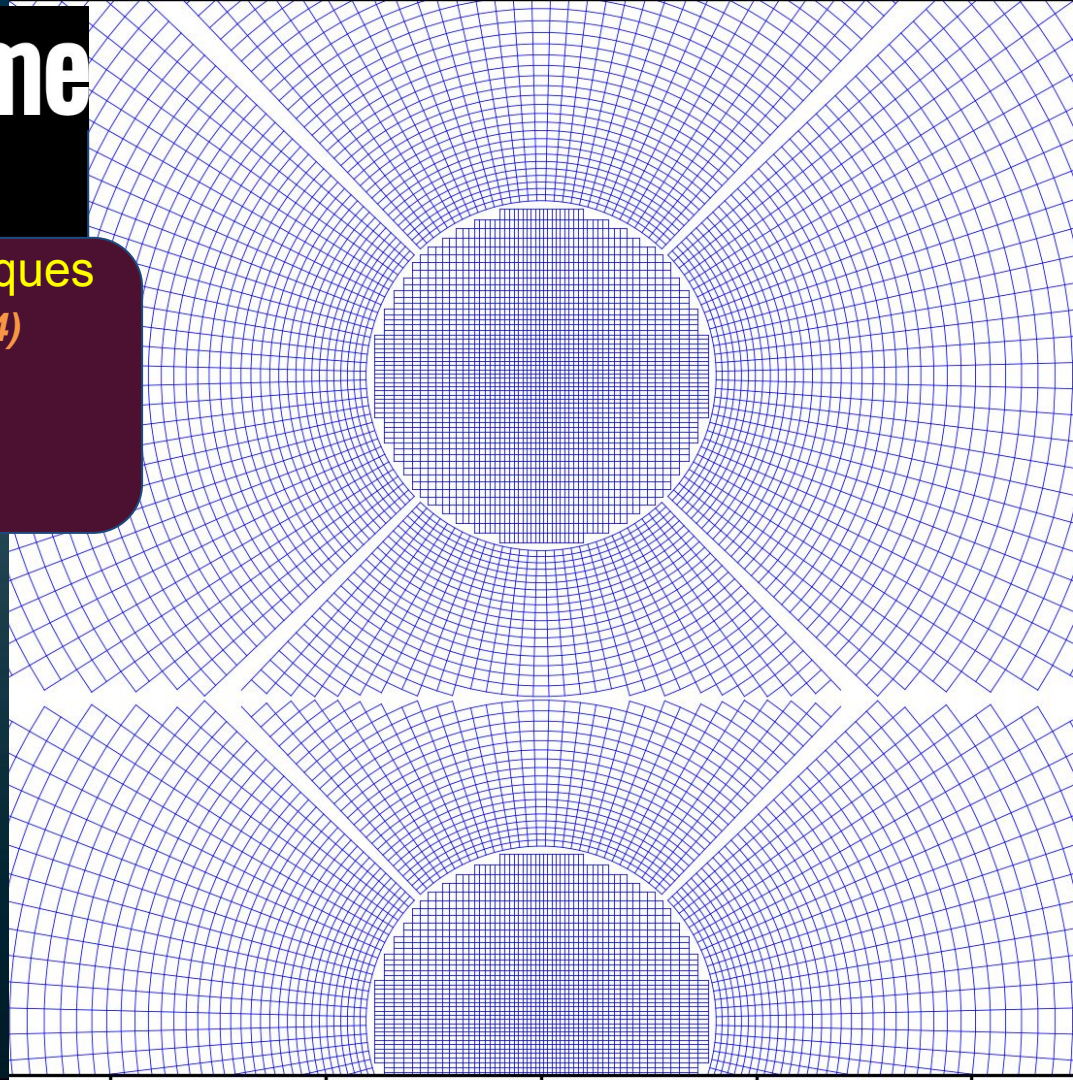




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Progress over past year

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Etienne arXiv:2404.01137 (2024)
2. Open-sourcing BH@H in NRPy 2
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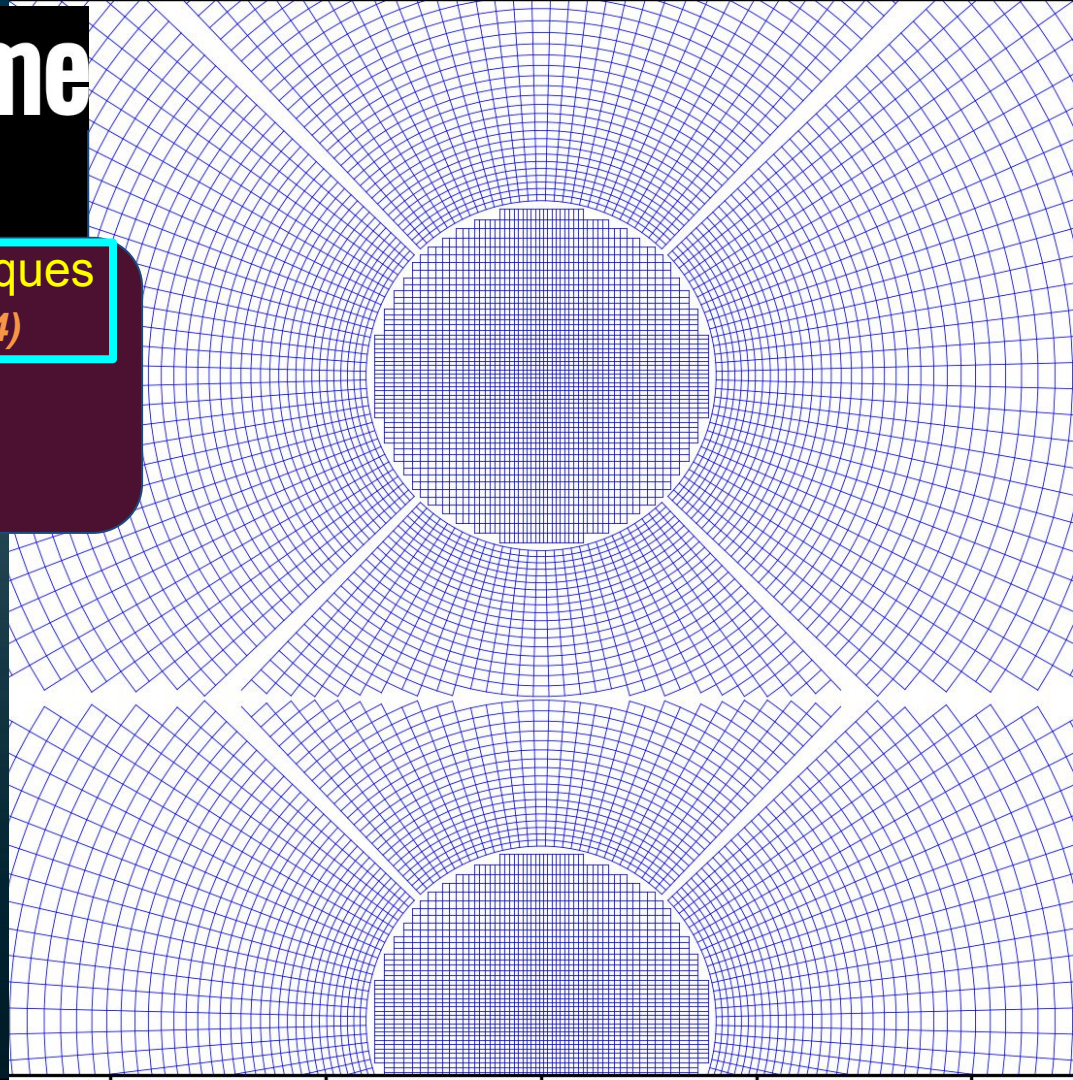




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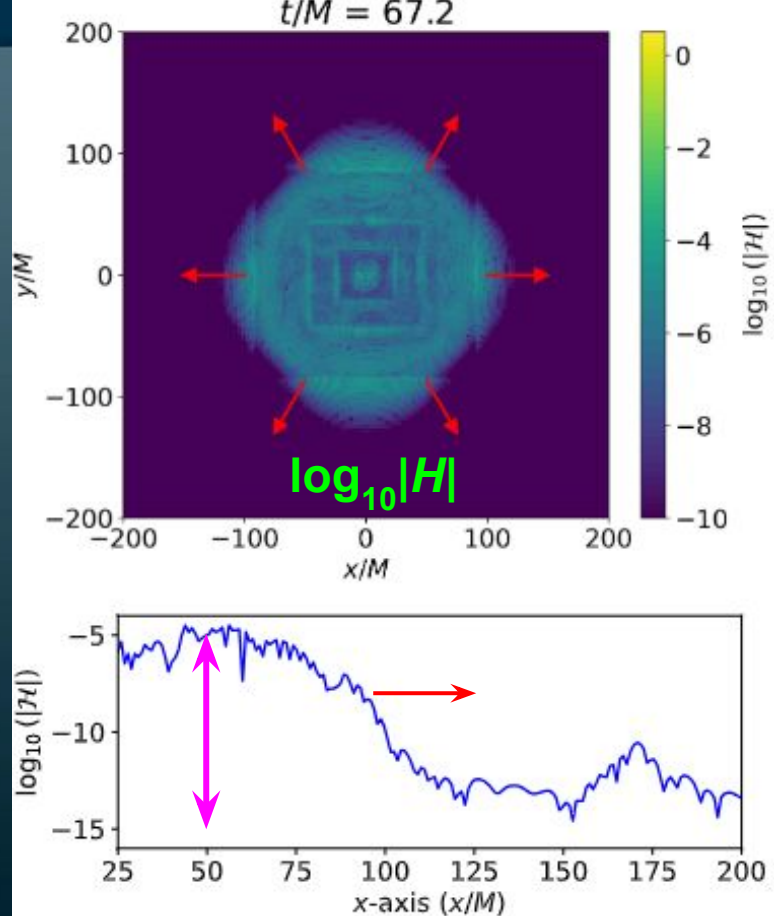


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Improved Moving-Puncture Techniques

Etienne *arXiv:2404.01137 (2024)*

- Improved moving puncture techniques
 - Developed in BH@H
 - Minimal cost, infrastructure agnostic
 - Demonstrated efficacy in Einstein Toolkit (Carpet AMR) with BBH
 - Focus: curtail development & impact of sharp lapse feature
→ **$\sim 10^{10}$ amplification of H**



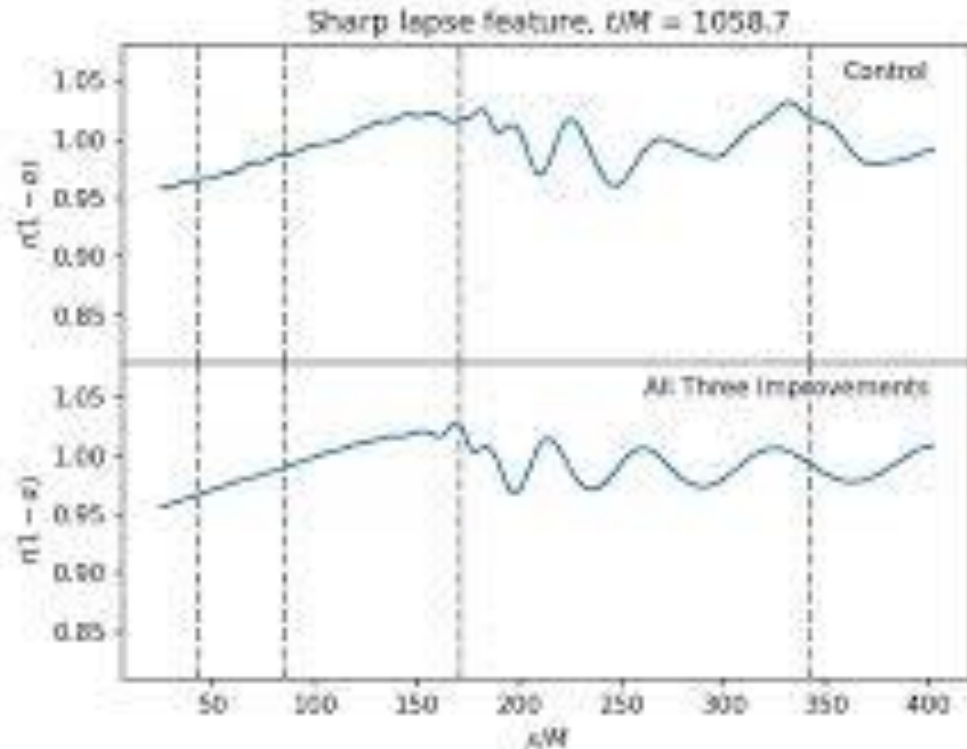
AMR BBH simulation

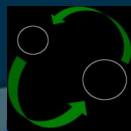
$$(1 - \alpha) \propto r^{-1}, \text{ so } r(1 - \alpha) \sim \text{const.}$$

Sharp lapse feature
propagates left-to-right
(strong-to-weak field)

Vertical-dashed lines:
AMR grid boundaries

Lapse (gauge) errors
contaminate physical
quantities



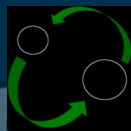


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Improved Moving-Puncture Techniques

Etienne arXiv:2404.01137 (2024)

- **Impact on Cartesian AMR BBH sims**
 - Strong field:
 - H violations $\sim 100\times$ smaller
 - M violations $\sim 1,000\times$ smaller

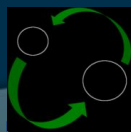


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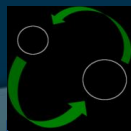


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 - Noise-obscured subdominant GW modes? **Some now visible!**

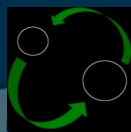


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 - → **factors improve with resolution**



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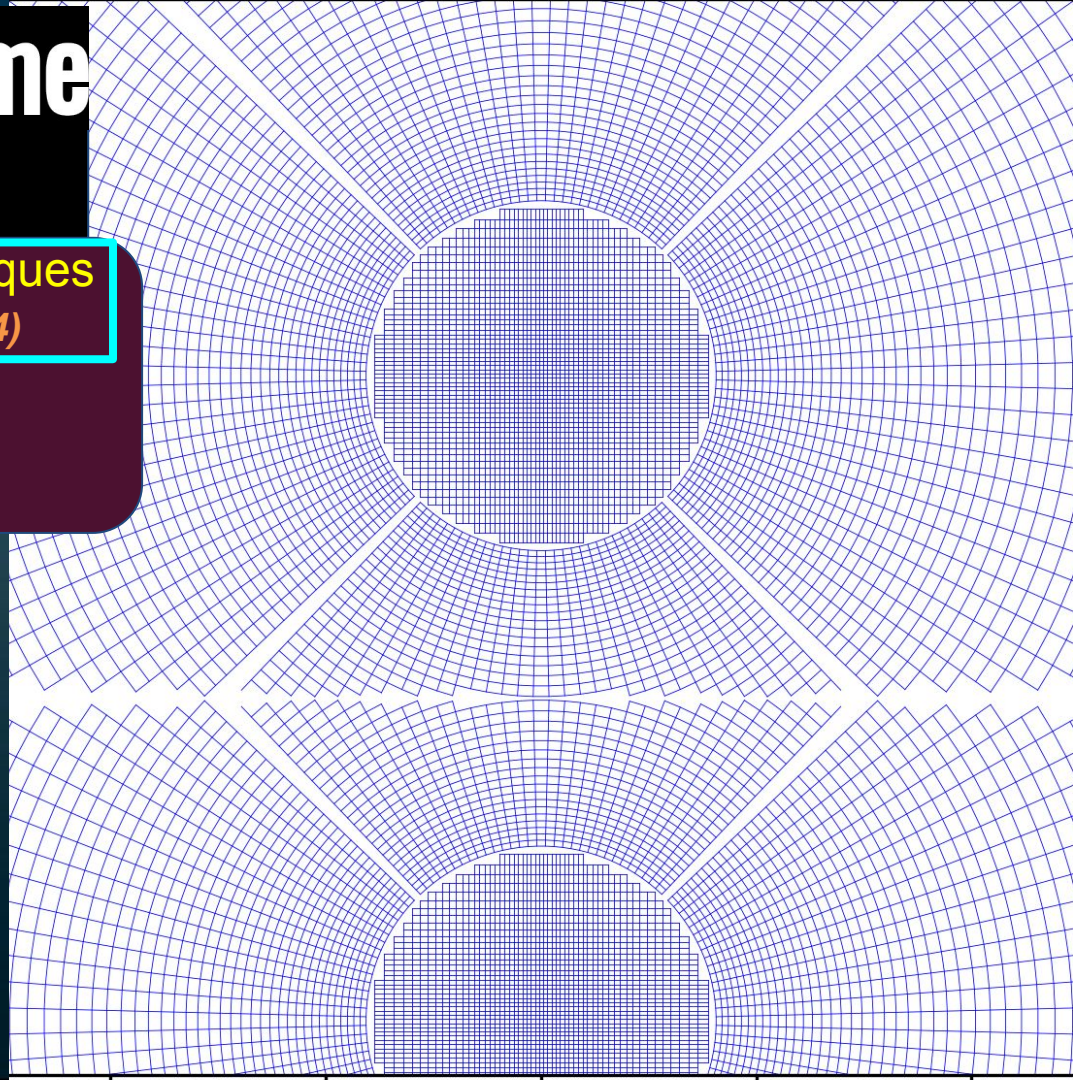
★ **Improved ET BSSN
code open sourced** ★



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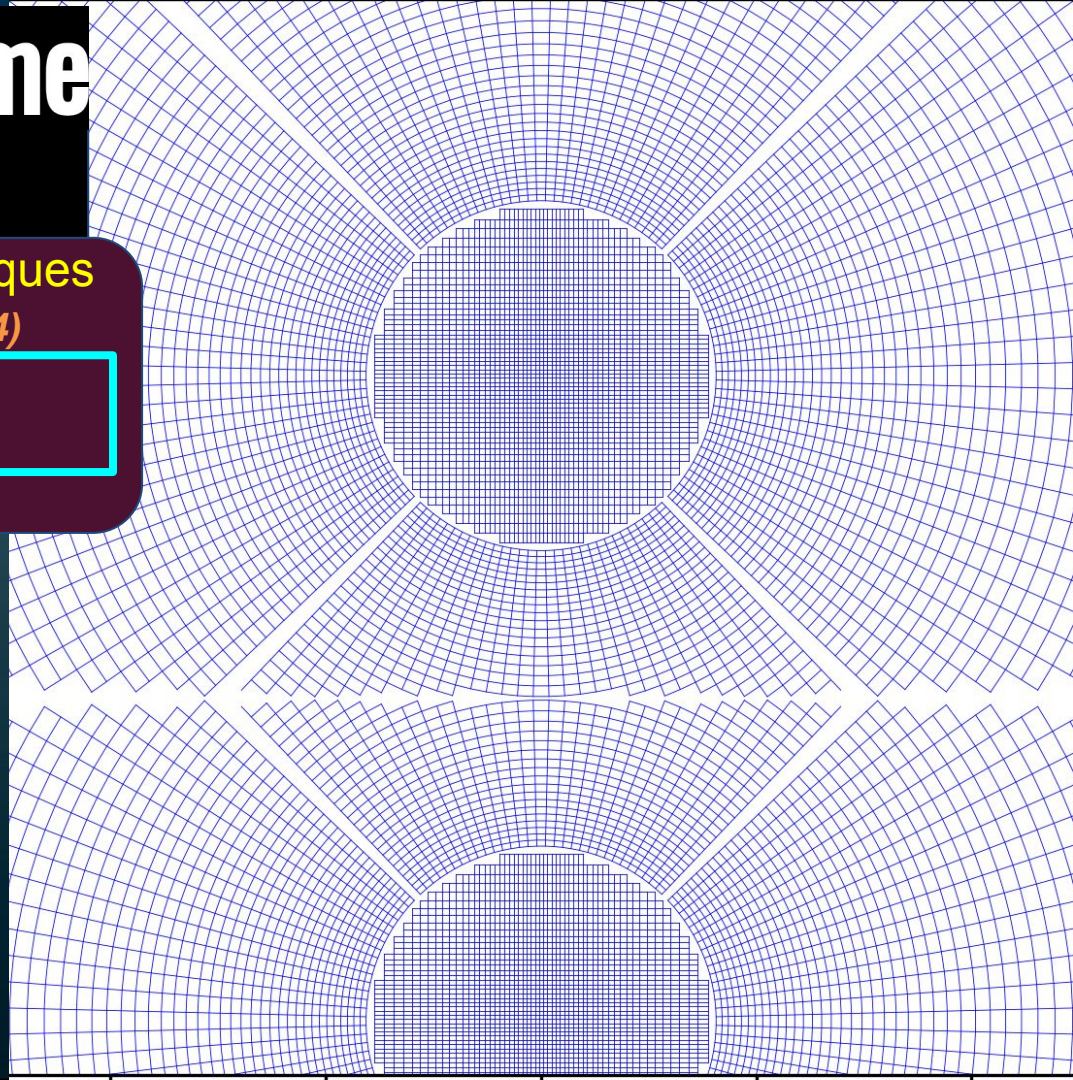




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Open-Source Progress

NRPy 2: github.com/nrpy/nrpy

BlackHoles@Home: A NRPy project

- NRPy = Python-based codegen for NR
- BH@H = NRPy-generated NR code



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NRPy 2: 2023 NRPy rewrite!

- Primary goals:
 - open source almost all of BH@H
 - modernize, address community feedback
 - `pip install nrpy`



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★ In development: “General Multipatch” ★

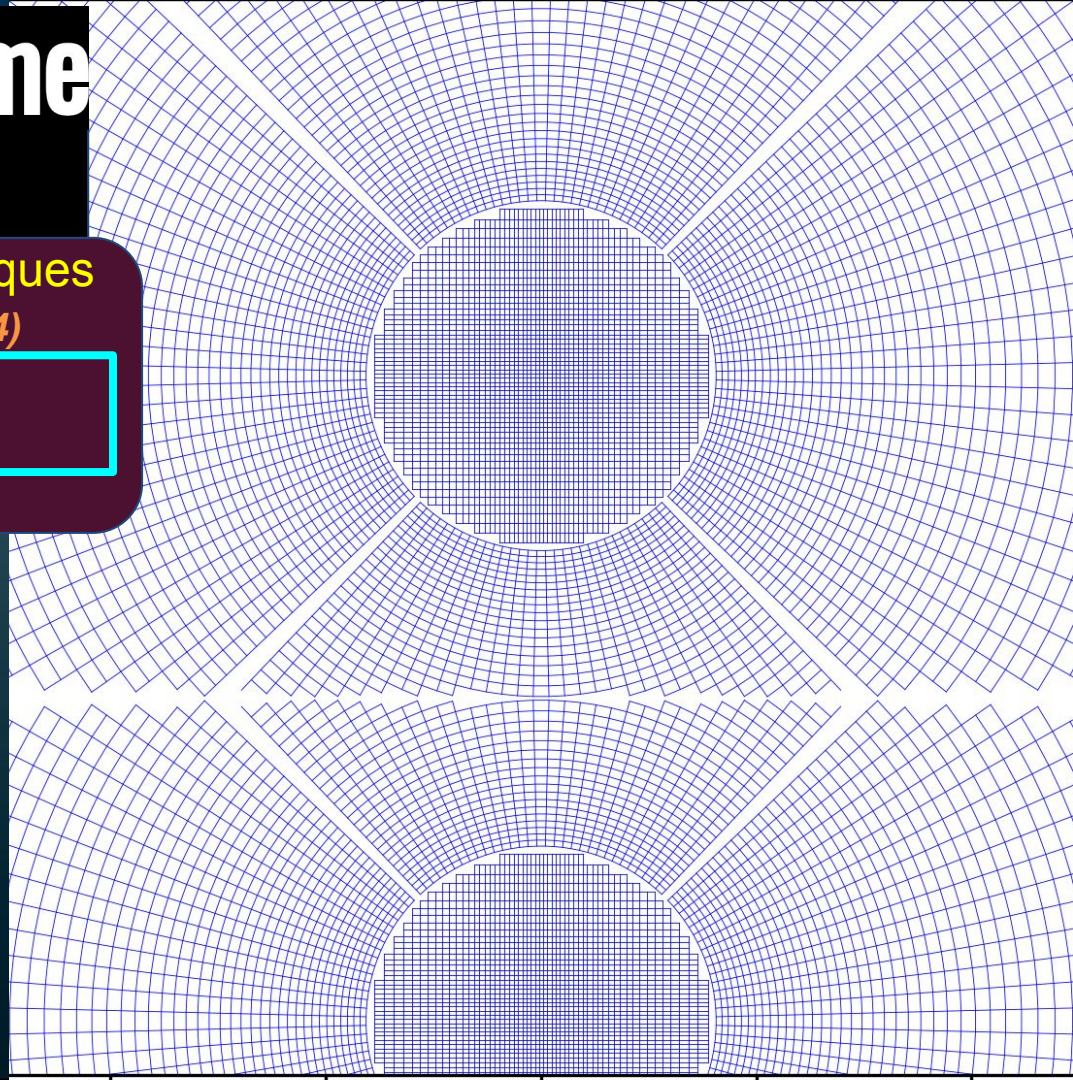




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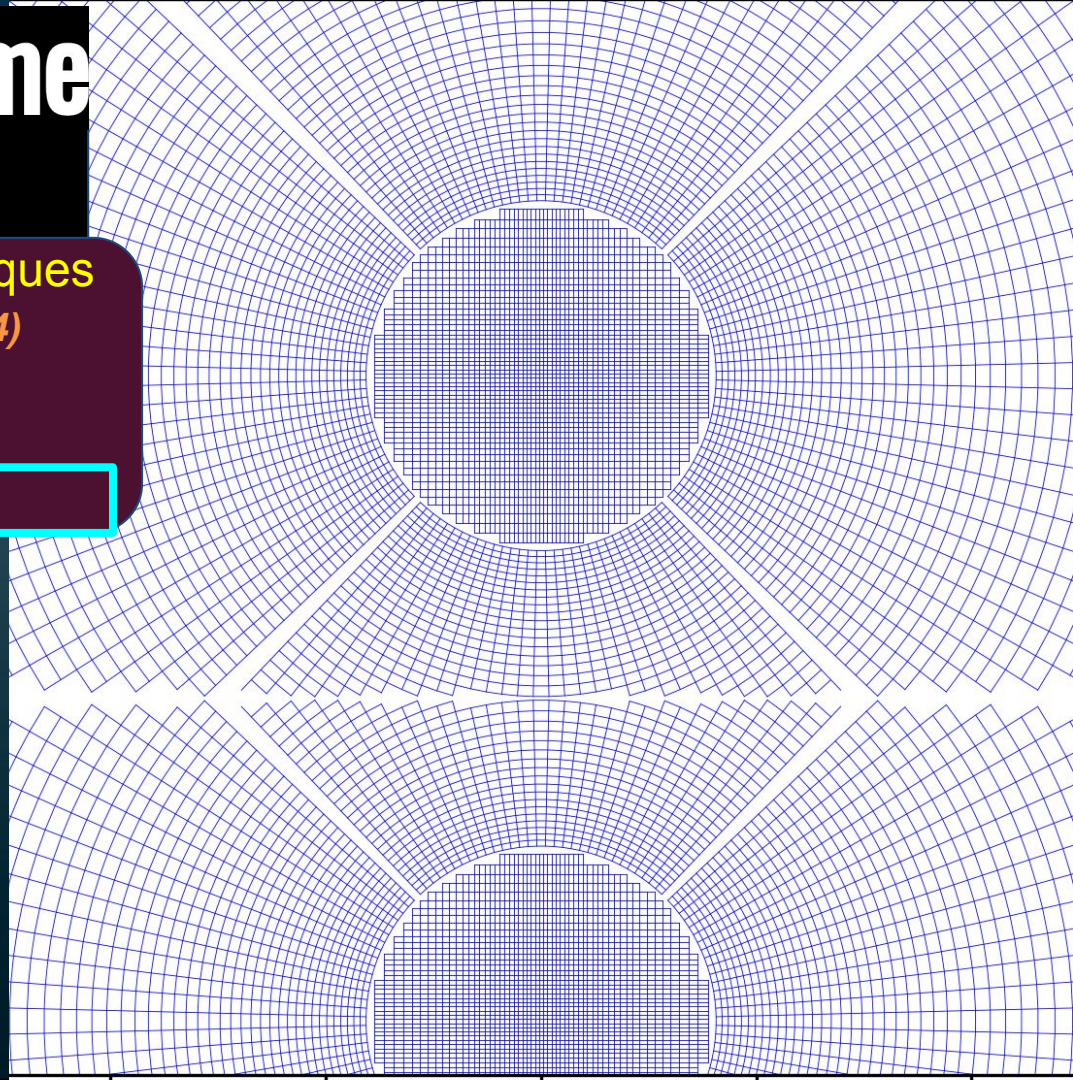




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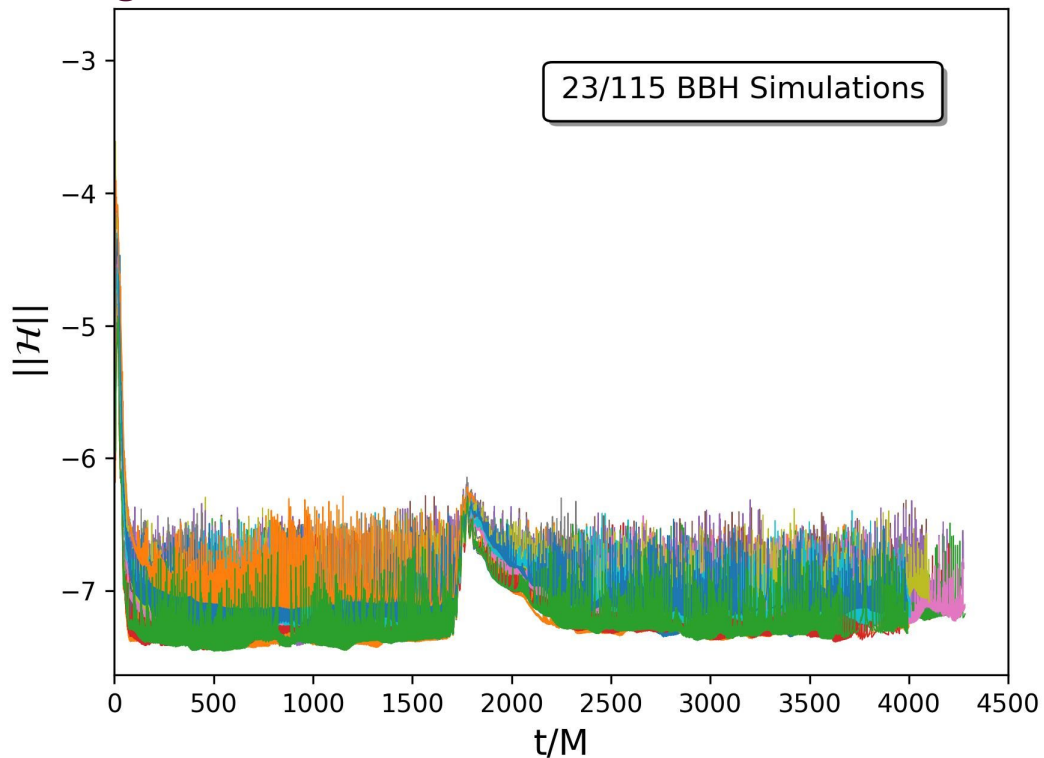




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Last year: First BBH Catalog

Strong field: $||\mathcal{H}||$ in the range $0M \leq r \leq 20M$



*Strong-field
evolution looks
great*

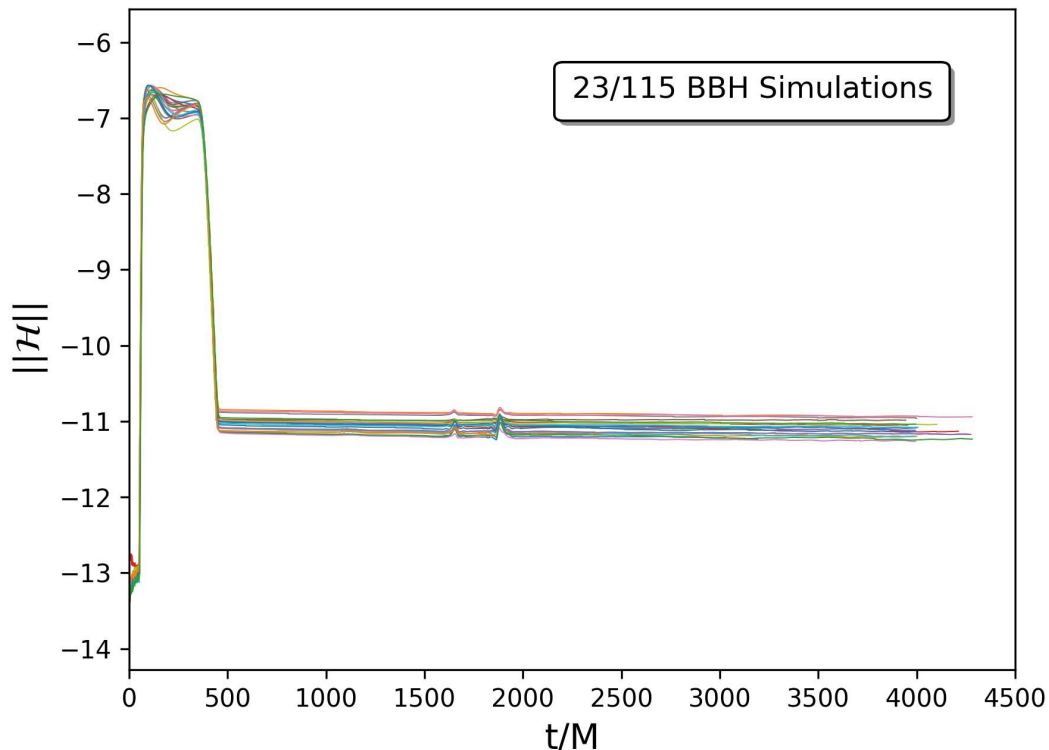
*Reproducing
115 SXS BBH
Catalog
Simulations*



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Last year: First BBH Catalog

Wave zone: $||\mathcal{H}||$ in the range $100M \leq r \leq 500M$



*Wave zone
evolution looks
great*

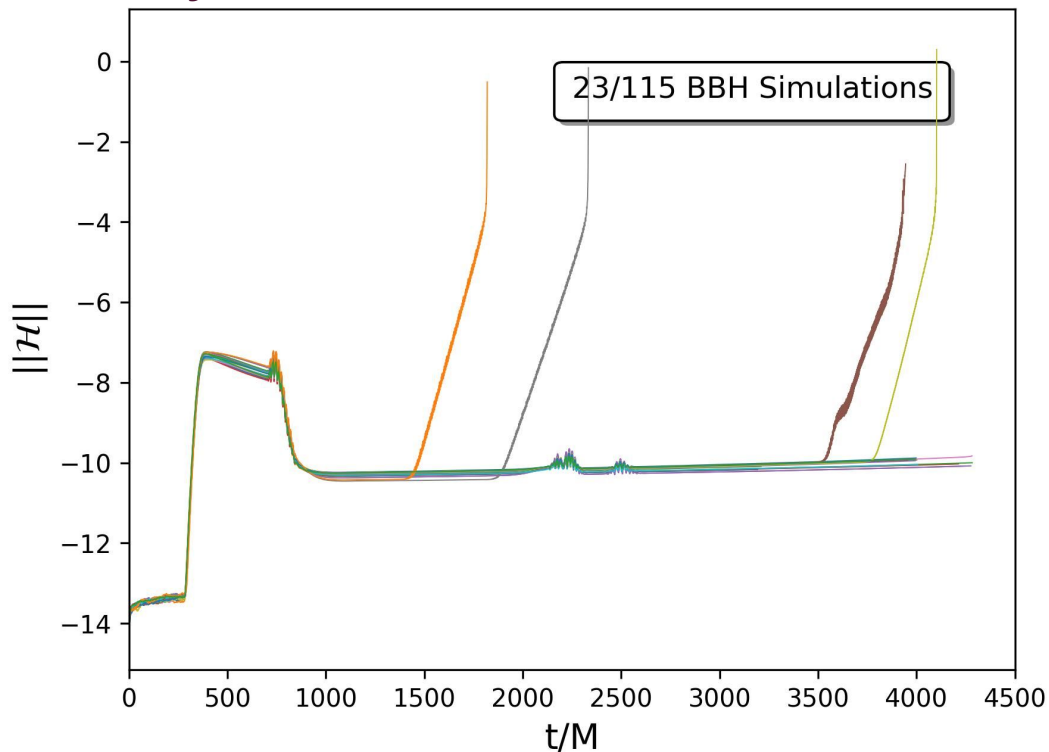
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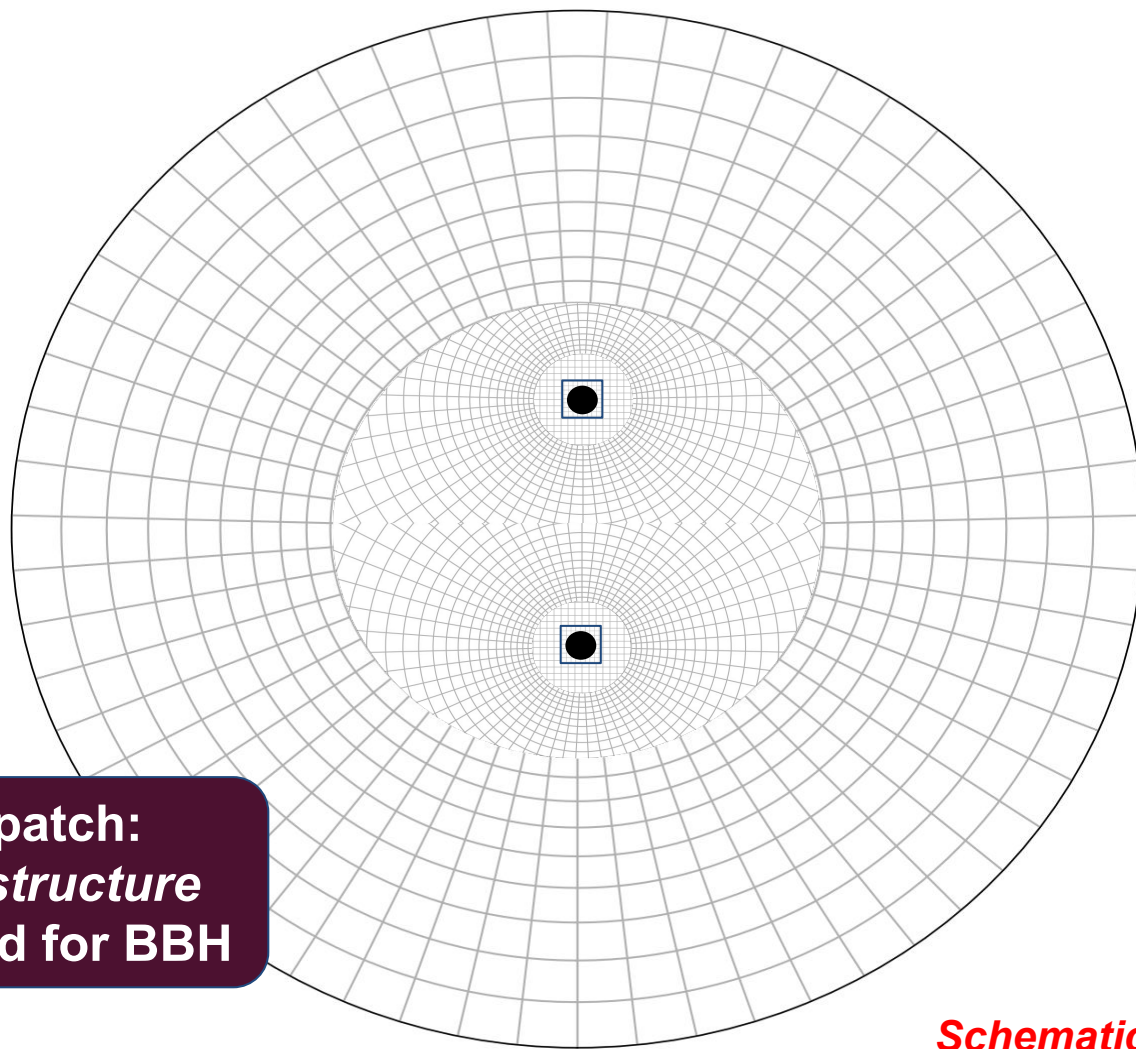
Last year: First BBH Catalog

Outer bdry: $||\mathcal{H}||$ in the range $500M \leq r \leq 1024M$



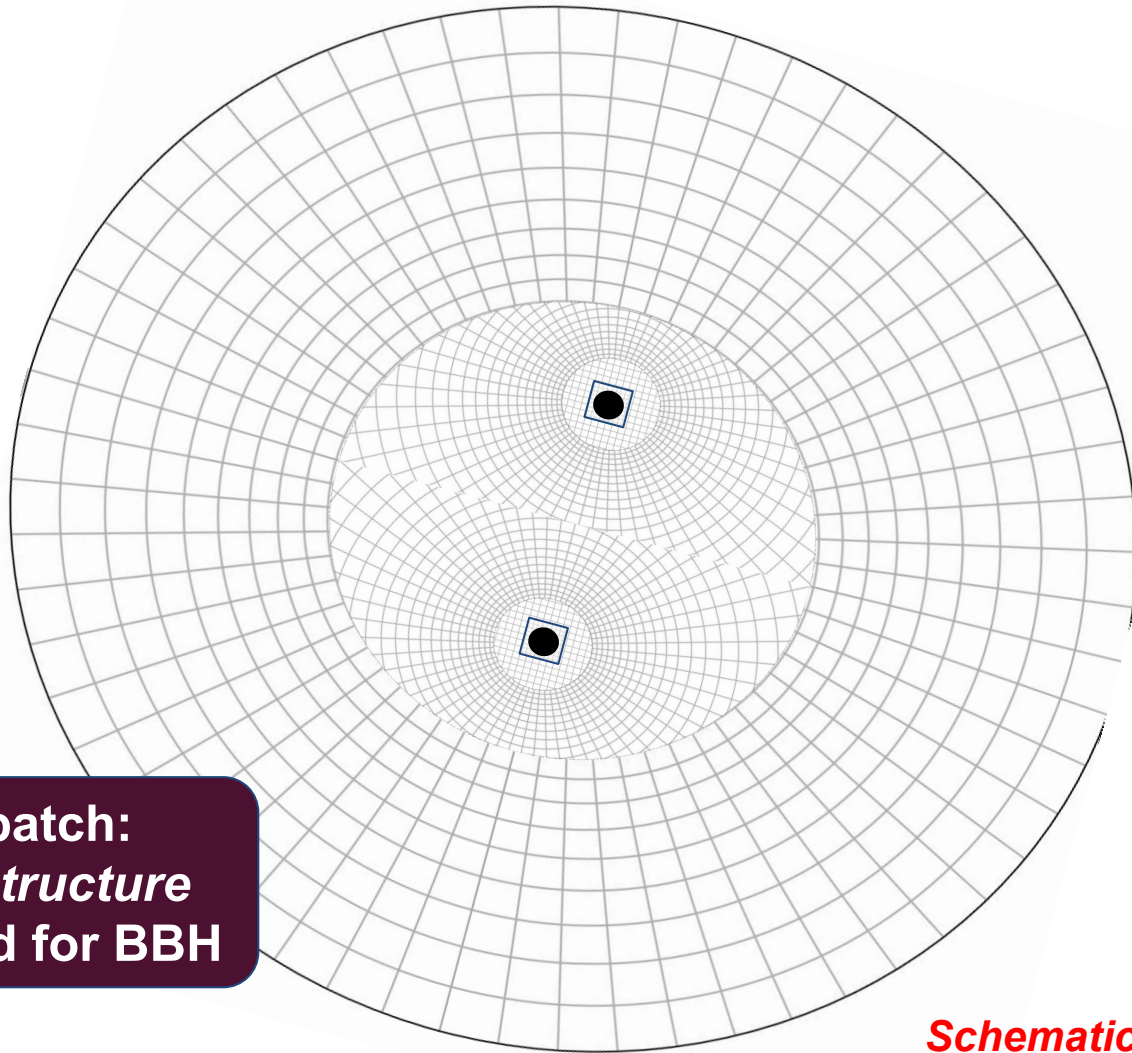
*Trouble at the
outer boundary*

*Reproducing
115 SXS BBH
Catalog
Simulations*



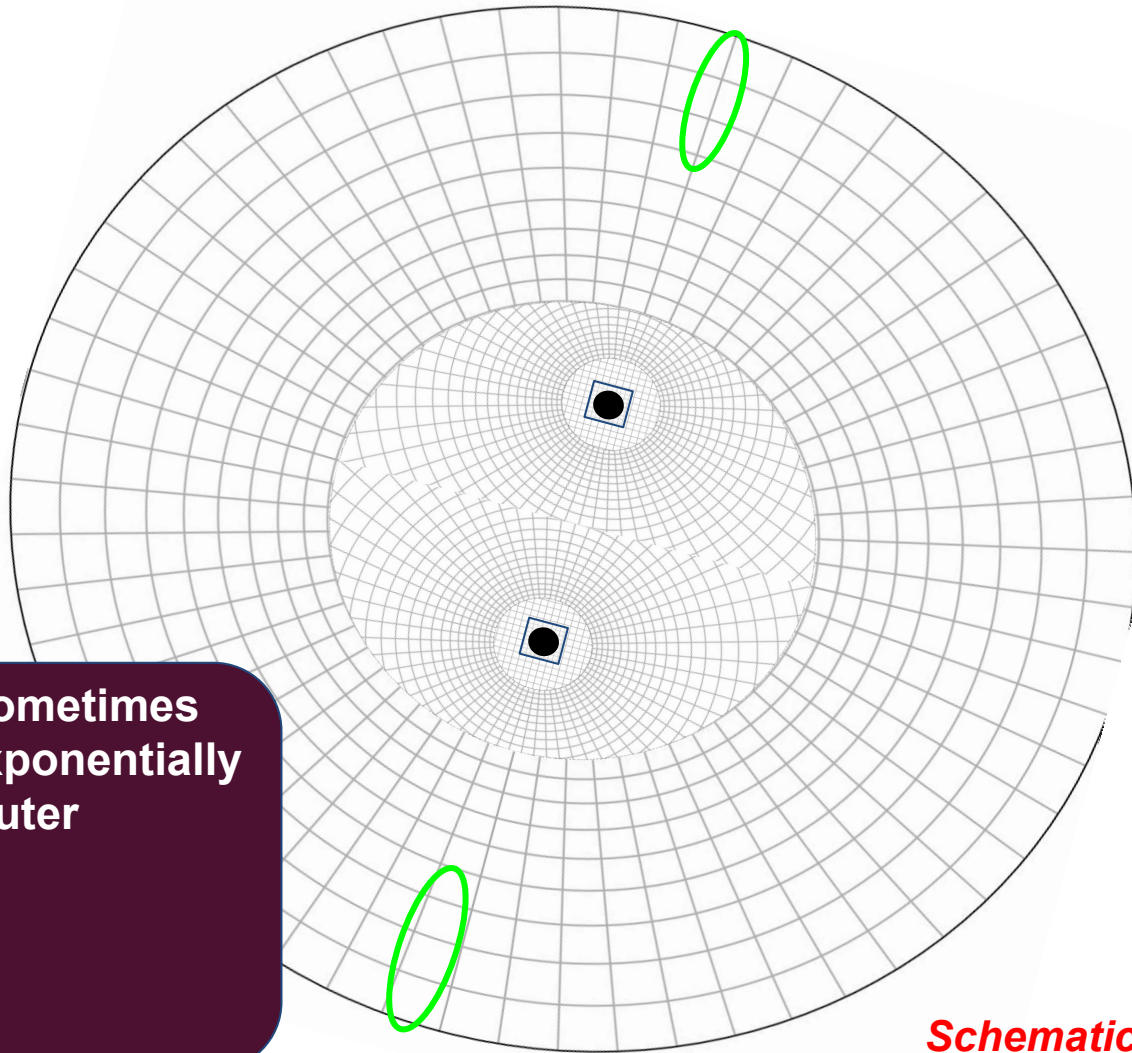
Old multipatch:
Entire grid structure
is interpolated for BBH

Schematic



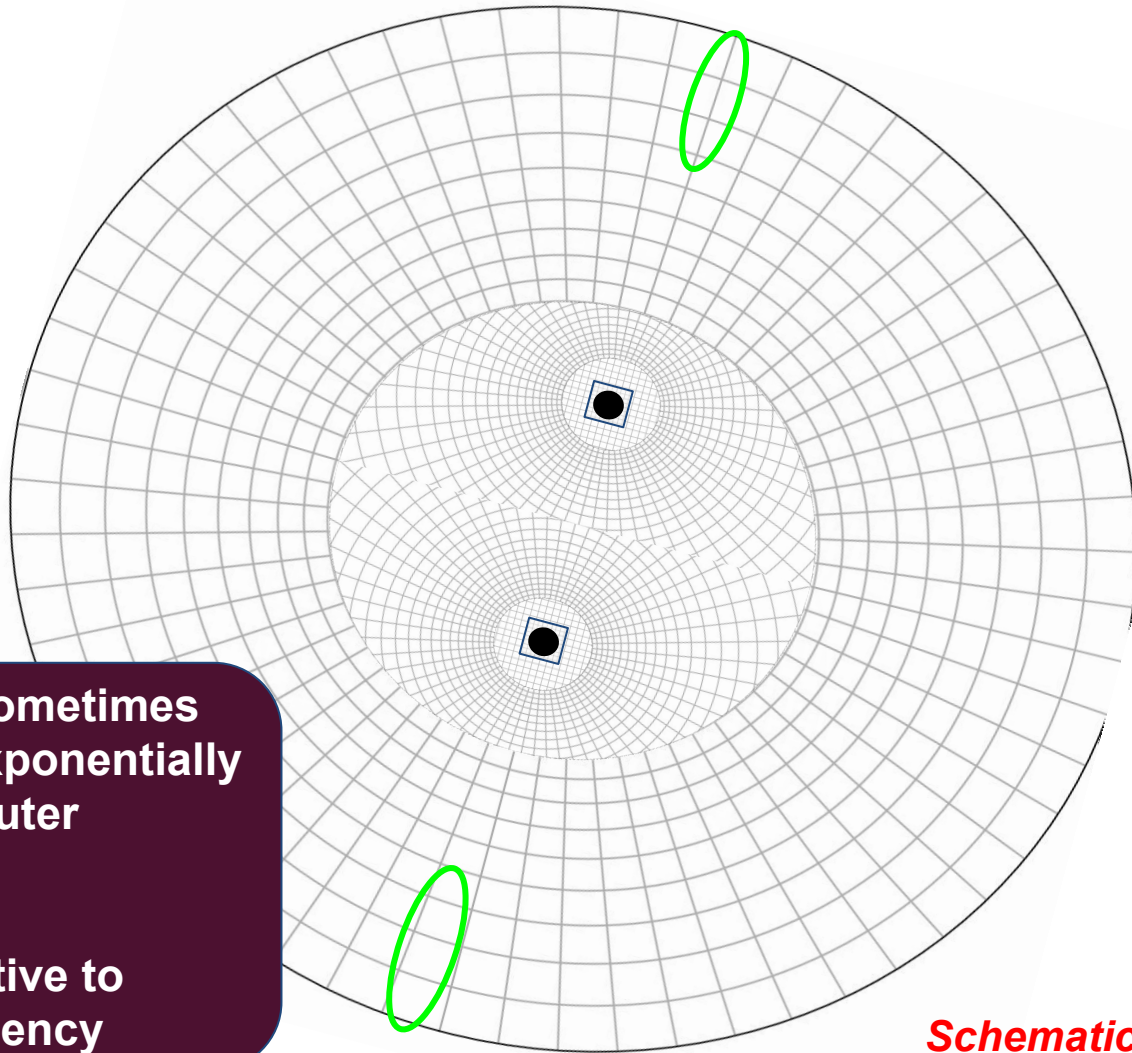
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Schematic



Interpolations sometimes
amplify noise exponentially
on z-axis near outer
boundary

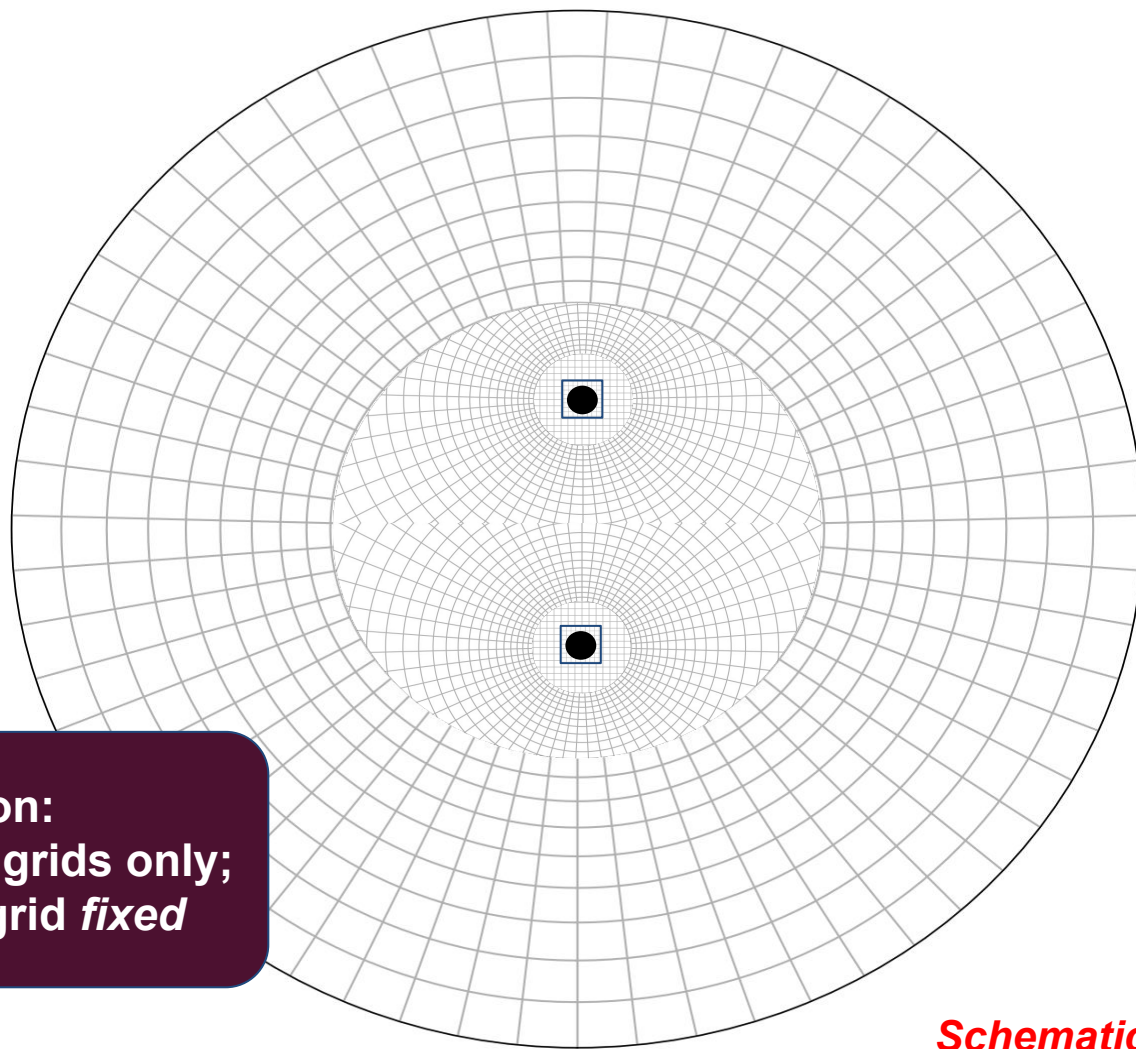
Schematic



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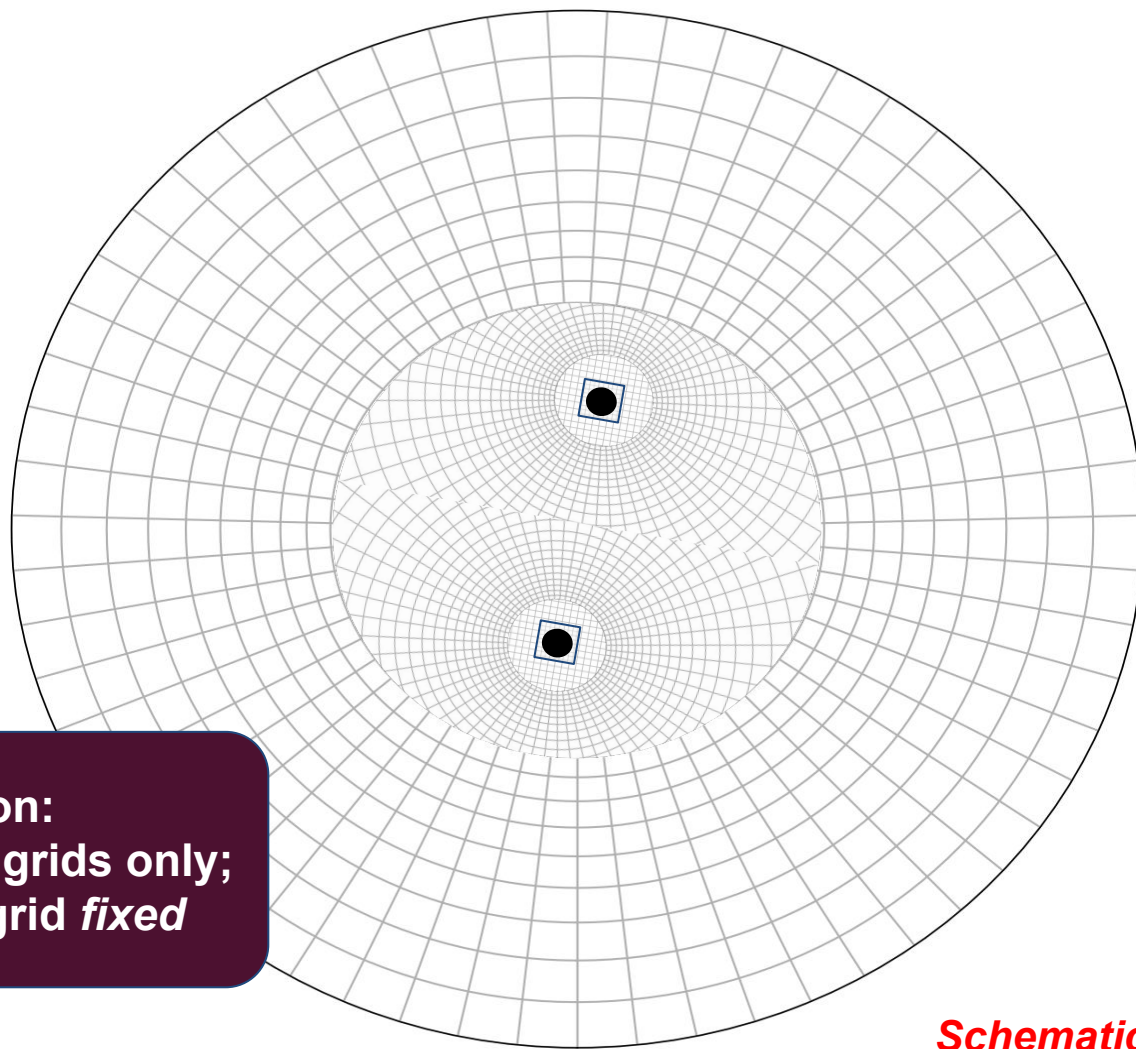
Instability sensitive to
regridding frequency

Schematic



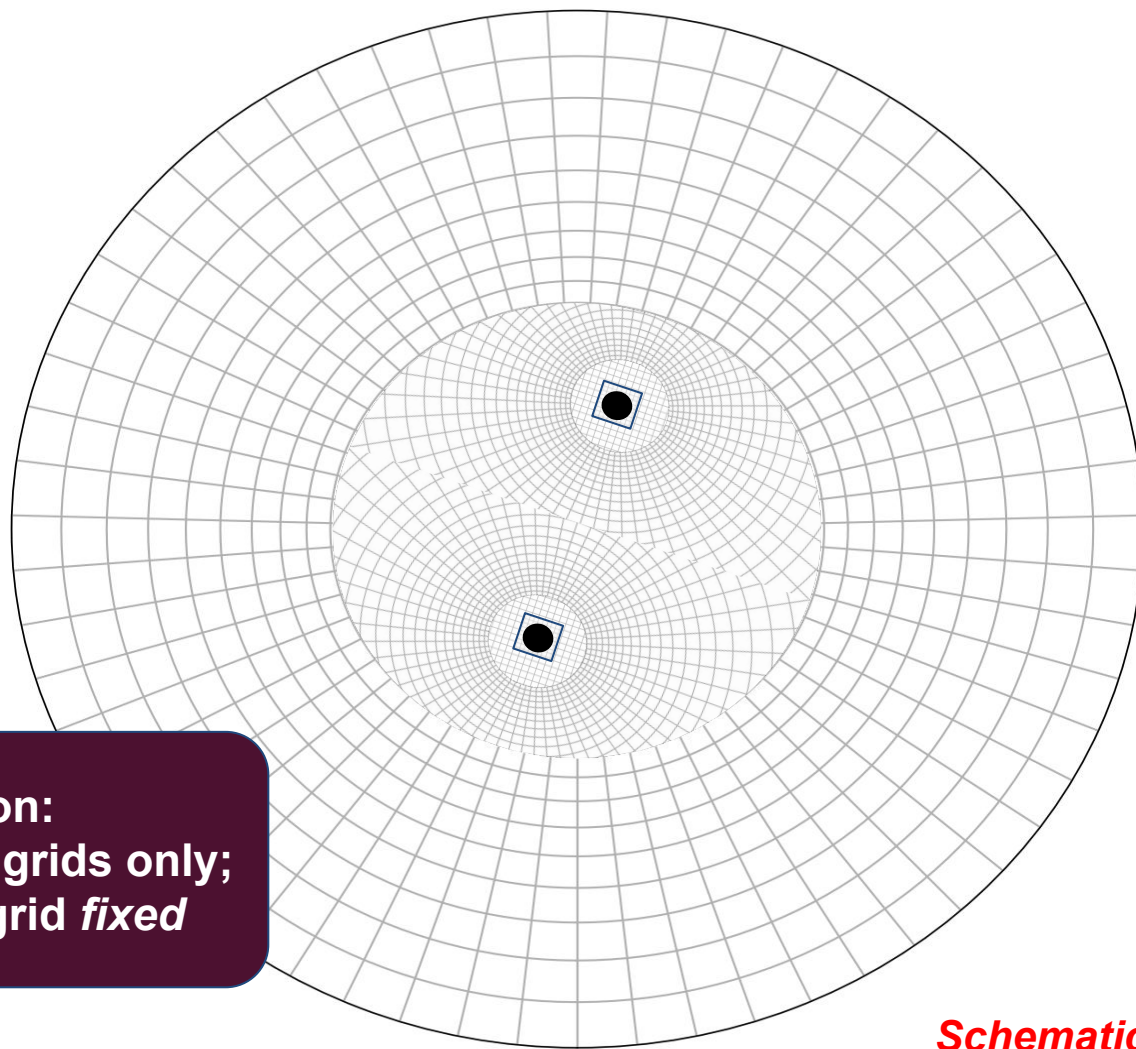
Solution:
Corotate inner grids only;
leave outer grid *fixed*

Schematic



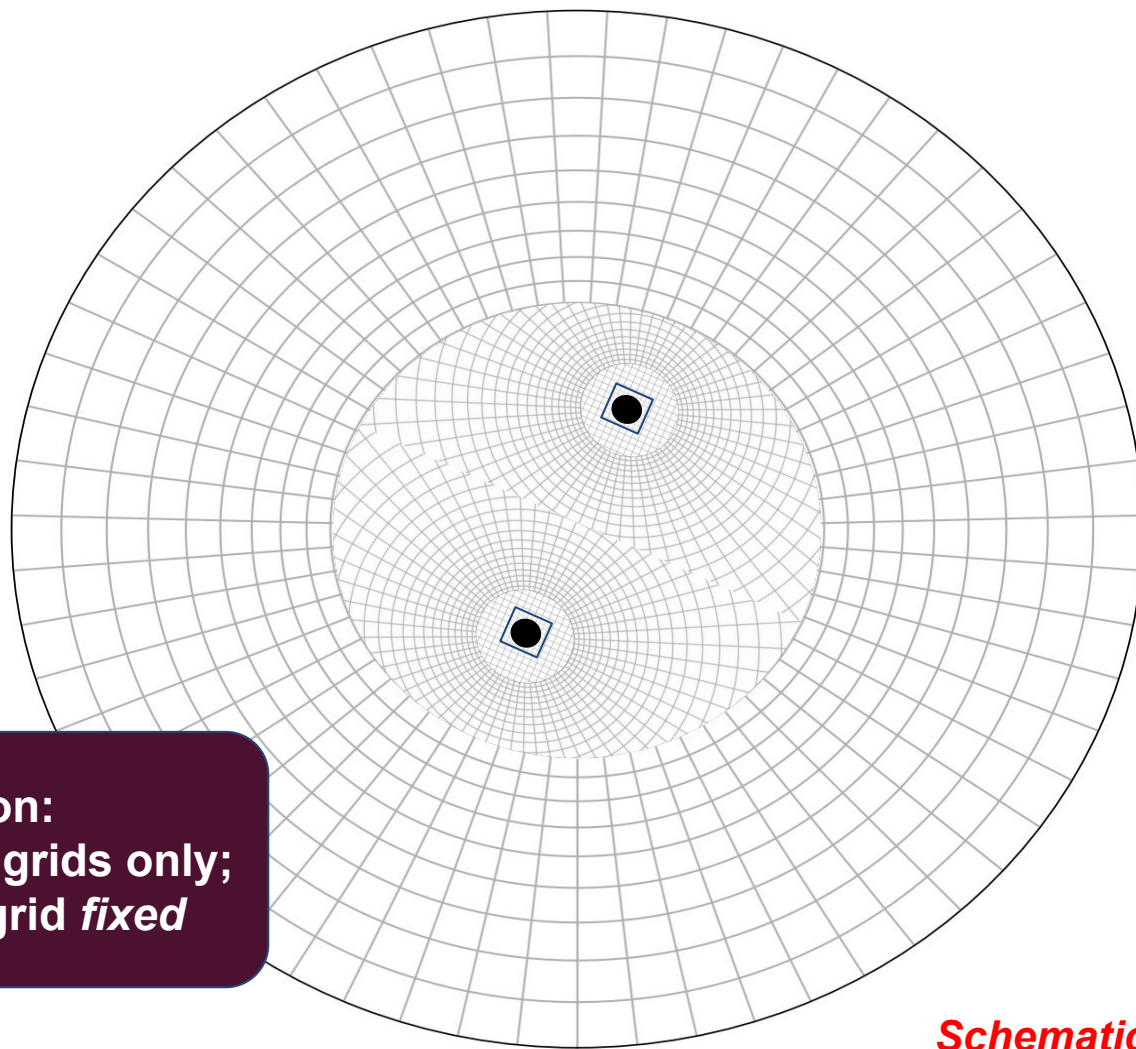
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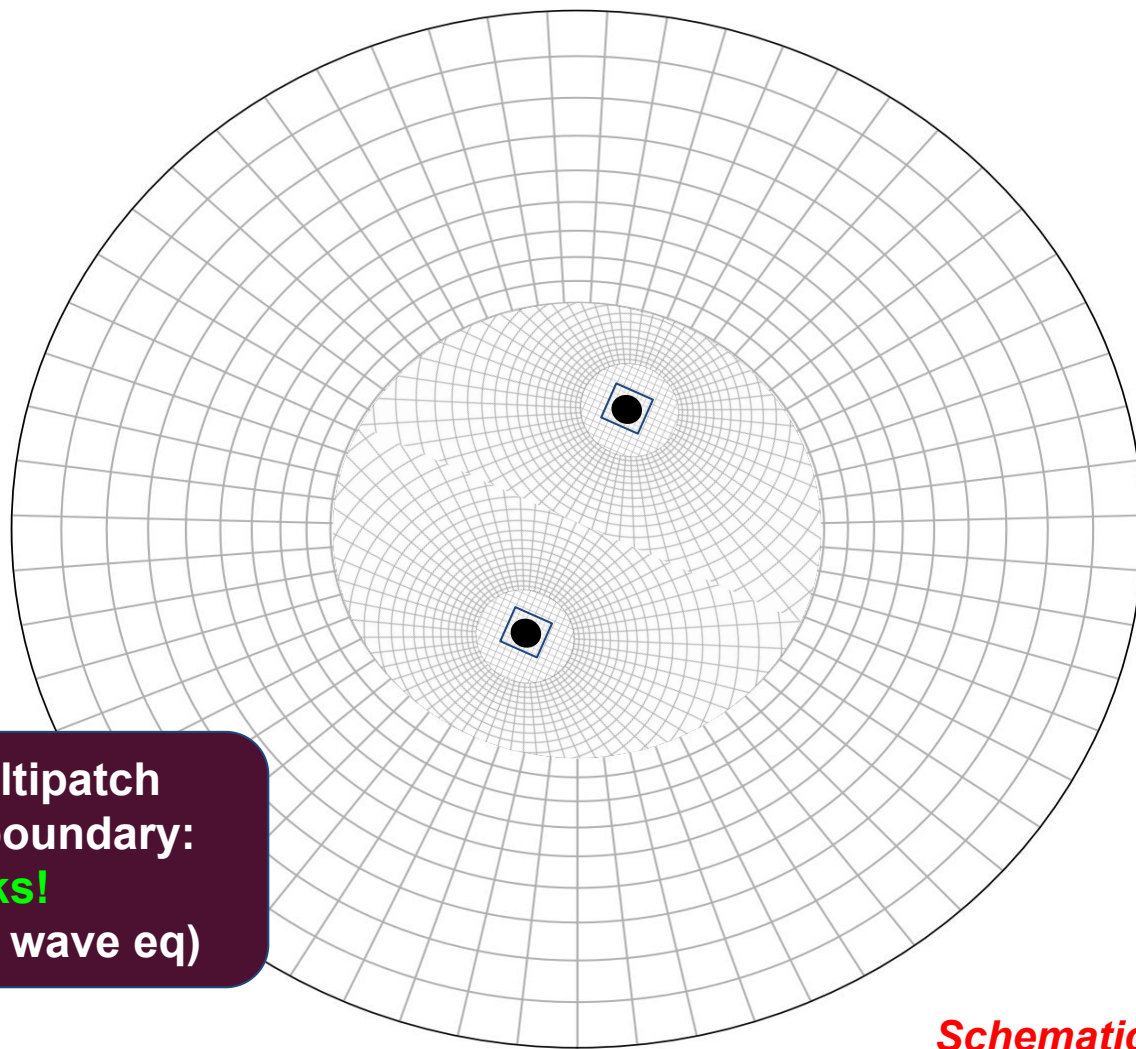
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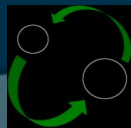
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Schematic



General Multipatch
sliding grid boundary:
It works!
(validated for wave eq)

Schematic

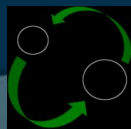


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Plans for Rest of 2024

1. Finish NRPy 2 implementation of BH@H
(90% complete)
2. BH@H paper + open source BH@H
3. Implement SpECTRE CCE
4. Start volunteer computing work
 - a. Top Priority: Reproduce SXS BBH catalog

