1 Introduction

We present a robust technique for Super-Helix (SH) modeling based on a variational and iterative fitting of SH curves to arbitrary regular parametric curves.

Recently Bertails et al. [2006] proposed using Cosserat curves with piecewise constant curvature - so called Super-Helices - for dynamic simulations of hair. This approach mainly has two advantages over previous work; it can easily handle curly hair, as well as preserve its length during deformations (i.e. prevent stretching). These are simple consequences of the underlying mathematical SH model. However, this all comes at the price of having to represent curves (or hair strands) in an unfamiliar and largely unintuitive parameter-space of curvatures and twists. Consequently the modeling and styling of SH curves is a challenging task that was not addressed in [Bertails et al. 2006].

In our method we employ data reduction and error-analysis known from mesh decimation algorithms as well as non-linear minimization, in order to fit SH curves to arbitrary parametric curves. This approach allows us to take advantage of the large body of existing work on parametric curve modeling. In particular, several commercially available packages already exist for grooming hair represented by NURBS curves. Our fitting procedure enables us to convert NURBS curves to SH curves, which in turn serve as input for subsequent dynamic simulation of the hair.

2 Adaptive Subdivision and Fitting

In general terms we wish to fit to a parametric (i.e. NURBS) curve, \( y(s) \), parameterized in arc length, \( s \), a SH curve, \( R_N(s) \), with \( N \) elements having constant curvatures \( q = \{q_0, q_1, q_2, \ldots\} \). A straightforward root-to-tip, per-segment quasi-Newton minimization suffer from severe stability problems due to the way error accumulates throughout the SH. On the other hand, exploiting the coupling between a segment and its predecessors - in an analogy to the physical system derived in [Bertails et al. 2006] - and performing global fitting, introduces a vast parameter space with an abundance of local, visually sub-optimal, minima. Working with a coupled Hessian also requires the solution of a \( 3N \times 3N \), dense, matrix, each iteration which further restricts the resolution of the Super-Helix as the number of iterations likely will be high.

Instead we propose a fitting technique inspired by standard mesh decimation schemes. First, we perform a high-resolution fit, where the variance in Frenet-Serret curvature of the NURBS curve drives an adaptive subdivision scheme. Each SH segment is fitted locally using an error metric based on its endpoint’s position, orientation and the average curvature over the segment. This provides an optimal starting point for subsequent segments and limits the accumulated error while maintaining a visually appealing shape. The Frenet-Serret analysis of the NURBS curve efficiently produces an initial SH curve with enough elements to accurately fit the NURBS curve. This is essential when we subsequently perform a global error minimization on the \( \hat{q} \)'s, since it reduces the risk of getting stuck in an unstable local minima as well as improves the rate of convergence. The non-linear optimization is performed using a modified Levenberg-Marquardt algorithm with a Preconditioned Conjugate Gradient solver.

3 Decimation

To reduce our high-resolution fit to a resolution feasible for simulation, we employ various sampling techniques to determine which segment introduces the least amount of error when removed, and its arc length is distributed over neighboring segments before it is removed and an adjusting fit is performed. This procedure is repeated until sufficiently many segments have been removed. Again, due to the high-resolution initial fit and subsequent optimization steps we remain sufficiently close to a visually pleasing minimum to enable global relaxation iterations using the full coupled system and a PCG solver.

References


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