1 Summary
In the interest of reproducibility, we describe here in detail the penalty functions we use in [Baran and Popović 2007] for discrete optimization and for embedding refinement. We plan to release the source code to Pinocchio in the future.

2 Skeleton Joint Attributes
The skeleton joints can be supplied with the following attributes to improve quality and performance (without sacrificing too much generality):

- A joint may be marked symmetric with respect to another joint. This results in symmetry penalties if the distance between the joint and its parent differs from the distance between the symmetric joint and its parent.
- A joint may be marked as a foot. This results in a penalty if the joint is not in the bottommost position.
- A joint may be marked as “fat.” This restricts the possible placement of the joint to the center of the σ largest spheres. We use σ = 50. In our biped skeleton, hips, shoulders and the head are marked as “fat.”

3 Discrete Penalty Basis Functions
The discrete penalty function measures the quality of a reduced skeleton embedding into the discretized character volume. It is a linear combination of basis penalty functions (Pinocchio uses nine).

The weights were determined automatically by the maximum-margin method described in the paper and are (0.27, 0.23, 0.07, 0.46, 0.14, 0.12, 0.72, 0.05, 0.33) in the order that the penalties are presented.

The specific basis penalty functions were constructed in an ad hoc manner. They are summed over all embedded bones (or joints), as applicable. Slight changes in the specific constants used should not have a significant effect on results. We use the notation $s_{-a}^{b}(x)$ to denote the bounded interpolation function that is equal to 0 if $x < a$, $d$ if $x > c$, and $b + \frac{(d - b)(x - a)}{(c - a)}$ otherwise.

Pinocchio uses the following discrete penalty functions:

1. It penalizes short bones: suppose a reduced skeleton bone is embedded at vertices $v_1$ and $v_2$, whose spheres have radii $r_1$ and $r_2$, respectively. Let $d$ be the shortest distance between $v_1$ and $v_2$ in the graph, and let $d'$ be the distance between the joints in the unreduced skeleton. If $d + 0.7(r_1 + r_2) < 0.5d'$, the penalty is infinite. Otherwise, the penalty is

$$\left(\frac{2}{5} - 0.3 \left(\frac{d'}{d + 0.7(r_1 + r_2)}\right)\right)^3$$

2. It penalizes embeddings in which directions between embedded joints differ from those in the given skeleton. More precisely, for every pair of joints that are either adjacent or share a common parent in the reduced skeleton, we compute $c$, the cosine of the angle between the vectors $v_2 - v_1$ and $s_2 - s_1$ where $v_1$ and $v_2$ are the joint positions in the embedding and $s_1$ and $s_2$ are the joint positions in the given skeleton. The penalty is then infinite if $c < \alpha_1$, and is $0.5 \max(0, \alpha_2 \cdot (1 - c)^2 - \alpha_3)$ otherwise. If the joints are adjacent in the reduced skeleton, we use $(\alpha_1, \alpha_2, \alpha_3 = (0, 0.16, 0.1))$ and if they share the parent, we use $(\alpha_1, \alpha_2, \alpha_3 = (-0.5, 4, 0.5))$, a weaker penalty.

3. It penalizes differences in length between bones that are marked as symmetric on the skeleton. Suppose that two bones have been marked symmetric and have been embedded into $v_1$–$v_2$ and $v_3$–$v_4$ with these vertices having sphere radii $r_1$, $r_2$, $r_3$, and $r_4$, respectively. Suppose that the distance along the graph edges between $v_1$ and $v_2$ is $d_1$ and the distance between $v_3$ and $v_4$ is $d_2$. Let

$$q = 0.2 \max\left(\frac{d_1}{d_2}, \frac{d_2}{d_1}\right) +$$

$$+ 0.8 \max\left(\frac{d_1}{d_2 + 0.7(r_3 + r_4)}, \frac{d_2}{d_1 + 0.7(r_1 + r_2)}\right).$$

Then the penalty for this pair of bones is $\max(0, q^3 - 1.2)$.

4. It penalizes bone chains sharing vertices. If two or more bone chain embedding share a vertex whose distance to the surface is smaller than 0.02, the penalty is infinite. If a bone chain is embedded into a path $v_1, \ldots, v_k$ such that $v_1$ is the child joint and $v_k$ is the parent joint, and if $S$ is the subset of these joints occupied by a previously embedded bone chain, the penalty is $0.5 + \sum_{v_i \in S} \frac{d}{27r}$ if $S$ is not empty.

5. It penalizes joints that are marked as feet if they are not in the bottommost possible position. For each such joint, the penalty is the $y$ coordinate difference between the graph vertex with these vertices having sphere radii $r_1$ and the vertex into which the joint is embedded.

6. It penalizes bone chains of zero length. This penalty is equal to 1 if a joint and its parent are embedded into the same vertex.

7. It penalizes bone segments that are improperly oriented relative to the given bones. This penalty is calculated for the unreduced skeleton, as we do before embedding refinement: we reinsert degree-two joints by splitting the shortest paths in the graph and proportion to the given skeleton. The penalty is then the sum of penalties over each unreduced bone. Let $\vec{u}$ be the vector corresponding to the embedded bone and let $\vec{d}^*$ be the vector of the bone in the given skeleton. The penalty per unreduced bone is

$$50\|\vec{d}^*\|^2 \left(1 - c\right)^{0.5 - \alpha_5}$$

where $c = \frac{\|\vec{d} - \vec{d}^*\|}{\|\vec{d}\|}$.

8. It penalizes degree-one joints that could be embedded farther from their parents and are not. Suppose a degree-one joint is embedded into $v_2$ and its parent is embedded into $v_1$ (different from $v_2$). This penalty is equal to 1 if there is a vertex $v_3$ adjacent to $v_2$ in the extracted graph whose sphere is at least 1/2 the radius of the sphere at $v_2$ and the following two conditions hold:

$$\frac{(v_2 - v_1) \cdot (v_4 - v_1)}{\|v_2 - v_1\| \|v_4 - v_1\|} \geq 0.95.$$
\[(v_2 - v_1) \cdot (v_3 - v_2) \geq 0.8.\]

Moreover, to improve optimization performance, we never try embedding a degree-one joint into a vertex \(v_1\) if for every adjacent vertex \(v_2\) there is a vertex \(v_3\) adjacent to \(v_1\) such that the sphere around \(v_3\) is at least 1/2 the radius of the sphere around \(v_1\) and:

\[
\frac{(v_3 - v_1) \cdot (v_1 - v_2)}{\|v_3 - v_1\|\|v_1 - v_2\|} \geq 0.8.
\]

9. It penalizes joints that are embedded close to each other in the graph, yet are far along bone paths. More precisely, for every pair of joints \(v_1\) and \(v_2\) (that are not adjacent in the reduced skeleton), this penalty is 1 if

\[
2d(v_1, v_2) + r_1 + r_2 < d(v_1, v_L) + d(v_2, v_L)
\]

where \(d\) is the distance along graph edges, \(r_1\) and \(r_2\) are the radii of spheres into whose centers \(v_1\) and \(v_2\) are embedded, and \(v_L\) is the embedding of the least common ancestor (in the oriented reduced skeleton) of the two embedded joints.

\section{4 Embedding Refinement Penalty Function}

This penalty function is used to refine the discrete embedding. It was also constructed ad hoc. It is the weighted sum of the following four penalty functions over all bones. The weights we use are \((\alpha_S, \alpha_L, \alpha_O, \alpha_A) = (15000, 0.25, 2, 1)\) for the respective penalties.

1. Pinocchio penalizes bones that are not near the center of the object. The penalty is the average of

\[
r(0.003, \min(m(q_i), 0.001 + \max(0, 0.05 + s(q_i))))
\]

over 10 samples \(q_i\) on the bone, where \(r(a, x) = 0\) if \(x < a\) and is \(x^2\) otherwise. \(m(p)\) is the distance from \(p\) to the nearest sampled medial surface point, and \(s(p)\) is the signed distance from \(p\) to the object surface (positive when \(p\) is outside).

2. It penalizes bones that are too short when projected onto their counterparts in the given skeleton. Suppose a bone has endpoints \(q_1\) and \(q_2\) in the embedding and endpoints \(s_1\) and \(s_2\) in the given skeleton. The penalty is:

\[
\max(0.5, \frac{\|s_2 - s_1\|^2}{(||q_2 - q_1|| \cdot ||s_2 - s_1||)^2/\|s_2 - s_1\|^2}).
\]

3. It penalizes improperly oriented bones. Suppose a bone has endpoints \(q_1\) and \(q_2\) in the embedding and endpoints \(s_1\) and \(s_2\) in the given skeleton. Let \(\theta\) be the angle between the vectors \(q_2 - q_1\) and \(s_2 - s_1\). The penalty is \((0.3 + 0.5\theta)^3\) if \(\theta\) is positive and \(10 \cdot (0.3 + 0.5\theta)^3\) if \(\theta\) is negative.

4. It penalizes asymmetry in bones that are marked symmetric. If a bone has endpoints \(q_1\) and \(q_2\) and its symmetric bone has endpoints \(q_3\) and \(q_4\) then the penalty is:

\[
\max\left(1.05, \frac{\|q_1 - q_2\|^2}{\|q_3 - q_4\|^2} \cdot \frac{\|q_3 - q_2\|^2}{\|q_1 - q_2\|^2}\right).
\]

This penalty appears in the sum once for every pair of symmetric bones.

\section{References}