Origamizing 3D Surface by Symmetry Constraints

Tomohiro Tachi*
The University of Tokyo

1 Introduction

We present a new method for “origamizing” or designing the origami crease pattern to construct a given arbitrary polyhedral surface through user interaction. The author [2006a; 2006b] previously proposed a method for manually designing 3D origami surface based on aligning facets and “tucking molecules.” However, crease pattern generated by such method is supposed to be super-complex and hard to be folded due to the general tucking molecules without symmetry.

Our new method is based on equality constraints that make molecules symmetrical to reduce the complexity of resulting crease pattern. These symmetry constraints can be represented as a linear equation by separately optimizing rotational positions and translational positions of the facets. By initially calculating the projection matrix to the constraint space, the user can interactively edit the alignment of facets while keeping the conditions satisfied.

2 Tucking Molecules

Crease pattern is designed by tessellating a convex area of a plane into surface polygons and “tucking molecules” (Figure 1). Surface polygons of a given polyhedral model is isometrically mapped onto the plane, and then tucking molecules are generated between mapped polygons. By being folded flat, tucking molecules “paste” corresponding segments and vertices of the polygons together and construct the 3D surface.

3 Symmetry Constraints and Initial Mapping

A general edge-tucking molecule is composed of 7 different short fold lines. This can be simplified by adding a constraint to make it symmetrical, and the number of crease lines be reduced to 1 (Figure 2(a)). A symmetry constraint gives 1 equation per edge-tucking molecule, which is represented as follows (Figure 2(b)).

\[ v_{\text{adj}} \cdot v_{\text{mid}} = 0 \quad (1) \]

Initial mapping of the elements are determined by first optimizing rotational positions and then calculating translational positions of the facets so that the whole polygons are mapped onto a convex region of a plane while the symmetry constraints for the entire model are kept satisfied. By separately calculating rotational and translational positions of the facets, each configuration is given as the minimum norm solution of a linear equation.

4 Interactive System

Since the symmetry constraints are given by a linear equation, a valid configuration of the facets is calculated just by multiplying a projection matrix to the linear subspace. This enables users to interactively edit the surface configuration by directly editing facets or by adding equality constraints between facets. Also, the system calculates the configuration which avoids intersection of the polygons and keeps the inequality conditions shown in [Tachi 2006a] to origamize the surface.

We have implemented an interactive system for designing an origami crease pattern based on this method (Figure 3). The system precalculates the matrix and gets an initial configuration in about 2 seconds for 630 triangles model on a PC with a Pentium 4/2.0GHz, and the interactive manipulation after the initialization is done in real time.

5 Conclusion

We proposed a method for designing an origami crease pattern to fold a given polyhedral surface based on symmetry constraints, which can be represented as a linear equation. The method enables an interactive system in which a user can edit the alignment of facets and constraints to design origami crease pattern.

References
