Math 4997-1

Lecture 2: Monte Carlo, Vectors, and Algorithms

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Reminder

Lecture 1

What you should know from last lecture

- Structure of a C++ program
- `std::string`
  - Reading strings `std::cin`
  - Writing strings `std:cout`
- Looping and counting
  - `while`
  - `for`
- Conditionals `if`
- Operators
- Built-in types

Monte Carlo methods

Monte Carlo method

Monte Carlo methods are computational algorithms which rely on repeated random sampling to obtain numerical results. More details [4].

Three problem classes [2]:

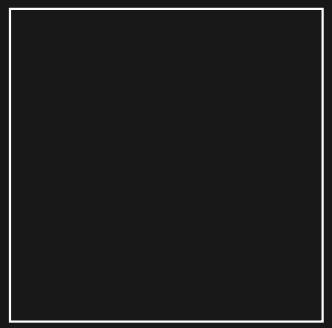
1. Optimization
2. Numerical integration
3. Probability distributions

General pattern:

1. Define the input parameters
2. Randomly chose input parameters
3. Do deterministic computations on the inputs
4. Aggregate the results

Example: Estimate the value of π

Ingredients



- Unit square 1×1
- circle with the radius of $r = {}^{1}/_{2}$
- Area of the circle $A_c = \pi r^2 = {}^{\pi}/_{4}$
- Area of the square $A_s = 1 \cdot 1 = 1$
- Recall: $A_c = {}^{\pi}/_{4} \rightarrow \pi = 4A_c$
- Hint: ${}^{A_c}/_{A_s} = A_c$ because $A_s = 1$

Now compute PI by using the two areas:

$$\pi \approx 4{}^{A_c}/_{A_s}$$

The areas can be approximated by using N random samples (x, y) and count the points inside the circle N_c

$$\pi \approx 4{}^{N_c}/_{N}$$

Example: Estimate the value of π

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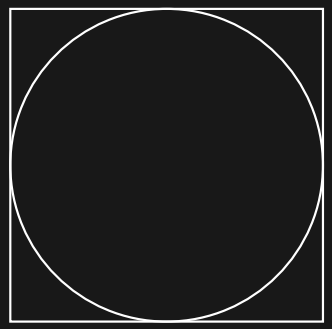

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Algorithm

1. Generate random coordinates $(x, y) \in \mathbb{R}^2$
2. Check if $x^2 + y^2 \leq 1$
   - Update N_c if ≤ 1
3. Increment n the interval
4. If $n < n_{\text{total}}$ go to 1
5. Calculate $\pi \approx 4N_c/n_{\text{total}}$
6. Print result

Interactive question:

Which features of C++ do we need?

Algorithm

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Interactive question:

Which features of C++ do we need?

`for`, `if`, `std::cout`, and random numbers

Random numbers[1]

```cpp
// Include for using rand
#include <cstdlib>
#include <iostream>
//Include for getting the current time
#include <ctime>

int main()
{
    // Use the current time as random seed
    std::srand(std::time(0));
    // Get one random number
    int random_variable = std::rand();
    std::cout
        << "Random value on [0 " << RAND_MAX << "]: "
        << random_variable << '\n';
}
```

More details: Section 3.2 [1] and [3]

[1] https://en.cppreference.com/w/cpp/numeric/random/srand

Uniform distributed random numbers[2] [3]

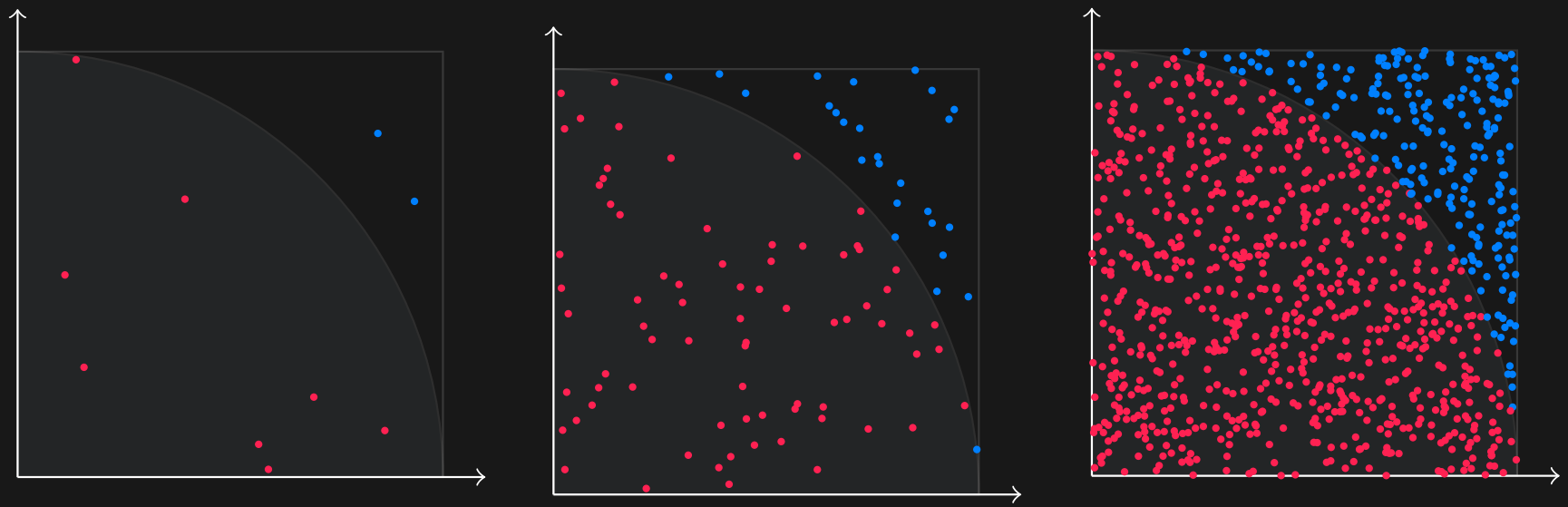
```cpp
// Include for advanced random numbers
#include <random>
#include <iostream>

int main()
{
    //Generate a random number device
    std::random_device rd;
    //Set the standard mersenne_twister_engine
    std::mt19937 gen(rd());
    //Specify the interval [1,6]
    std::uniform_int_distribution<size_t> dis(1, 6);
    //Specifiy the interval [1.0,6.0]
    std::uniform_real_distribution<double> disd(1,6);
    std::cout << dis(gen) << " " << disd(gen) << '\n';
}
```

---

[2] https://en.cppreference.com/w/cpp/numeric/random/uniform_int_distribution

[3] https://en.cppreference.com/w/cpp/numeric/random/uniform_real_distribution

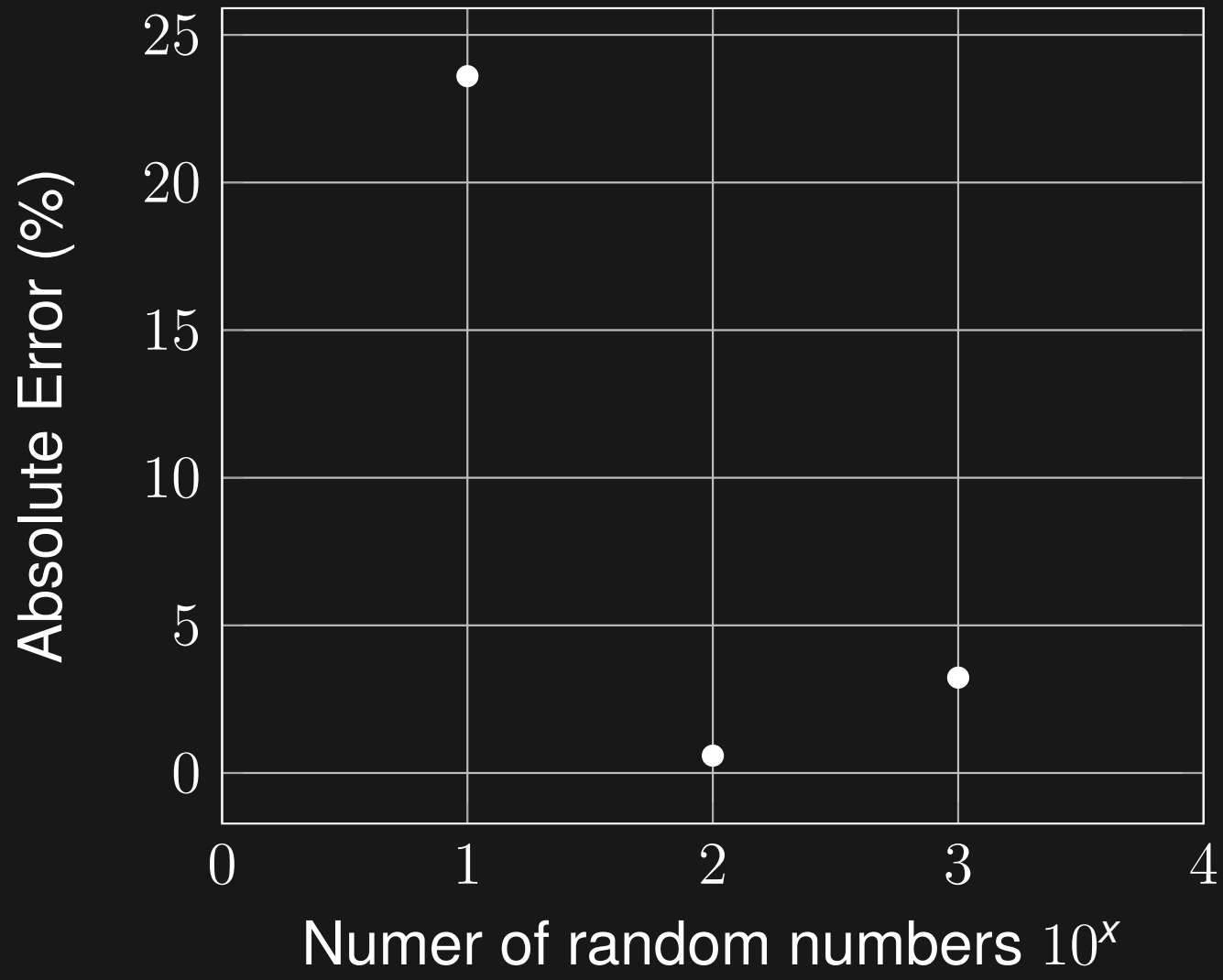
Sketches for for various random numbers



(a) $N = 10$ (b) $N = 100$ (c) $N = 1000$

Figure: Distribution of the point inside the circle (red) and outside of the circle (blue) for $N = 10$, $N = 100$, and $N = 1000$ random numbers.

How many random numbers are needed?



Containers & Algorithms

Computation of the average[4]

```
#include <iostream>
#include <iomanip>

int main()
{
double sum = 0;
size_t count = 0;
double x = 0;
    while (std::cin >> x)
    {
        sum += x;
        ++count;
    }
std::cout << "Average: "
    << std::setprecision(3)
    << sum / count << std::endl;
}
```

[4] https://en.cppreference.com/w/cpp/io/manip/setprecision

Reading multiple values

Recall: Reading one value

```
std::cin >> x
```

Reading multiple values

```
size_t count = 0;
double x = 0;
    while (std::cin >> x)
    {
        sum += x;
        ++count;
    }
```

Note that the `while` loops stops by inserting `\n`

What about computing the median?

We have to:

- We have to store all values without knowing the amount in advance
- We have to sort all the values
- We have to select the middle value(s) efficiently

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What we will use:

- Containers: `std::vector` to store the elements
- Algorithm, like sorting or accumulate
- Iterators for accessing elements

Advanced computing of the average

```
#include <iostream>
#include <vector>
#include <numeric>

int main()
{
std::vector<double> values;
double x;
    while (std::cin >> x)
    {
        values.push_back(x);
    }
double sum =
std::accumulate(values.begin(), values.end(), 0.0f);
std::cout << "Average: "
    << sum / values.size() << std::endl;
}
```

Initialization of Vectors[5]

Initialization of an empty vector

- `std::vector<double> values;`
- `values.empty()` will return `true`
- `values.size()` will return zero

Initialization

- `std::vector<double> values = {1, 2.5};`
- `values.empty()` will return `false`
- `values.size()` will return two

Note that C++ starts counting by 0 and the length is always number of elements minus one!

[5] https://en.cppreference.com/w/cpp/container/vector

Manipulating vectors

Inserting

- `values.push_back(3.5);` append 3.5 at the end
- `values[3] = 1.5;` replaces the third element by 1.5

Accessing

- `values[i]` returns element *i*
- `values.first()` returns the first element
- `values.last()` return the last element

Deleting

- `values.pop_back();` deletes the last element
- `values.erase(values.start()+i)` deletes the *i*-th element

Algorithms

Accumulate[6]

```
double sum = std::accumulate(v.begin(), v.end(), 0.0f);
```

- The first two arguments define the range of the vector
- The third is the initial value of the sum
- The header `#include <numerics>` provides this functionality

Sorting[7]

```
std::sort(s.begin(), s.end());
```

- The header `#include <algorithm>` provides this functionality

---

[6] https://en.cppreference.com/w/cpp/algorithm/accumulate

[7] https://en.cppreference.com/w/cpp/algorithm/sort

Looping over a vector

Long form

```
for( size_t i = 0; i < value.size() ; i++)
{
        values[i] *= 2;
}
```

Short form

```
for (auto& v : values)
{
        v *= 2;
}
```

- `auto` is equivalent to `double v : values`[8]

[8] https://en.cppreference.com/w/cpp/language/auto

Recall: Advanced computing of the average

```
#include <iostream>
#include <vector>
#include <numeric>

int main()
{
std::vector<double> values;
double x;
    while (std::cin >> x)
    {
        values.push_back(x);
    }
double sum =
std::accumulate(values.begin(), values.end(), 0.0f);
std::cout << "Average: "
    << sum / values.size() << std::endl;
}
```

Computing the median

Code

```
typedef std::vector<double>::size_type vec_sz;
vec_sz size = values.size();
double median = 0;
vec_sz mid = size/2;

if( size % 2 == 0 )
        median = 0.5*(values[mid] + values[mid-1]);
else
        median = values[mid];
```

Features

- `typedef`[9] is useful to not write large expression again
- Conditional parameter[10]

  ```
  median = size % 2 == 0 ? true : false;
  ```

[9] https://en.cppreference.com/w/cpp/language/typedef

[10]

Advanced computing of the median

```
#include <iostream>
#include <vector>
#include <algorithm>

int main(){
typedef std::vector<double>::size_type vec_sz;
std::vector<double> values;
double x;
    while (std::cin >> x)
    {
        values.push_back(x);
    }
std::sort(values.begin(),values.end());
vec_sz mid = values.size() / 2;
double median = values.size() % 2 == 0 ?
    0.5*(values[mid]+values[mid-1]) : values[mid];
std::cout << "Median: "
    << median << std::endl;
}
```

Functions

Function definition[11]

Example: maximum

```
int max(int a, int b)
{
return a>b?a:b;
}
```

Each function has

- a name `max`
- a return type `int max()`
- a return value `return a>b?a:b;`
- some function parameters `max(int a, int b)` or none `max(void)`

Function are defined between `#include` and `int main (void)`

[11] https://en.cppreference.com/w/c/language/function_definition

Function definition for the median

Function definition

```
double median(std::vector<double> values)
{
std::sort(values.begin(),values.end());
vec_sz mid = values.size() / 2;
double res = values.size() % 2 == 0 ?
    0.5*(values[mid]+values[mid-1]) : values[mid];

return res;
}
```

Example

```
double result = median(values)
```

Summary

Summary

After this lecture, you should know

- Monte Carlo Methods
- Random numbers
- Containers like `std::vector`
- Functions

References

References I

[1] Donald Ervin Knuth.
The art of computer programming: Seminumerical Algorithms, volume 2.
Pearson Education, 1968.

[2] Dirk P Kroese, Tim Brereton, Thomas Taimre, and Zdravko I Botev.
Why the monte carlo method is so important today.
Wiley Interdisciplinary Reviews: Computational Statistics, 6(6):386–392, 2014.

[3] Makoto Matsumoto and Takuji Nishimura.
Mersenne twister: a 623-dimensionally equidistributed uniform pseudo-random number generator.
ACM Transactions on Modeling and Computer Simulation (TOMACS), 8(1):3–30, 1998.

References II

[4] Ronald W Shonkwiler and Franklin Mendivil.
Explorations in Monte Carlo Methods.
Springer Science & Business Media, 2009.