The Final Spin of Black Hole Binaries

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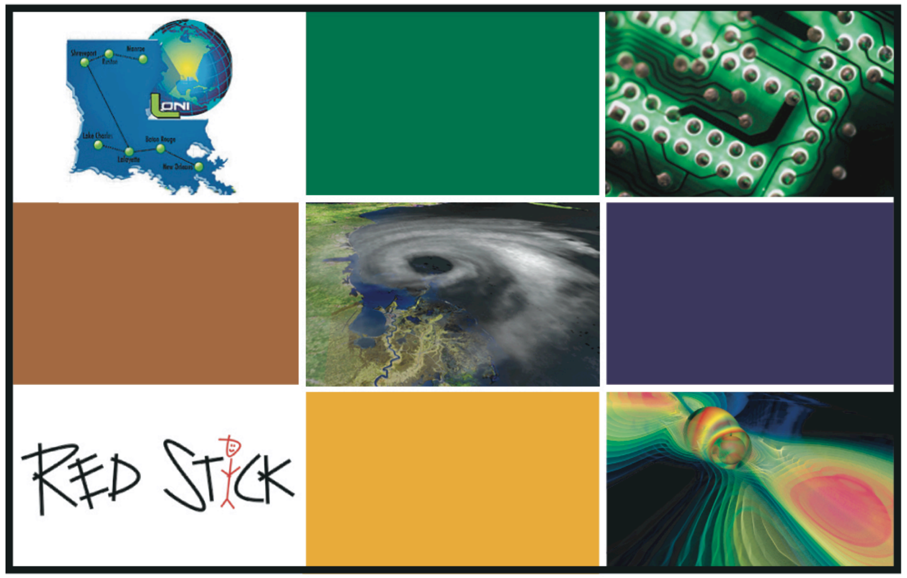
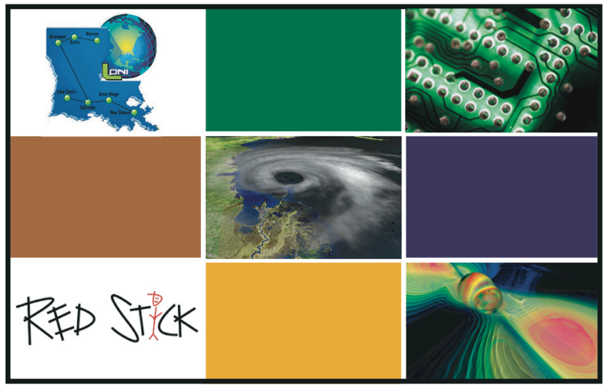


Outline

1. Introduction: Binary black hole systems
2. Modelling merger events as black box
   a. Unequal spin case
   b. Unequal mass case
3. Current work: generalisations
4. Peeking behind the scenes: numerical and computational infrastructure
5. Future infrastructure development



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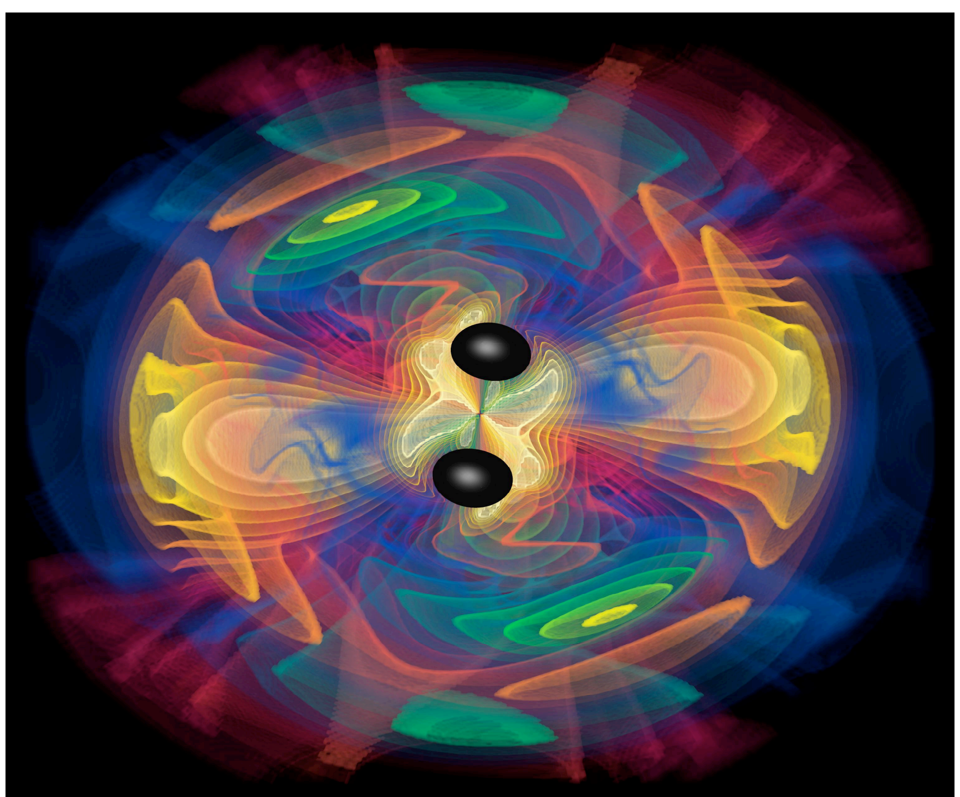
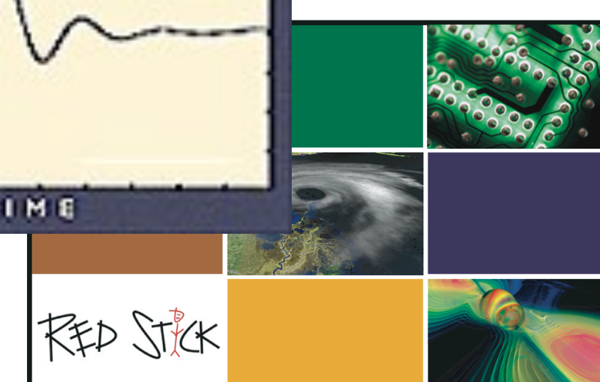
Gravitational Wave Physics

Observations

Models

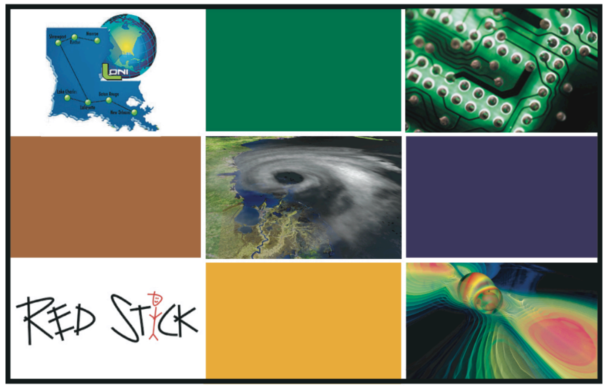




Predictions

Binary Black Holes

- Very "simple" system – only Einstein equations
- Interesting to relativists: two-body problem, determined by very few parameters; still open questions regarding trapped regions
- Interesting to astrophysicists: fine source of gravitational radiation for LIGO and LISA; first step towards binaries with matter



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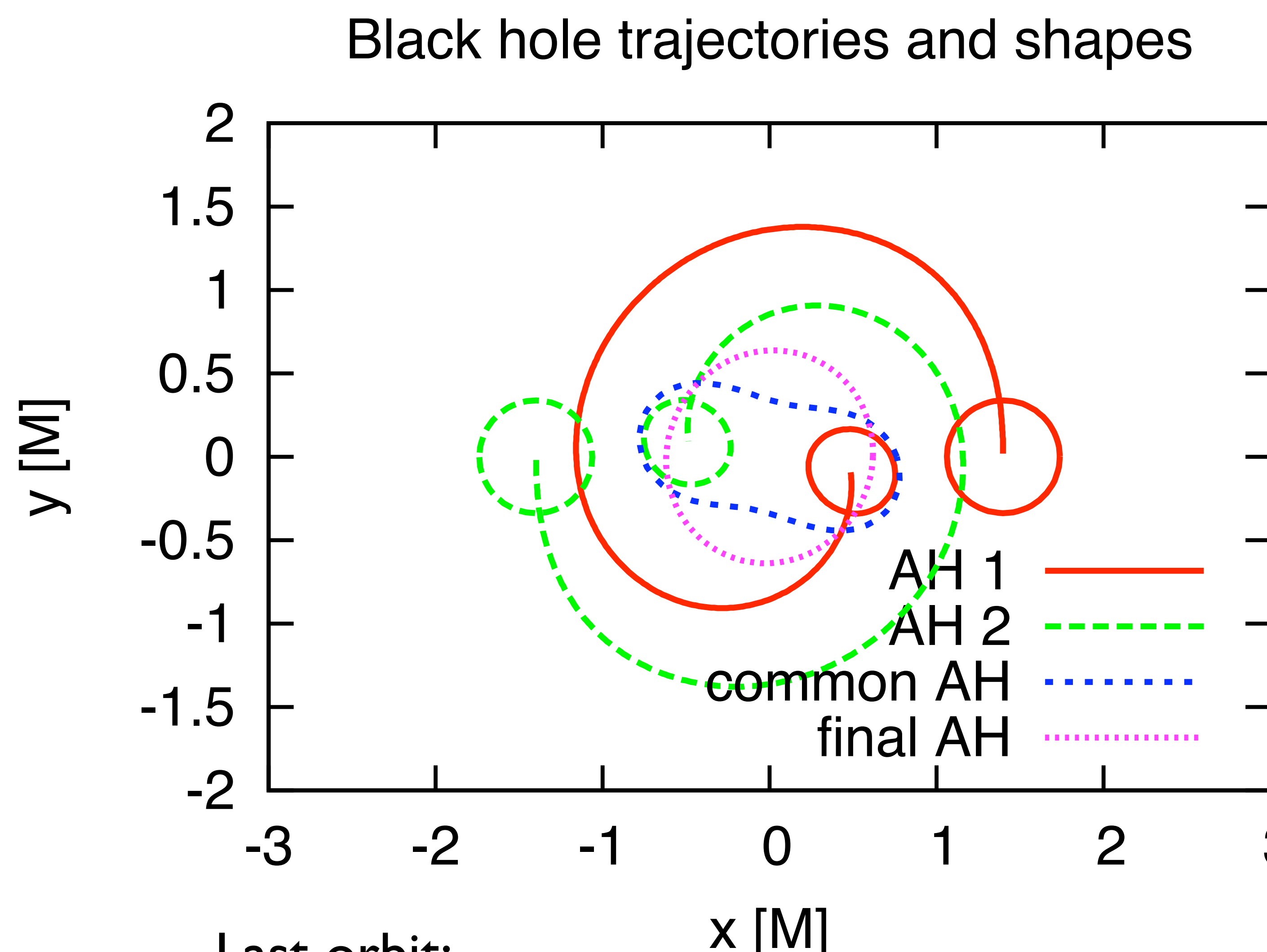
Phenomenology of Binary Black Hole Mergers

1. Inspiral (red/green), here: slightly eccentric

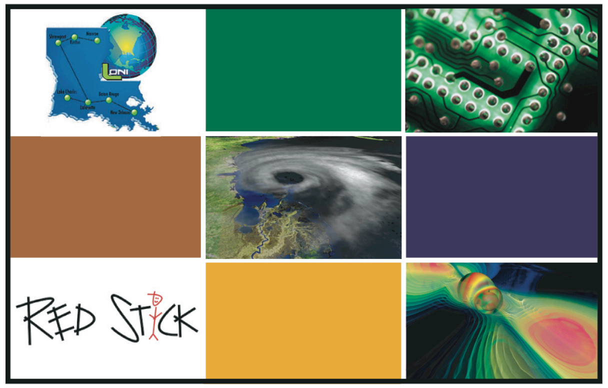
2. Common horizon (blue)

3. Final state (magenta)

- Not shown: possible recoil (“kick”), if system is not symmetric



Last orbit: Cook-Pfeiffer initial data



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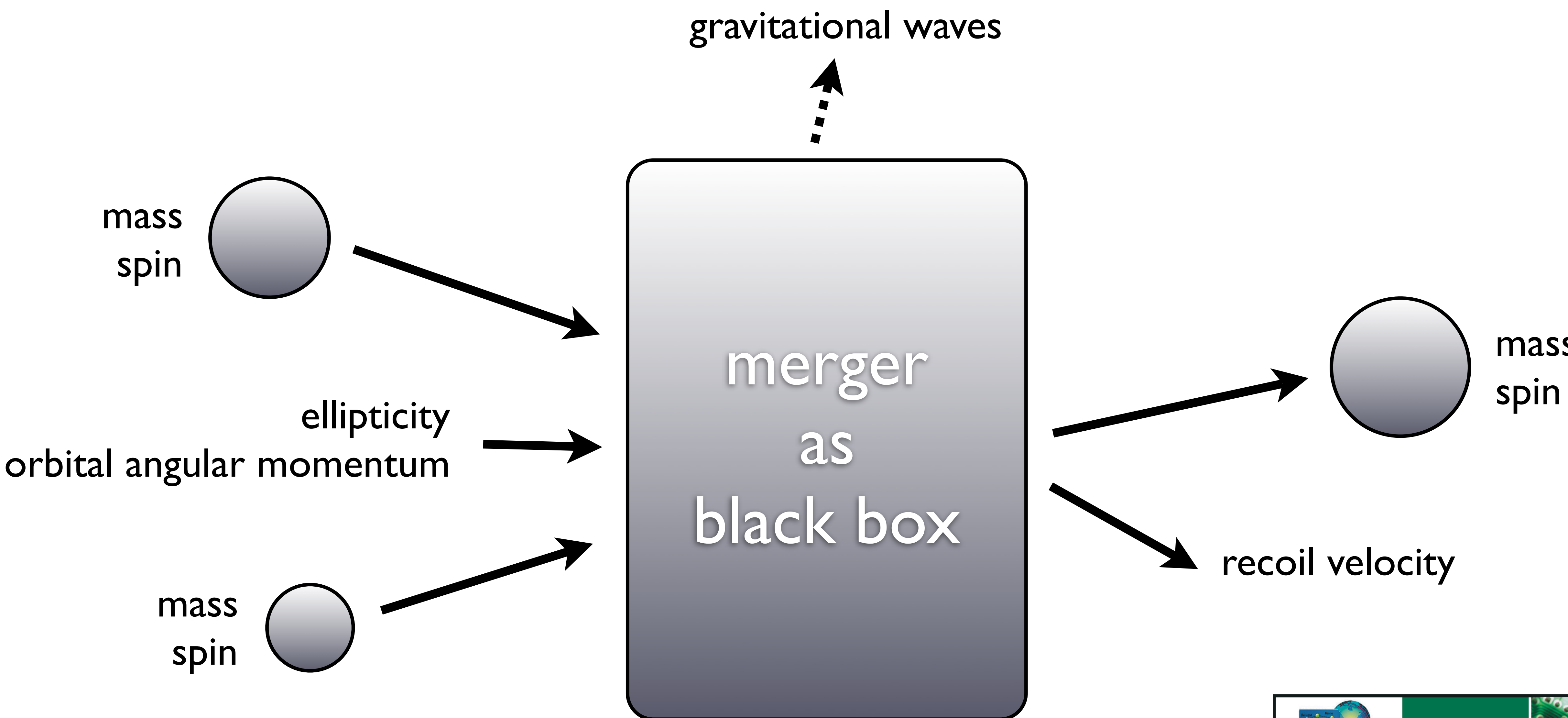
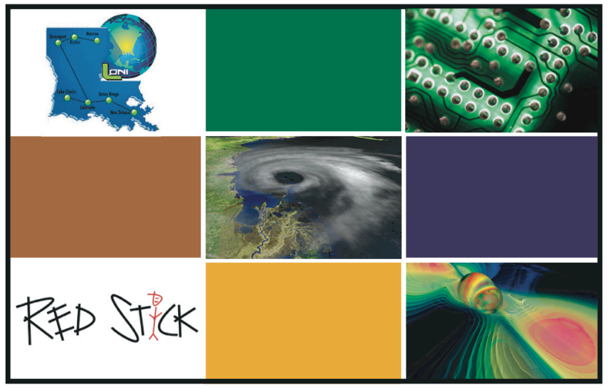
Binary Black Hole Mergers as Black Box



Merger determined by nine parameters

Goal: Find Analytic Description for Final State

- Full numerical treatment for all parameter values is too expensive – use fitting functions instead
- Previous work: Campanelli et al. 2007 (large recoils), González et al. 2007 (non-spinning)
- Take analytic approximations for special cases (e.g. extreme mass ratios) into account
- Initially, restrict parameter space



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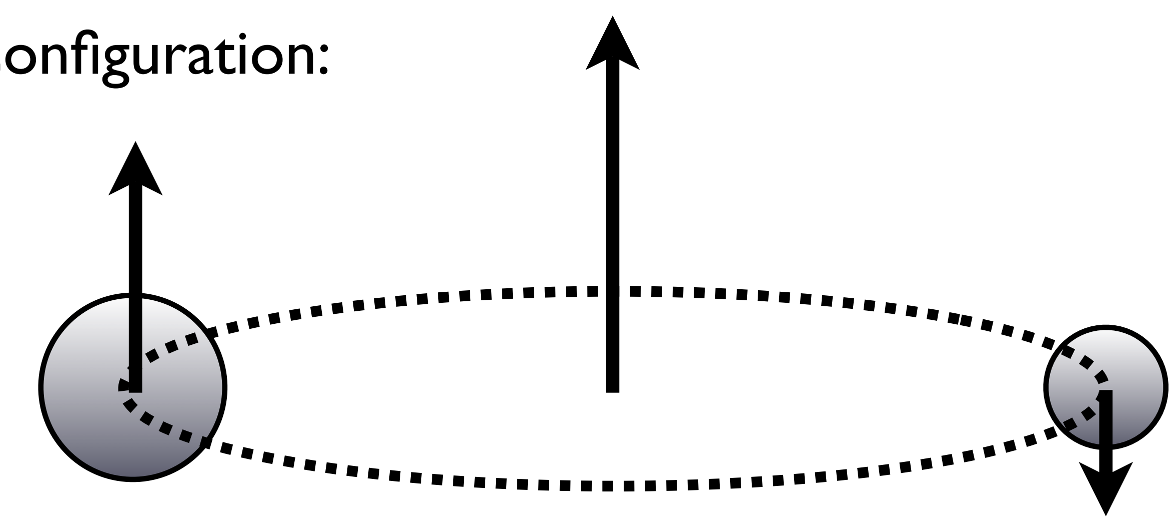
Assumptions

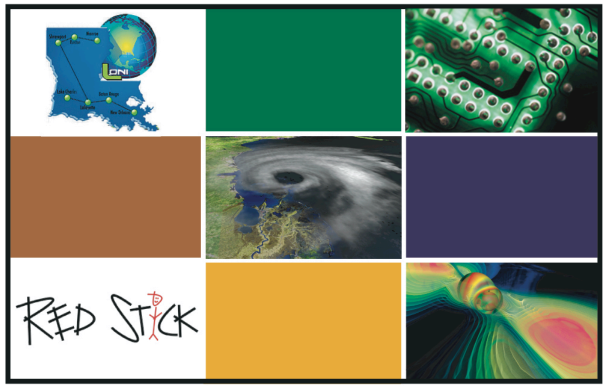
- Masses equal or unequal
- Circular orbits
- Spins aligned and/or anti-aligned with orbital angular momentum

Binary Configuration:



Some Arguments for these Assumptions

- Altogether, considering 3 out of 9 parameters
- However, these may well represent *preferred configurations*, especially for SMBH:
- PN spin-spin and spin-orbit interactions align spins for small inclinations (Schnittmann 2007)
- In discs (expected for SMBH), torques exerted by the matter produce alignment before merger (Bogdanović et al. 2007)
- Most galaxies have SMBH, i.e., there were no large recoils in mergers



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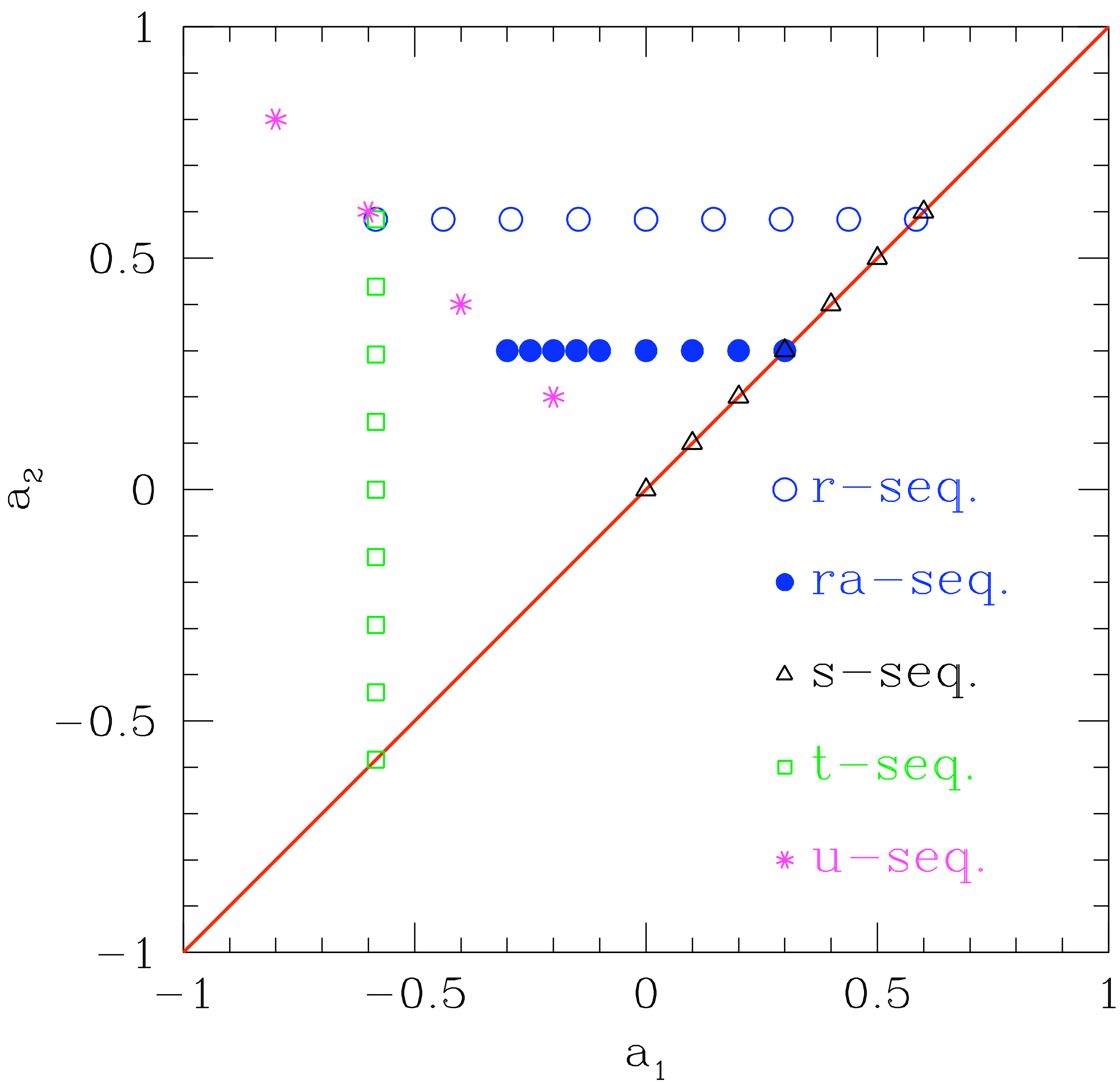
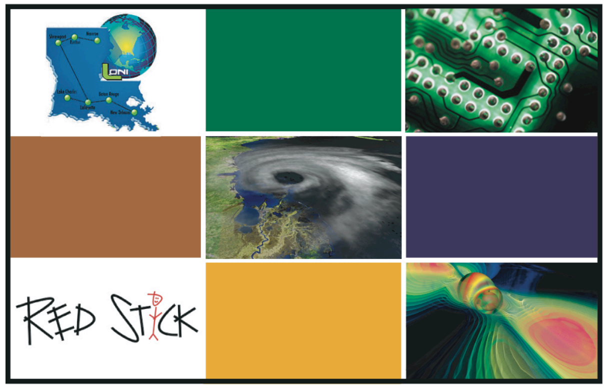
Equal Mass Case: Spin Diagrams



	$\pm x/M$	$\pm p/M$	m_1/M	m_2/M	a_1	a_2	$\widetilde{M}_{\rm ADM}$	$\widetilde{J}_{\rm ADM}$	$\lvert v_{\rm kick}\rvert$	$\lvert v_{\rm kick}^{\rm fit}\rvert$	err. (%)	$a_{\rm fin}$	$a_{\rm fin}^{\rm fit}$	err. (%)
$r0$	3.0205	0.1366	0.4011	0.4009	-0.584	0.584	0.9856	0.825	261.75	258.09	1.40	0.6891	0.6883	0.12
$r1$	3.1264	0.1319	0.4380	0.4016	-0.438	0.584	0.9855	0.861	221.38	219.04	1.06	0.7109	0.7105	0.06
$r2$	3.2198	0.1281	0.4615	0.4022	-0.292	0.584	0.9856	0.898	186.18	181.93	2.28	0.7314	0.7322	0.11
$r3$	3.3190	0.1243	0.4749	0.4028	-0.146	0.584	0.9857	0.935	144.02	146.75	1.90	0.7516	0.7536	0.27
$r4$	3.4100	0.1210	0.4796	0.4034	0.000	0.584	0.9859	0.971	106.11	113.52	6.98	0.7740	0.7747	0.08
$r5$	3.5063	0.1176	0.4761	0.4040	0.146	0.584	0.9862	1.007	81.42	82.23	1.00	0.7948	0.7953	0.06
$r6$	3.5988	0.1146	0.4638	0.4044	0.292	0.584	0.9864	1.044	45.90	52.88	15.21	0.8150	0.8156	0.07
$r7$	3.6841	0.1120	0.4412	0.4048	0.438	0.584	0.9867	1.081	20.59	25.47	23.70	0.8364	0.8355	0.11
$r8$	3.7705	0.1094	0.4052	0.4052	0.584	0.584	0.9872	1.117	0.00	0.00	0.00	0.8550	0.855	0.00
$ra0$	2.9654	0.1391	0.4585	0.4584	-0.300	0.300	0.9845	0.8250	131.34	132.58	0.95	0.6894	0.6883	0.16
$ra1$	3.0046	0.1373	0.4645	0.4587	-0.250	0.300	0.9846	0.8376	118.10	120.28	1.85	0.6971	0.6959	0.17
$ra2$	3.0438	0.1355	0.4692	0.4591	-0.200	0.300	0.9847	0.8499	106.33	108.21	1.77	0.7047	0.7035	0.17
$ra3$	3.0816	0.1339	0.4730	0.4594	-0.150	0.300	0.9848	0.8628	94.98	96.36	1.46	0.7120	0.7111	0.13
$ra4$	3.1215	0.1321	0.4757	0.4597	-0.100	0.300	0.9849	0.8747	84.74	84.75	0.01	0.7192	0.7185	0.09
$ra6$	3.1988	0.1290	0.4782	0.4602	0.000	0.300	0.9850	0.9003	63.43	62.19	1.95	0.7331	0.7334	0.04
$ra8$	3.2705	0.1261	0.4768	0.4608	0.100	0.300	0.9852	0.9248	41.29	40.55	1.79	0.7471	0.7481	0.13
$ra10$	3.3434	0.1234	0.4714	0.4612	0.200	0.300	0.9853	0.9502	19.11	19.82	3.72	0.7618	0.7626	0.11
$ra12$	3.4120	0.1209	0.4617	0.4617	0.300	0.300	0.9855	0.9750	0.00	0.00	0.00	0.7772	0.7769	0.03
$s0$	2.9447	0.1401	0.4761	0.4761	0.000	0.000	0.9844	0.8251	0.00	0.00	0.00	0.6892	0.6883	0.13
$s1$	3.1106	0.1326	0.4756	0.4756	0.100	0.100	0.9848	0.8749	0.00	0.00	0.00	0.7192	0.7185	0.09
$s2$	3.2718	0.1261	0.4709	0.4709	0.200	0.200	0.9851	0.9251	0.00	0.00	0.00	0.7471	0.7481	0.13
$s3$	3.4098	0.1210	0.4617	0.4617	0.300	0.300	0.9855	0.9751	0.00	0.00	0.00	0.7772	0.7769	0.03
$s4$	3.5521	0.1161	0.4476	0.4476	0.400	0.400	0.9859	1.0250	0.00	0.00	0.00	0.8077	0.8051	0.33
$s5$	3.6721	0.1123	0.4276	0.4276	0.500	0.500	0.9865	1.0748	0.00	0.00	0.00	0.8340	0.8325	0.18
$s6$	3.7896	0.1088	0.4002	0.4002	0.600	0.600	0.9874	1.1246	0.00	0.00	0.00	0.8583	0.8592	0.11
$t0$	4.1910	0.1074	0.4066	0.4064	-0.584	0.584	0.9889	0.9002	259.49	258.09	0.54	0.6868	0.6883	0.22
$t1$	4.0812	0.1103	0.4062	0.4426	-0.584	0.438	0.9884	0.8638	238.37	232.62	2.41	0.6640	0.6658	0.27
$t2$	3.9767	0.1131	0.4057	0.4652	-0.584	0.292	0.9881	0.8265	200.25	205.21	2.48	0.6400	0.6429	0.45
$t3$	3.8632	0.1165	0.4053	0.4775	-0.584	0.146	0.9879	0.7906	174.58	175.86	0.73	0.6180	0.6196	0.26
$t4$	3.7387	0.1204	0.4047	0.4810	-0.584	0.000	0.9878	0.7543	142.62	144.57	1.37	0.5965	0.5959	0.09
$t5$	3.6102	0.1246	0.4041	0.4761	-0.584	-0.146	0.9876	0.7172	106.36	111.34	4.68	0.5738	0.5719	0.33
$t6$	3.4765	0.1294	0.4033	0.4625	-0.584	-0.292	0.9874	0.6807	71.35	76.17	6.75	0.5493	0.5475	0.32
$t7$	3.3391	0.1348	0.4025	0.4387	-0.584	-0.438	0.9873	0.6447	35.36	39.05	10.45	0.5233	0.5227	0.11
$t8$	3.1712	0.1419	0.4015	0.4015	-0.584	-0.584	0.9875	0.6080	0.00	0.00	0.00	0.4955	0.4976	0.42
$u1$	2.9500	0.1398	0.4683	0.4685	-0.200	0.200	0.9845	0.8248	87.34	88.39	1.20	0.6893	0.6883	0.15
$u2$	2.9800	0.1384	0.4436	0.4438	-0.400	0.400	0.9846	0.8249	175.39	176.78	0.79	0.6895	0.6883	0.17
$u3$	3.0500	0.1355	0.3951	0.3953	-0.600	0.600	0.9847	0.8266	266.39	265.16	0.46	0.6884	0.6883	0.01
$u4$	3.1500	0.1310	0.2968	0.2970	-0.800	0.800	0.9850	0.8253	356.87	353.55	0.93	0.6884	0.6883	0.01



Fitting Functions (Recoil)

Generic quadratic ansatz:

$$|v_{\rm kick}| = |c_0 + c_1 a_1 + c_2 a_1^2 + d_0 a_1 a_2 + d_1 a_2 + d_2 a_2^2|$$

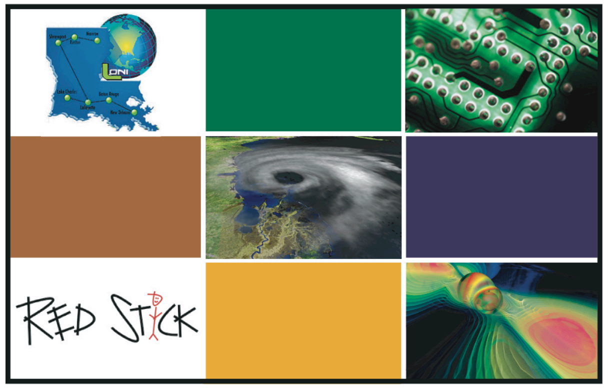
Using symmetries of the system,
and assuming that no kick is produced when the spins are equal and opposite:

$$c_0 = 0\,, \quad c_1 = -d_1\,, \quad c_2 = -d_2\,, \quad d_0 = 0\,,$$

$$|v_{\rm kick}| = |c_1 (a_1 - a_2) + c_2 (a_1^2 - a_2^2)|$$

Least square fitting result:

$$c_1 = -220.97 \pm 0.78\,, \quad c_2 = 45.52 \pm 2.99$$



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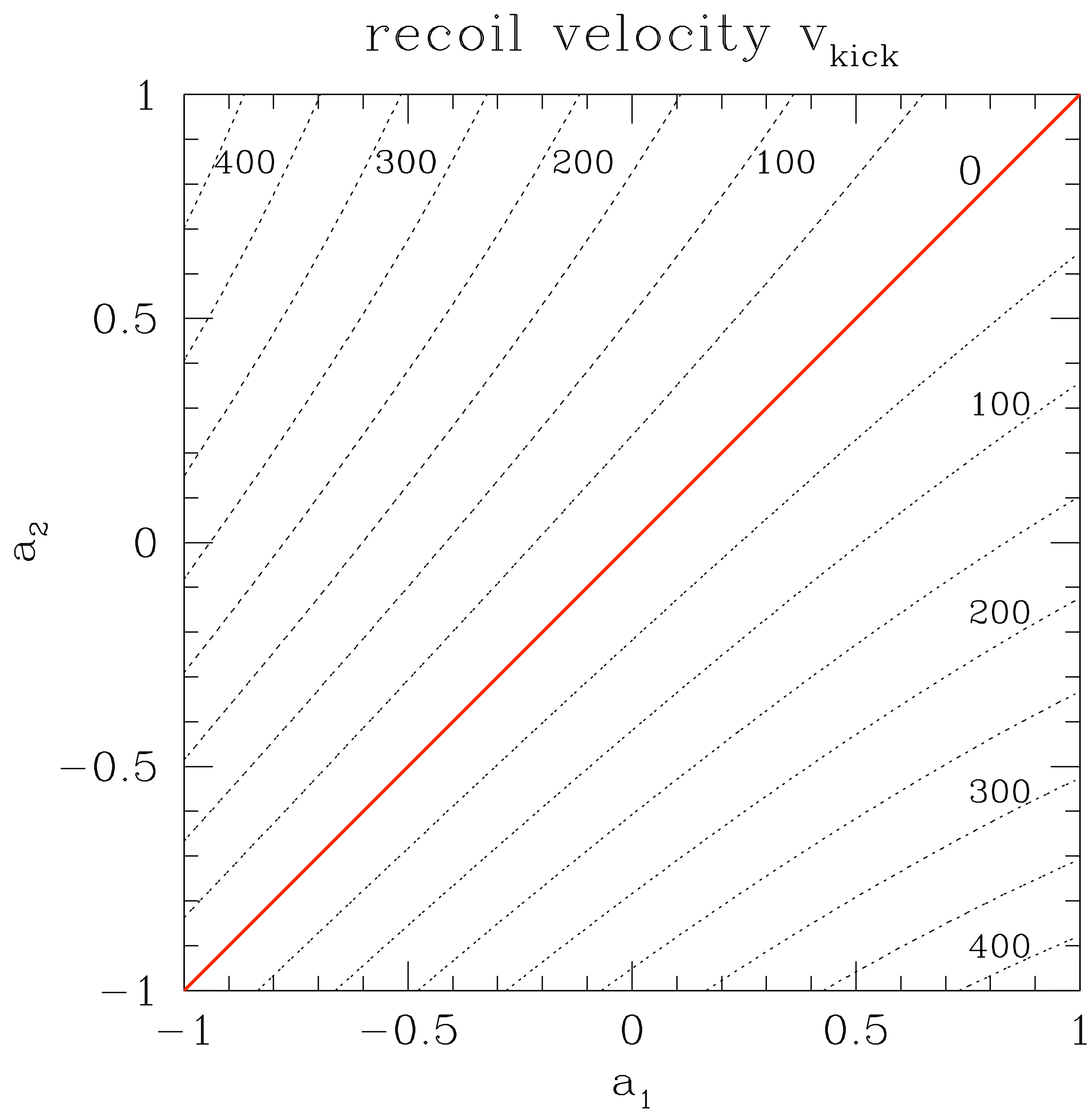
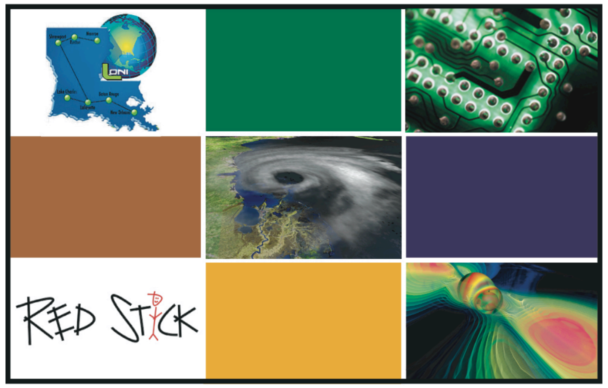
Fitting Functions (Recoil)

- Results consistent with brute-force fitting functions
- Maximum recoil about 440 km/s for anti-aligned spins
- Quadratic behaviour: improvement over linear PN predictions
- Very good agreement with other numerical studies





Fitting Functions (Final Spin)

Final spin can be treated equivalently:

$$a_{\rm fin} = p_0 + p_1 a_1 + p_2 a_1^2 + q_0 a_1 a_2 + q_1 a_2 + q_2 a_2^2$$

Using symmetries,
and assuming that equal and opposite spins cancel:

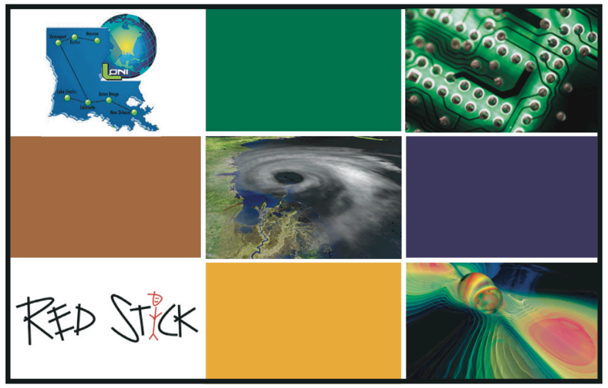
$$p_1 = q_1\,, \qquad p_2 = q_2 = q_0/2$$

$$a_{\rm fin} = p_0 + p_1(a_1 + a_2) + p_2(a_1 + a_2)^2$$

Least square fitting result:

$$p_0 = 0.6883 \pm 0.0003\,, \qquad p_1 = 0.1530 \pm 0.0004\,,$$

$$p_2 = -0.0088 \pm 0.0005\,,$$



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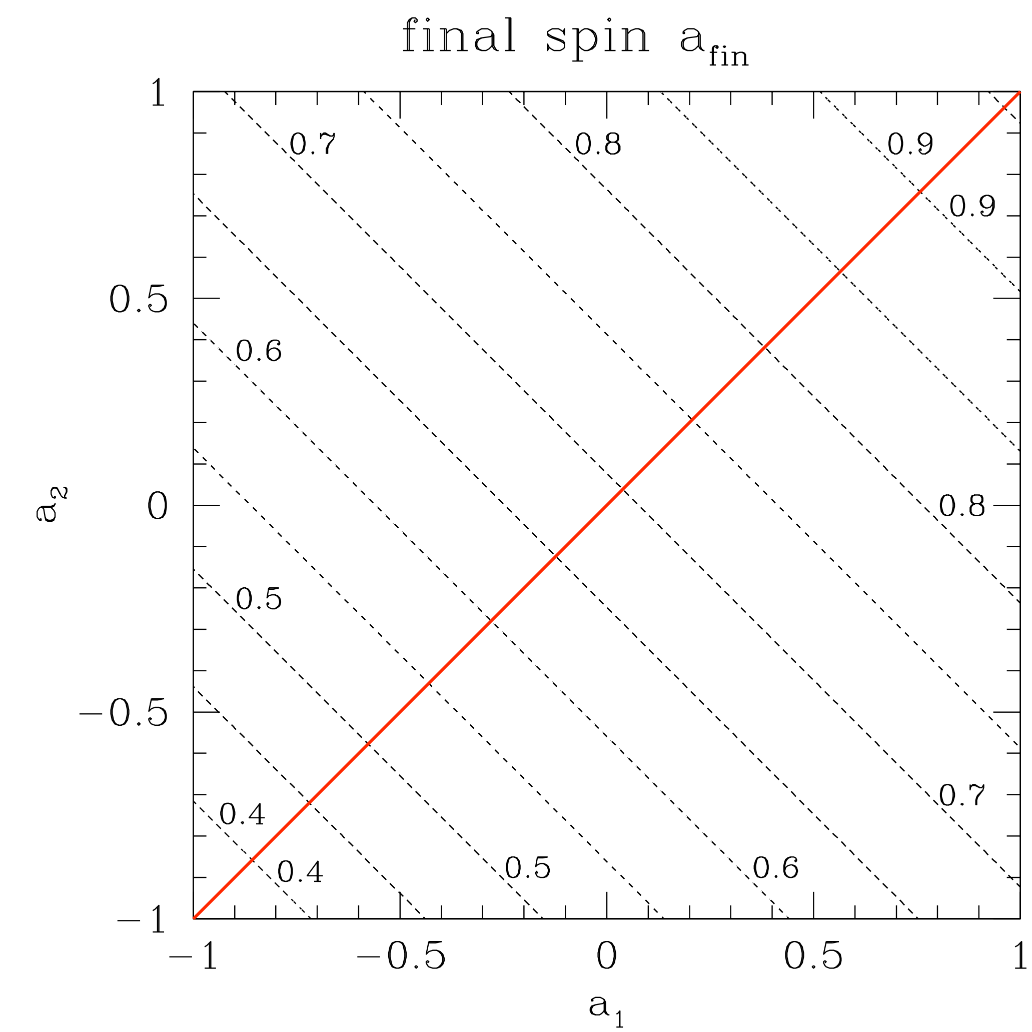
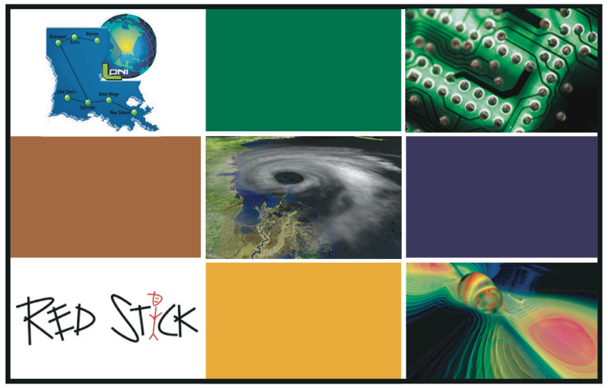
Fitting Functions (Final Spin)

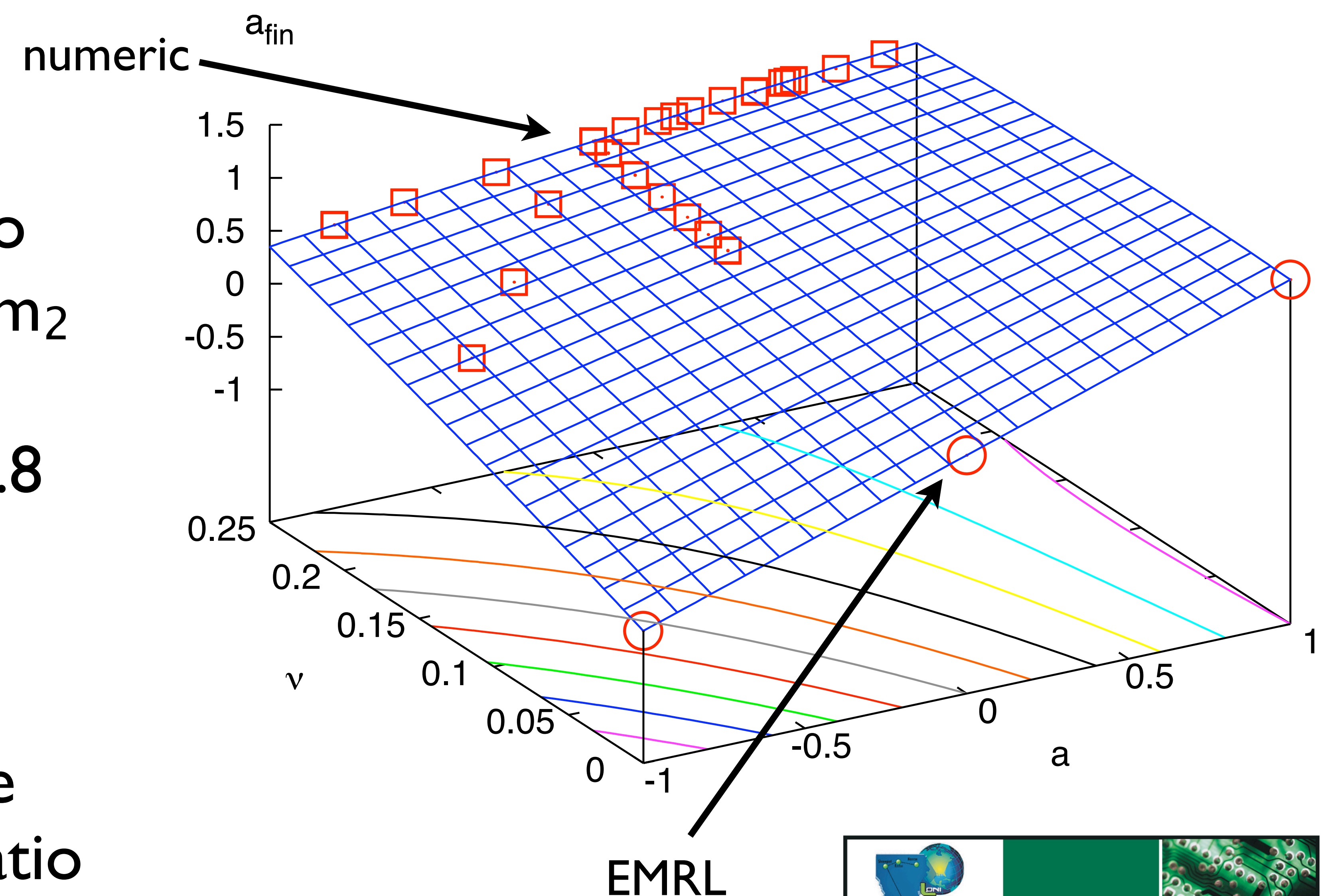
- Again, results consistent with brute-force fitting functions
- Possible final spins approximately in [0.35 ... 0.96]
- Quadratic term possibly zero
- No local maxima as suggested by EOB (Damour 2001)

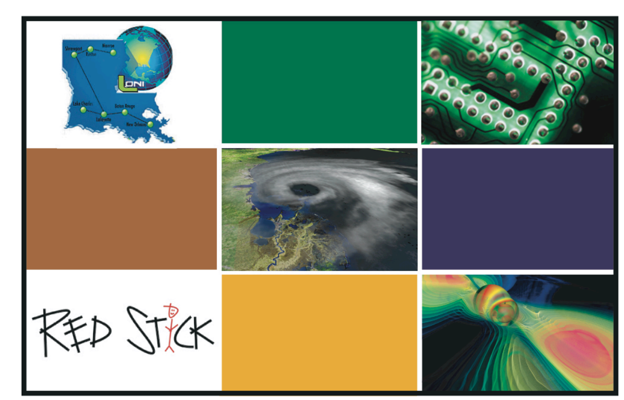




Unequal Mass, Equal Spin Case

- ν: symmetric mass ratio
  $\nu = q/(1+q)^2, \quad q = m_1/m_2$
- $\nu <\sim 0.16$ and $|a| >\sim 0.8$ difficult to access for numerical relativity
- Use analytic knowledge about extreme mass ratio limit (ν=0)





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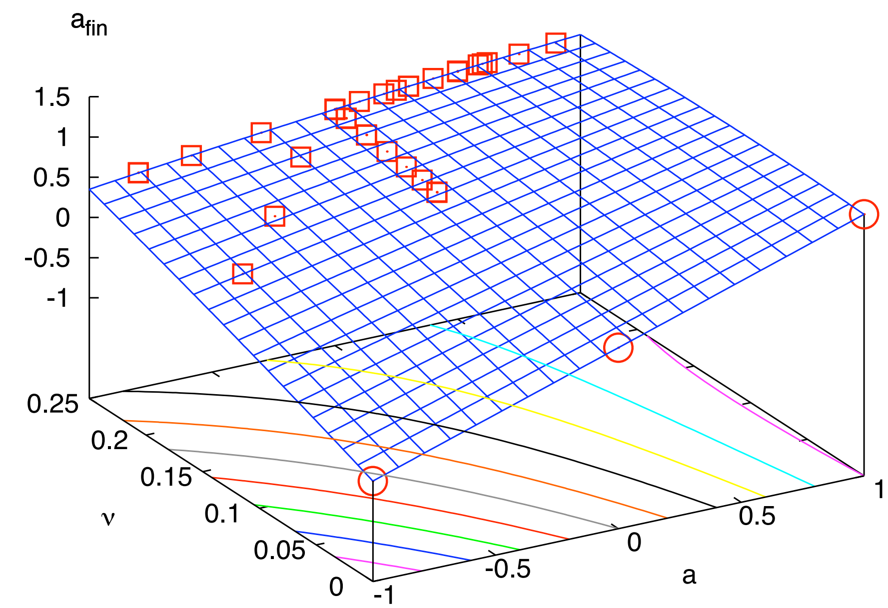
Unequal Mass Case (Final Spin)



Generic cubic ansatz:

$$a_{\rm fin} = s_0 + s_1 a + s_2 a^2 + s_3 a^3 + s_4 a^2 \nu + s_5 a \nu^2 + t_0 a \nu + t_1 \nu + t_2 \nu^2 + t_3 \nu^3 \,.$$

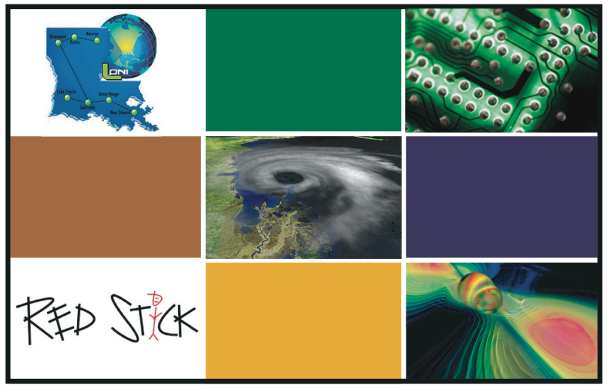
EMRL: $a_{\rm fin}(a, \nu = 0) = a$

$$s_0 = s_2 = s_3 = 0 \,, \qquad s_1 = 1$$

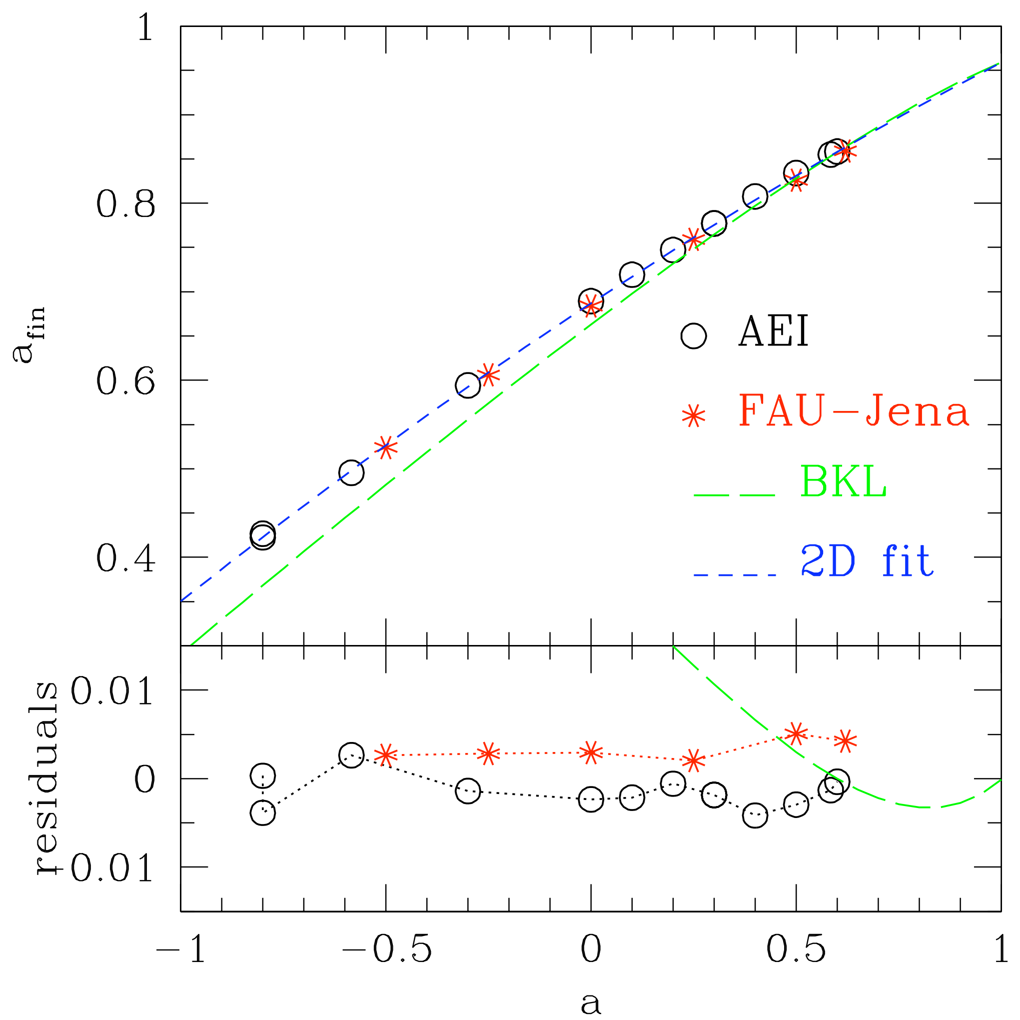
Choose also t_1 from EMRL via Taylor expansion about $\nu=0$

$$a_{\rm fin} = a + s_4 a^2 \nu + s_5 a \nu^2 + t_0 a \nu + 2\sqrt{3}\nu + t_2 \nu^2 + t_3 \nu^3$$

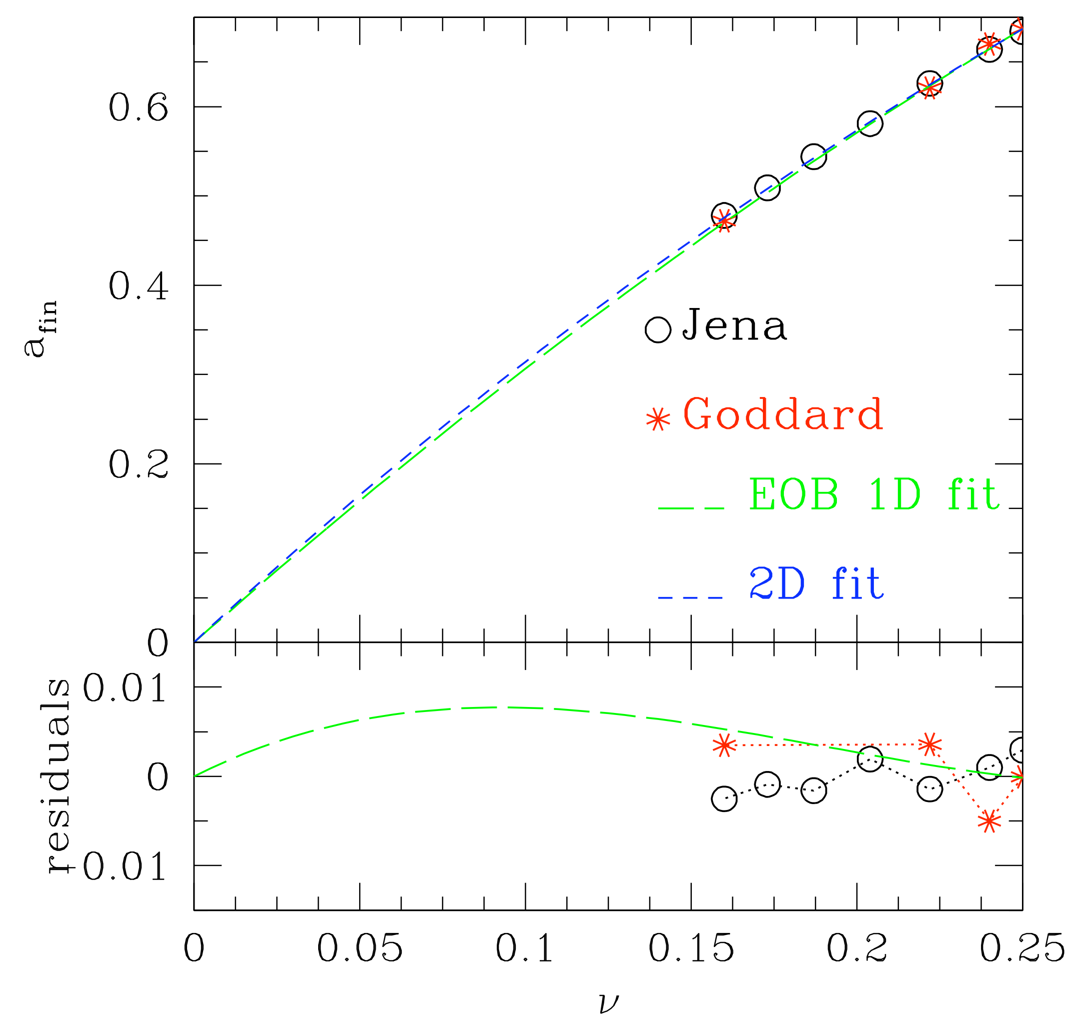
- Cubic spin dependence drops out
- Possible final spins approximately in [0.35 ... 0.96]



Comparison with Numerical Results

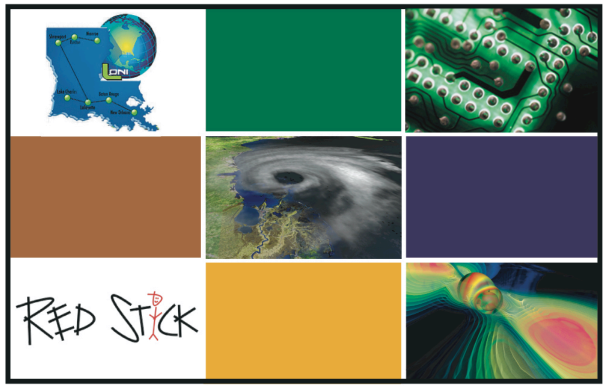


equal mass



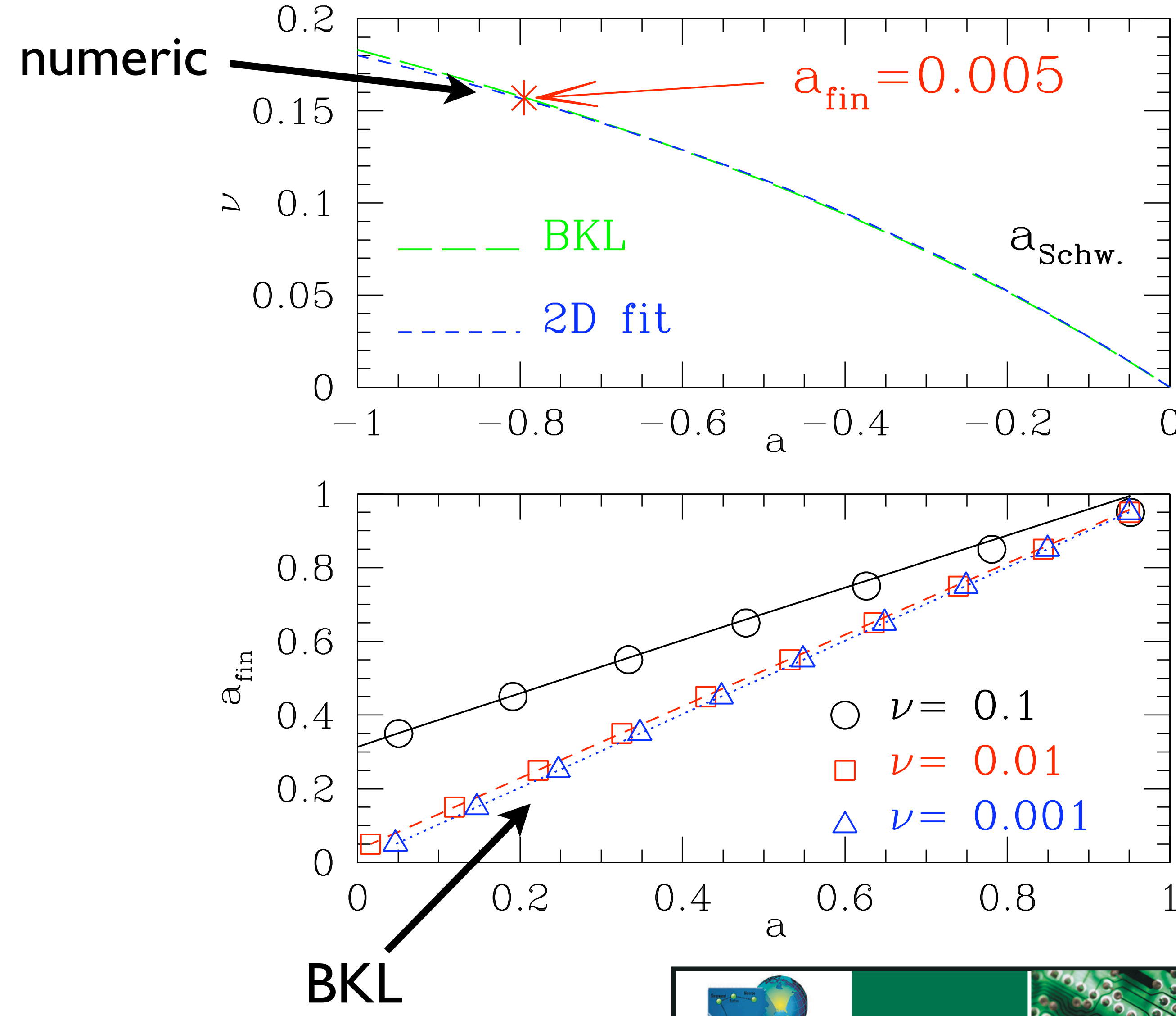
non-spinning

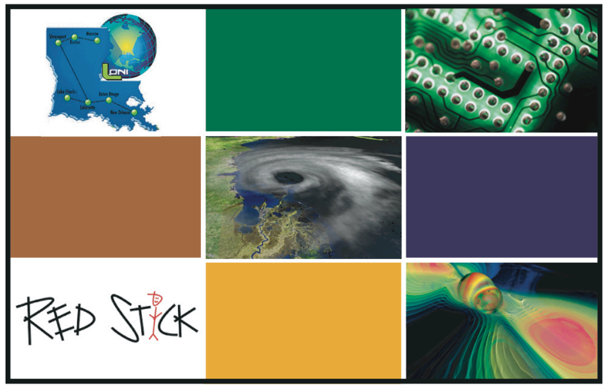
Impressive accuracy!



Producing a Schwarzschild Black Hole

- Just ask for *a*[fin] = 0 in fitting function
- Producing a non-spinning final black hole requires both unequal masses and initial non-zero spin
- Confirmed by Berti et al. 2007





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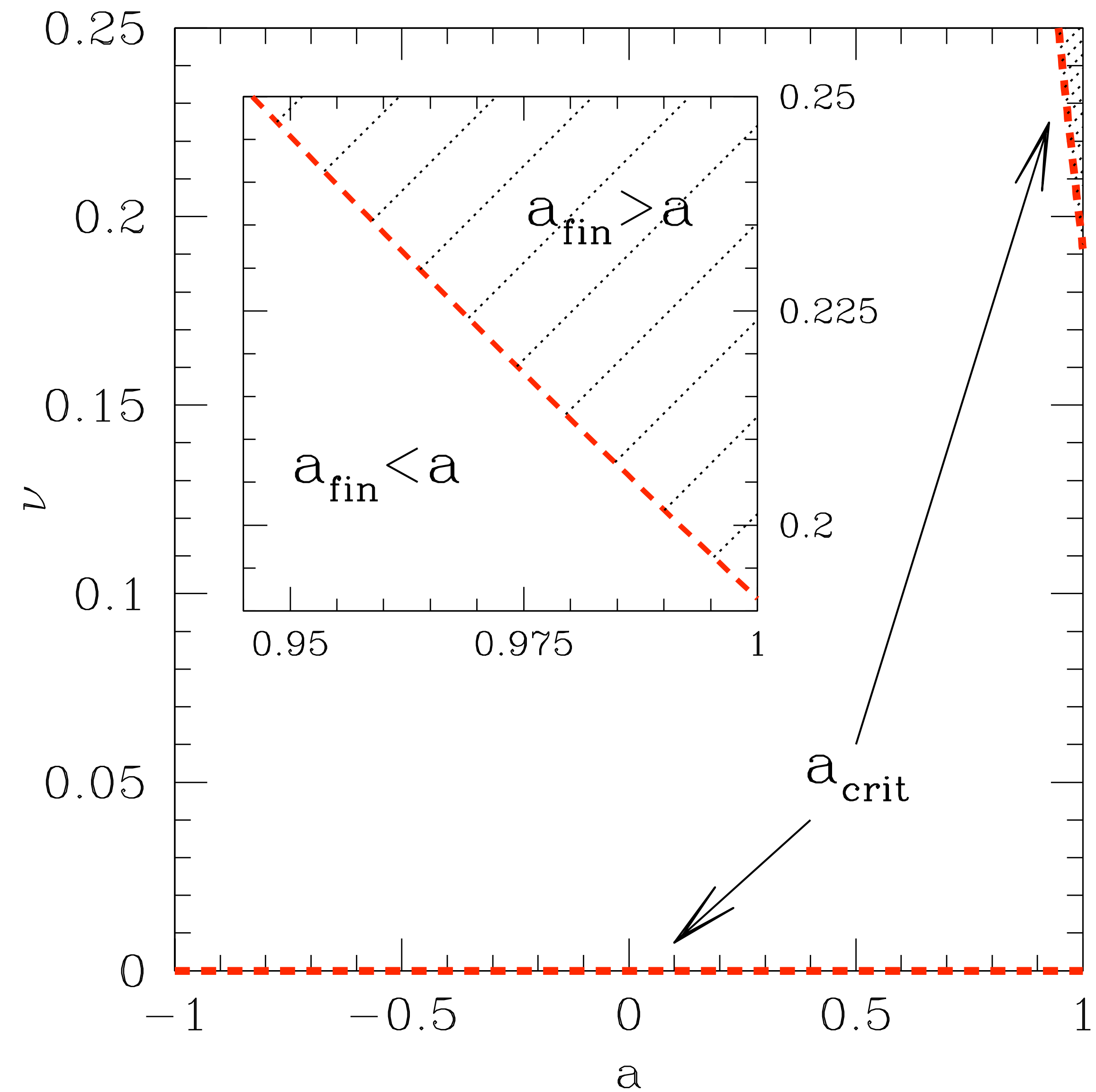
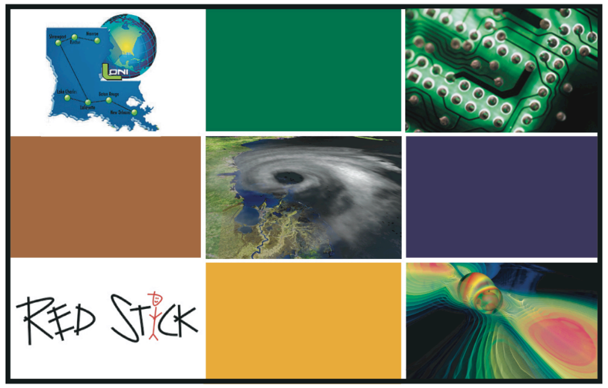
Spin-Up or Spin-Down?

- Just ask for *a*[fin] = *a* in fitting function
- Almost all equal-spin configurations are *spun down*
- Note: fitting function is unphysical near |*a*| = 1, probably due to extrapolation





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Consistency Check

We have now two expressions for the final spin:

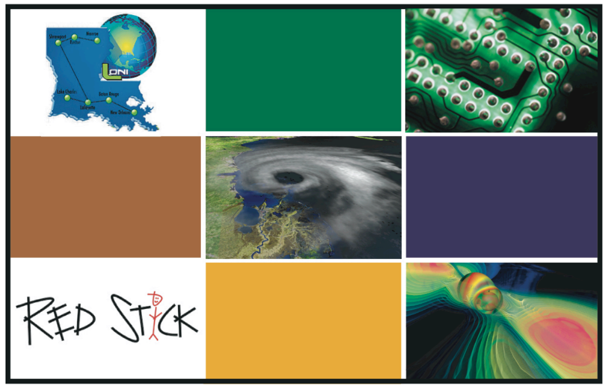
Equal mass (ν=0.25):

$$a_{\rm fin} = p_0 + p_1(a_1 + a_2) + p_2(a_1 + a_2)^2$$

Equal spin (a_1=a_2):

$$a_{\rm fin} = a + s_4 a^2 \nu + s_5 a \nu^2 + t_0 a \nu + 2\sqrt{3}\nu + t_2 \nu^2 + t_3 \nu^3$$

Are they consistent?
Answer: *yes*, to within the numerical error bars!



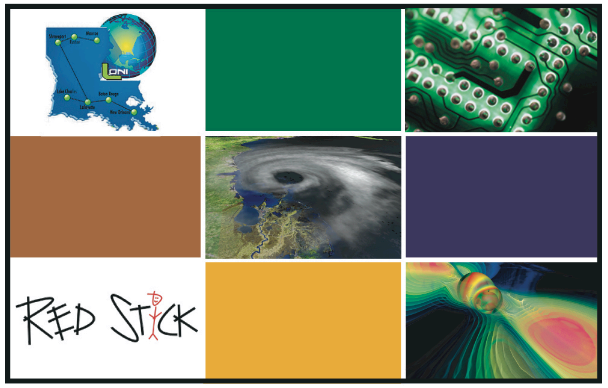
Generalisation: Unequal Mass and Unequal Spin Case

- Simplest ansatz to handle both unequal masses and unequal spins, and which recovers the correct limits:

  Replace a by $(a_1 + a_2 q^2) / (1+q^2)$, $q = m_1/m_2$

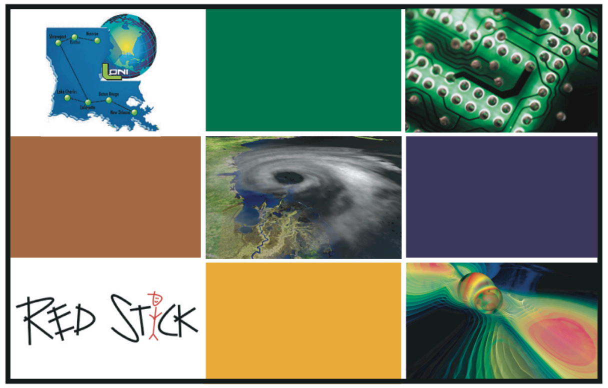
- Work underway to verify this ansatz

- Work underway to handle misaligned spins (see arXiv:0712.3541 [gr-qc])



Summary: Black Box Binaries

- Consider binary black hole mergers as black box
- Model final state using generic fitting function depending on initial state – cheaper and more accurate than other approximations
- Could e.g. be used for N-body simulations
- Excellent agreement between fitting function and numerical results for unequal (aligned) spins and for unequal masses
- Work underway to generalise this to arbitrary configurations



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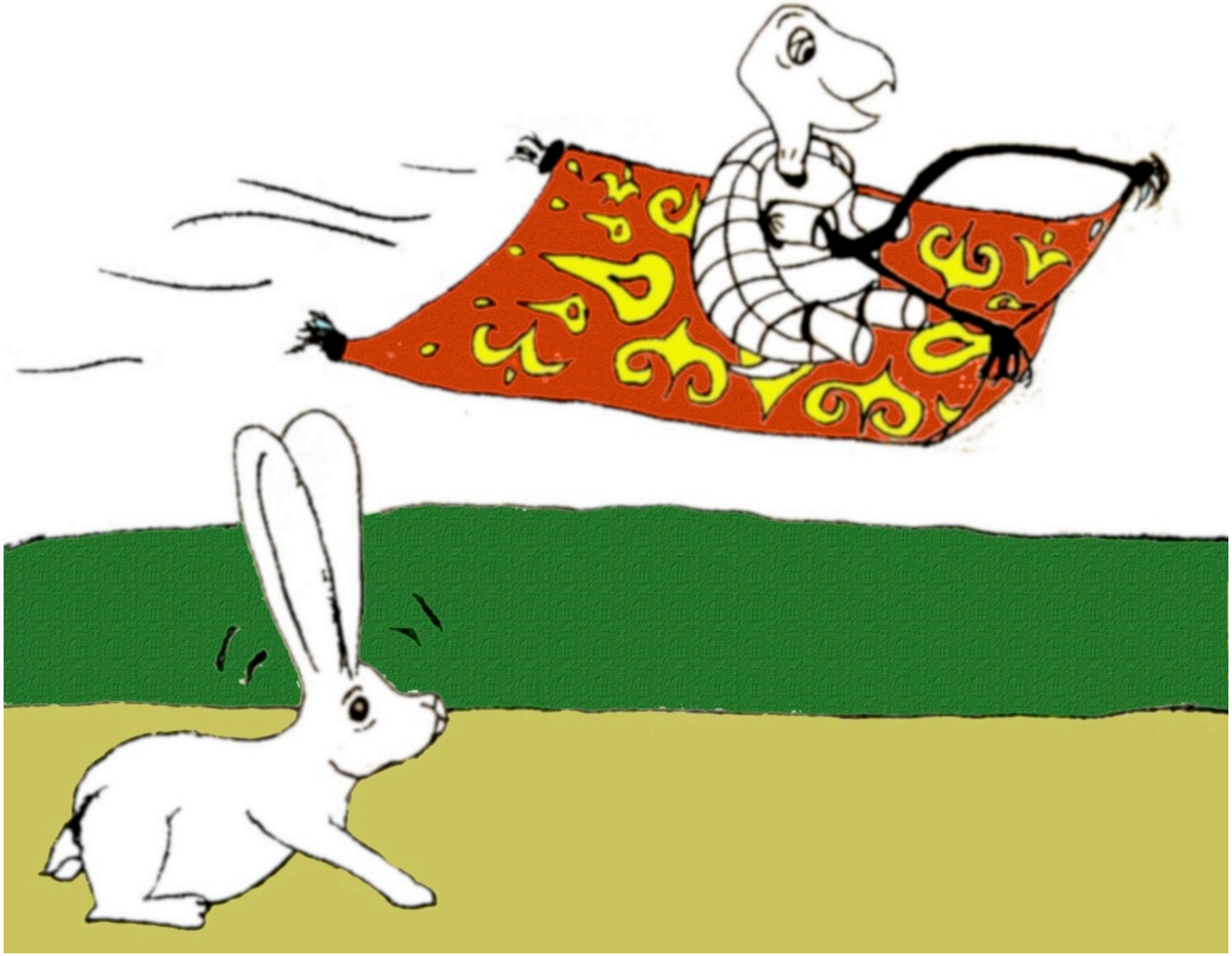
Behind the Scenes



J. Seiler

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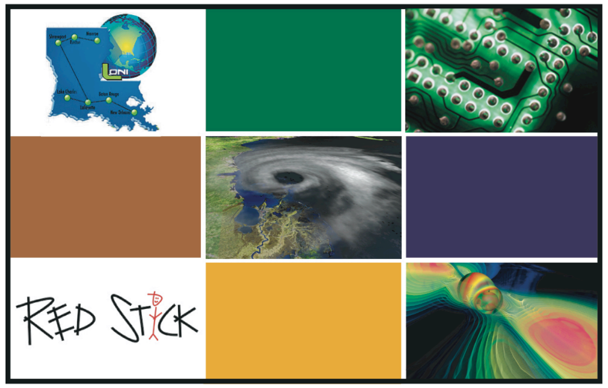
Behind the Scenes: Numerical Infrastructure

- Initial data solver for quasi-circular configuration
- CCATIE: Efficient 4th order BSSN code for time evolution
- Horizon finding, measuring horizon quantities
- Wave extraction (not used here)
- Much of the above is public,
  part of the *Einstein Toolkit*



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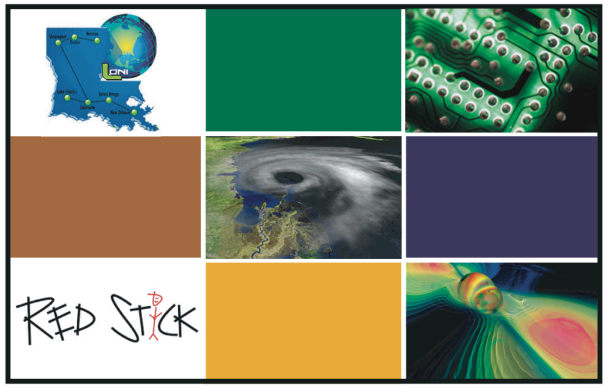
Behind the Scenes: Computational Infrastructure

- Large scale differences and moving objects require adaptive mesh refinement (AMR)
[typical: L=1000, h=0.02, using 9 refinement levels]
- Long time evolutions and desired accuracy require high order methods (4th order or higher)
- Same infrastructure (Cactus, Carpet) also used for GRMHD simulations
- Computation time/efficiency still an issue



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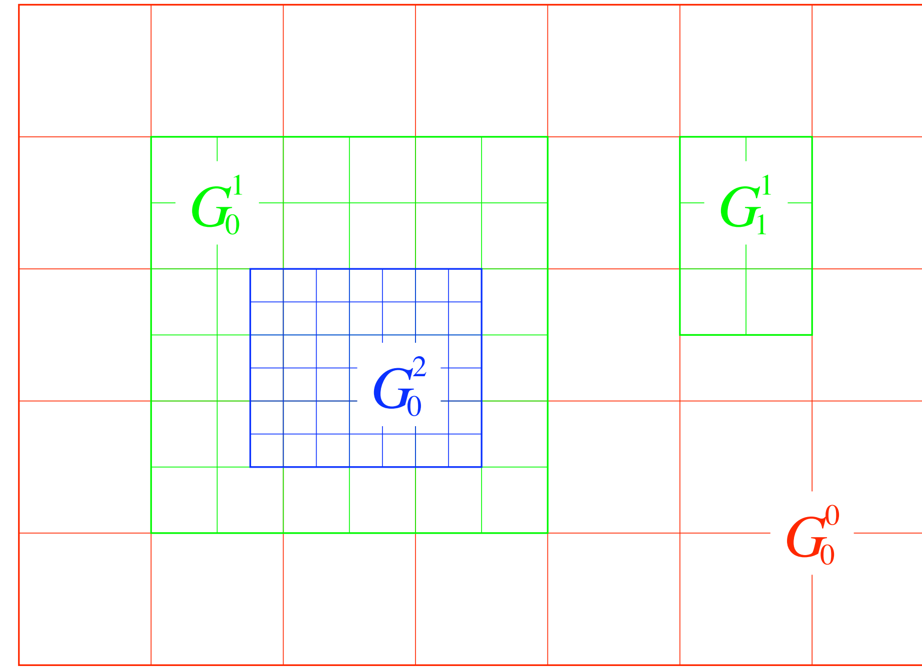
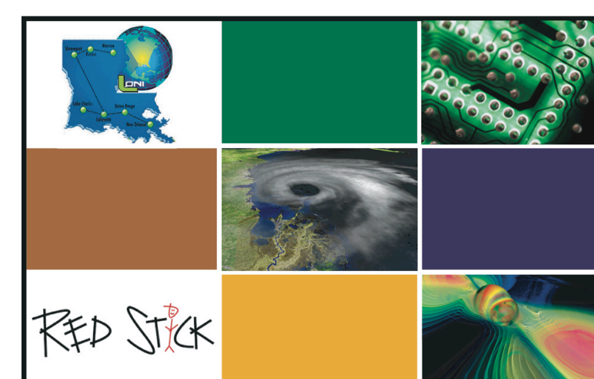
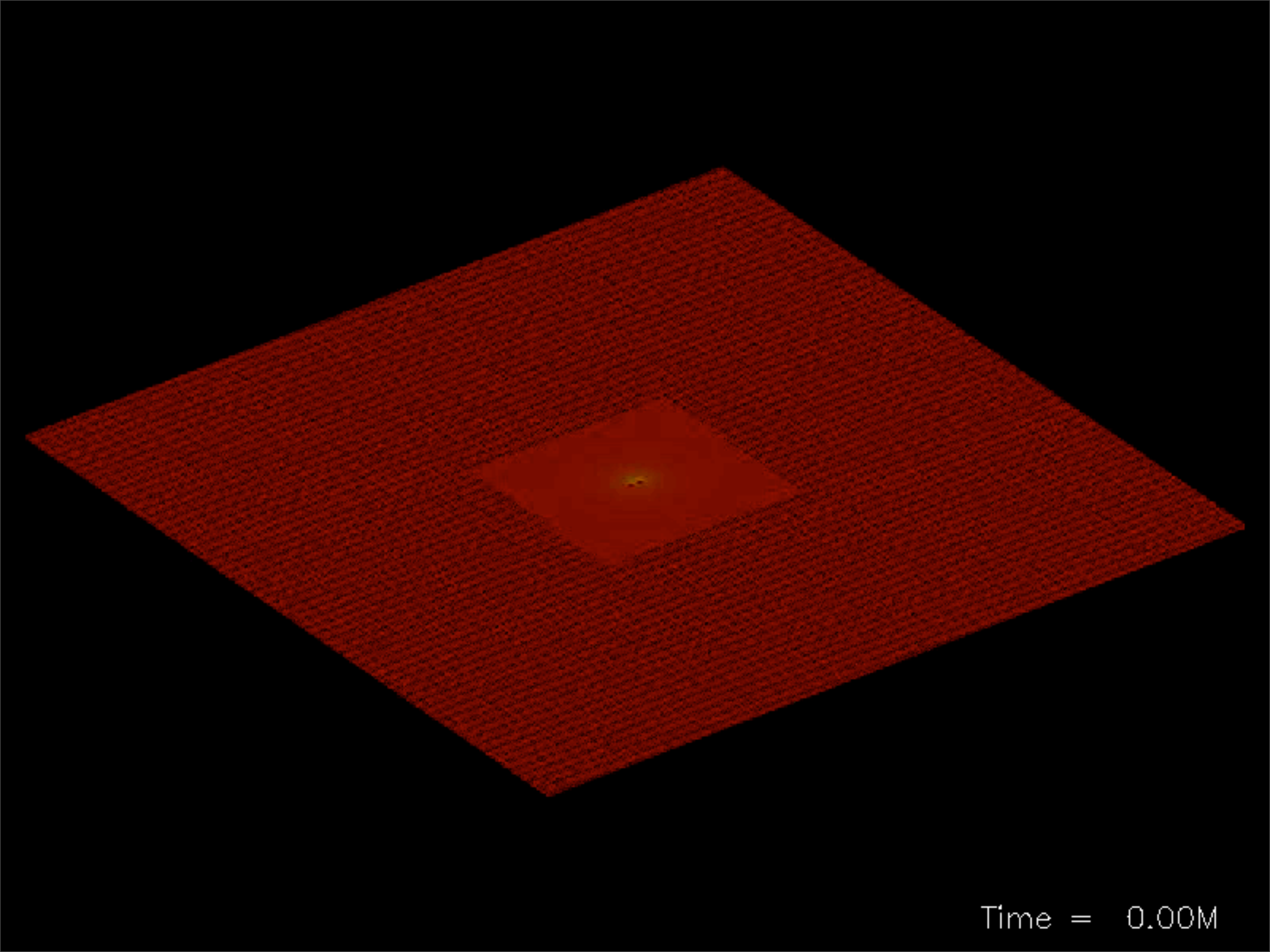
Carpet: Mesh Refinement



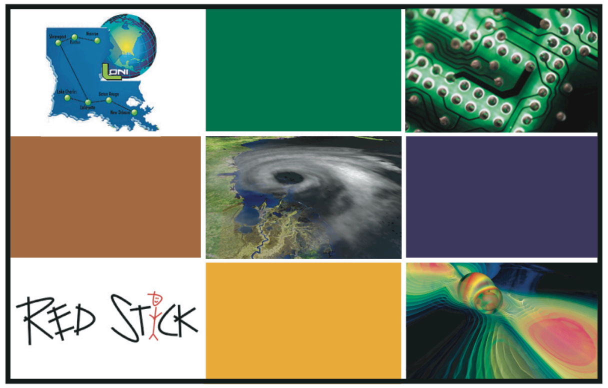
- Berger-Oliger AMR with subcycling in time
- Using *buffer zones* for stable AMR boundaries
- Domain decomposition parallelisation (typically 3 ghost zones – expensive!)
- [Using *tapered grids* to improve accuracy]
- AMR tracks physics features explicitly, refining e.g. around black holes




Time = 0.00M

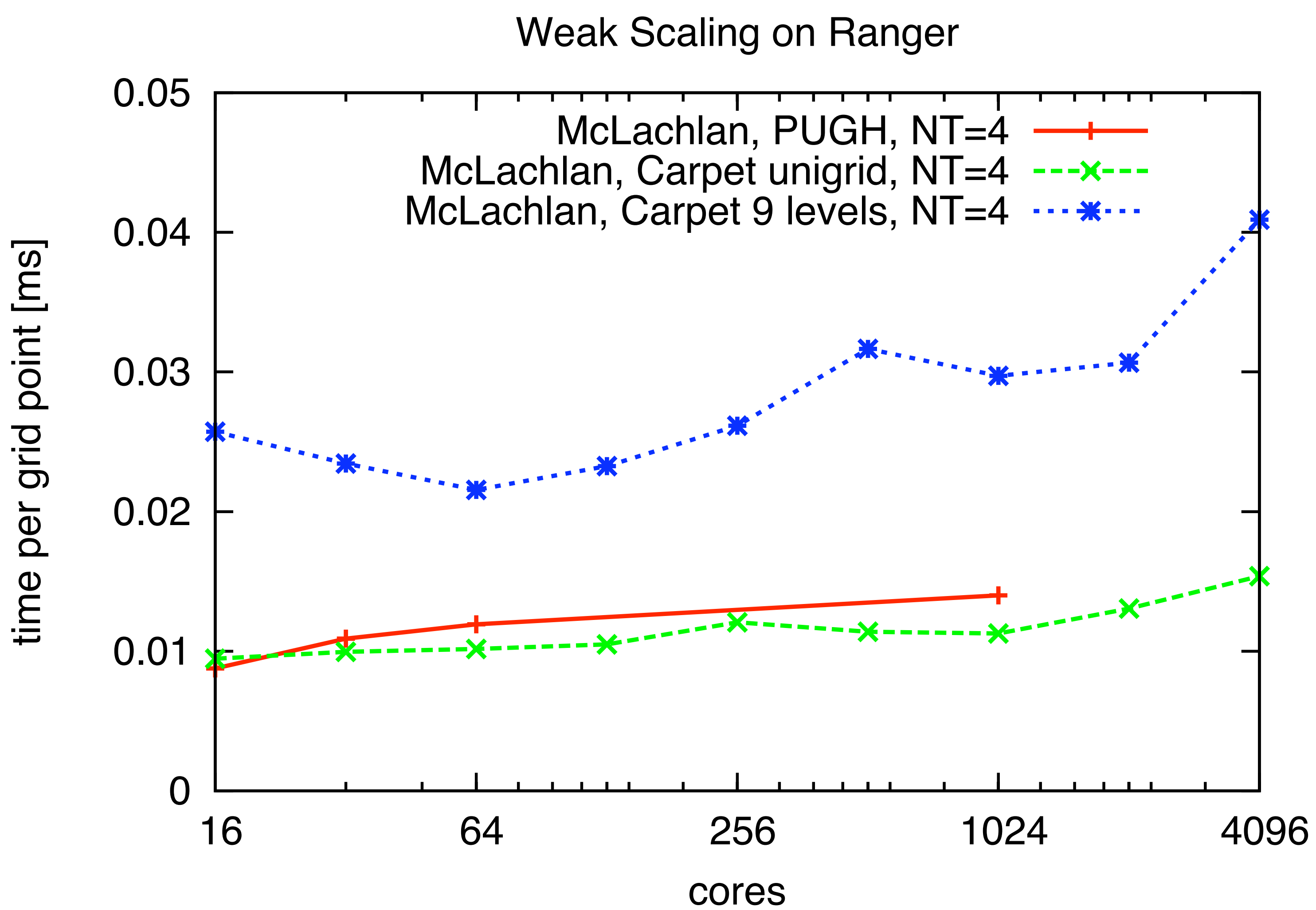
Efficiency and Scalability

- Many variables (~25 evolved, ~250 in total, many ghost zones (higher order):
  - requires much memory
  - few evolved grid points per node – inefficient
- Reduce amount of communication or interpolation
- [Use multi-threading on multi-core machines]
- Currently, a very simple AMR testcase requires already 8 GByte memory

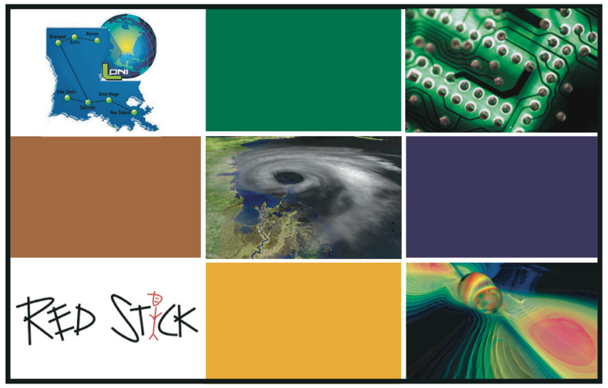


AMR Parallel Scaling on Ranger (TACC)

- Ranger is new petascale machine at TACC with 60,000 cores, still in friendly user mode
- Using OpenMP with 4 threads to reduce parallelisation overhead
- McLachlan: New BSSN code for experimenting with performance (XiRel)



Ranger produces 45 MSU/month!



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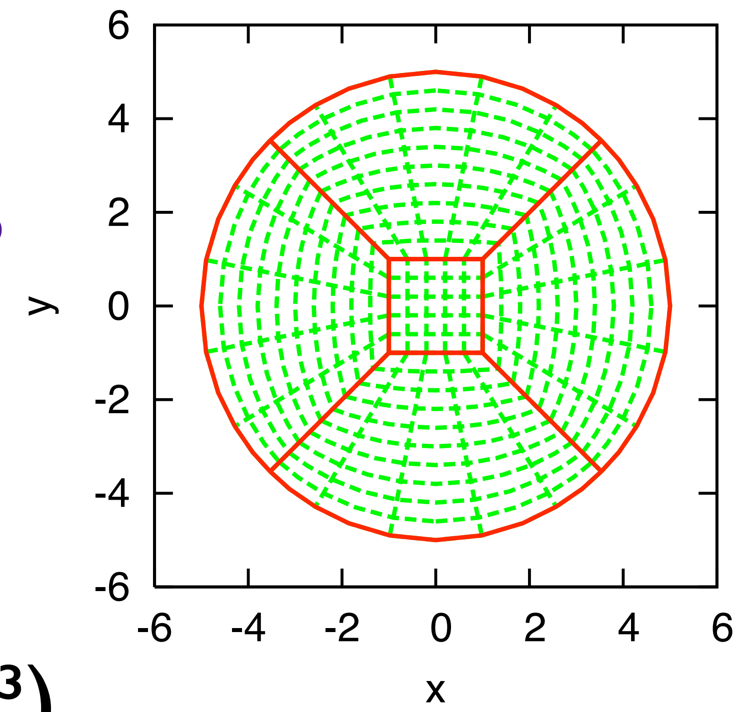
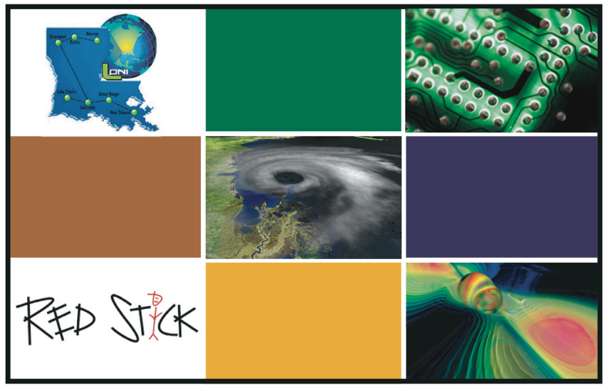
Multi-Patch Methods

not just for spectral codes!



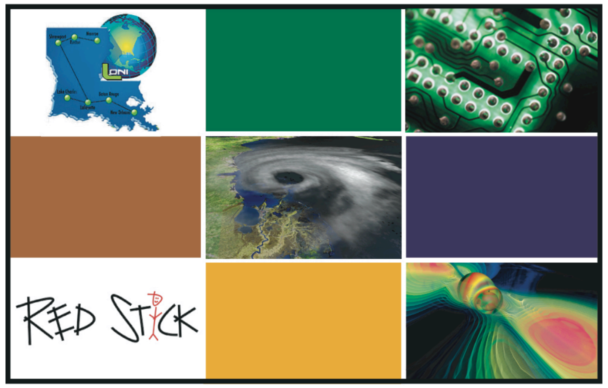
- Increasing AMR domain size scales with $O(L^3)$, increasing resolution scales with $O(1/h^4)$
- Multi-patch systems separate *radial* and *angular* resolutions: significantly more efficient
- With MP: increasing domain size is only $O(L)$, increasing radial resolution is only $O(1/h^2)$
- Plus, outer boundary is spherical – no useless corners, better boundary conditions, better wave extraction ...



Multi-Patch Methods

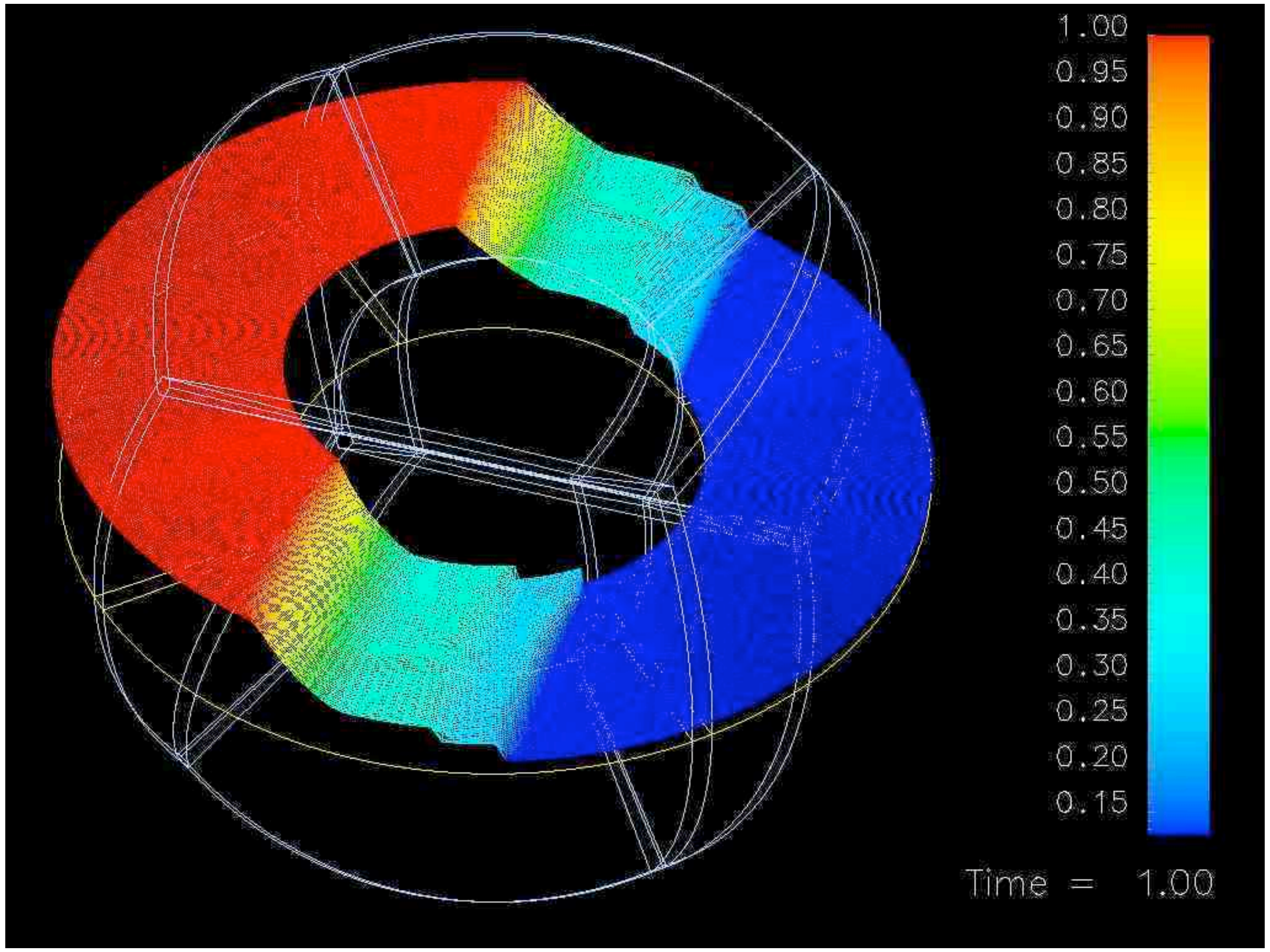
you can do spectral, too!

- Finite differencing is independent of AMR and MP infrastructure
- FD also independent of formulation, of singularity handling, of inter-patch boundary conditions
- Can use spectral methods with MP
- Could also combine spectral angular with FD radial derivatives



Multi-Patch Methods GRMHD

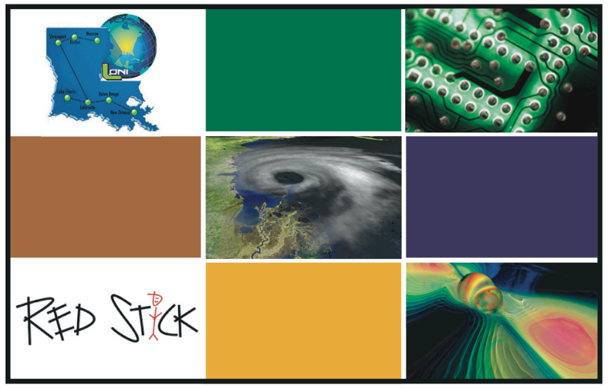
- MP-GRMHD code under development (B. Zink)
- Need to be careful with flux reconstruction on distorted grids and on interfaces



Sod test (1D shock) –
no artefacts at MP boundaries visible

Other Infrastructure

- BBH Factory: scripts/middleware to manage source trees and simulations on many different machines, with a common user interface
- LoopControl: automatic dynamic cache optimisations on multi-core machines
- Kranc (by I. Hinder, S. Husa): automatic code generation, enabling optimising transformations
- VisIt (LLNL): (remote) visualisation tool for time-dependent AMR data



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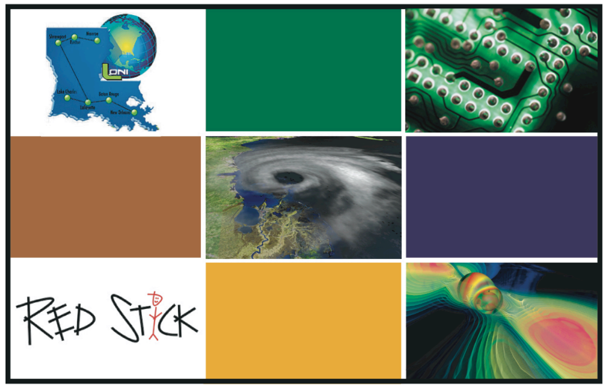
XiRel:
Cyberinfrastructure for Numerical Relativity

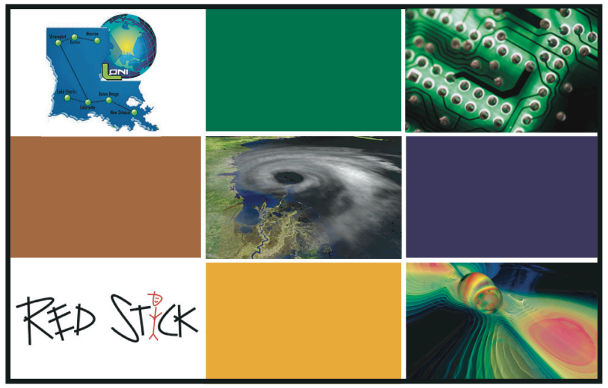
- Collaborative project including CCT, AEI, PSU, RIT
- 3 goals:
  - Improve efficiency of computational infrastructure (AMR etc.)
  - Design and implement data and metadata management infrastructure for large simulations
  - Work with community to update and expand the Einstein Toolkit



Alpaca:

Application Level Profiling and Correctness Analysis

- Observation: writing correct large programmes is very difficult, even when they are modular (Cactus)
- Observation: making parallel programmes efficient is not easy
- *Alpaca* (NSF SDCI, 3 years) will provide tools to profile and debug Cactus applications on a very high level (scheduled routines, variables declared in interface)
- Goal: debugging at the algorithmic level



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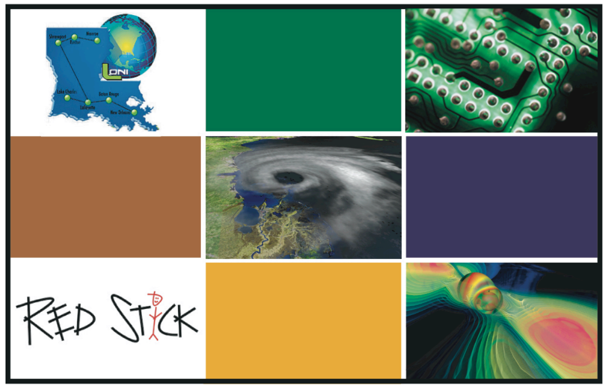
Summary

- Efficient numerical and computational infrastructure, used by many groups in the field (Einstein Toolkit, Cactus, Carpet)
- Collaboration on infrastructure, sharing the cost and beginning to create a "community feeling"
- Significant recent improvements in efficiency
- Ideas and projects for future methods (e.g. multi-patch, pseudo-spectral)



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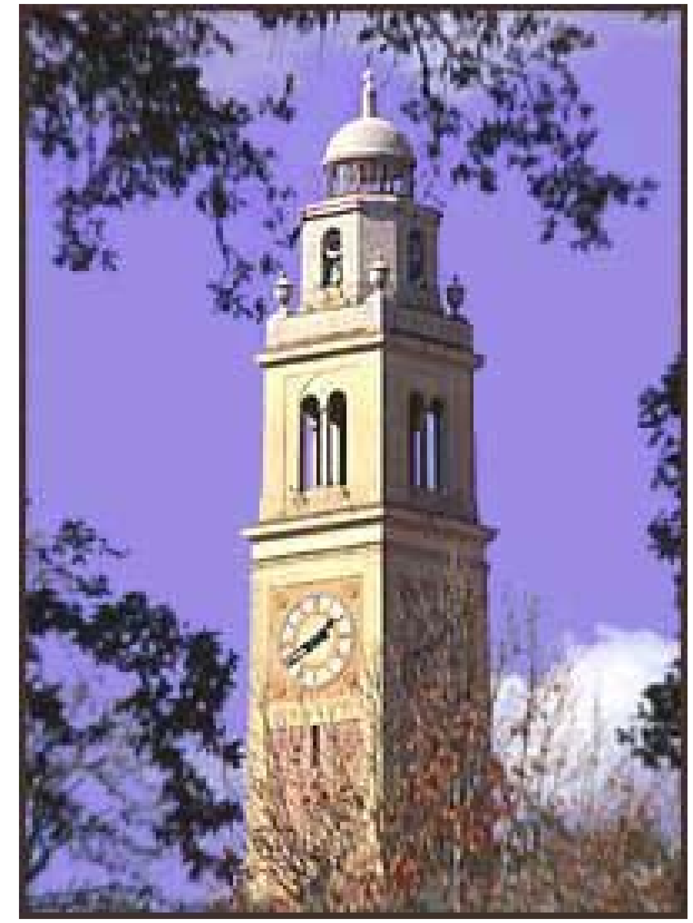
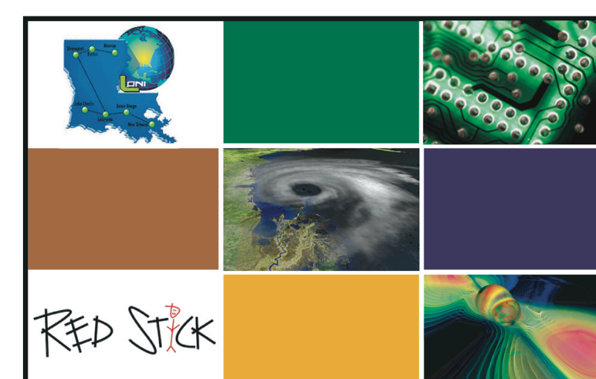


T. Sterling



References

- Spin Diagrams for Equal-Mass Black-Hole Binaries with Aligned Spins, arXiv:0708.3999 [gr-qc]
- The final spin from the coalescence of aligned-spin black-hole binaries, arXiv:0710.3345 [gr-qc]
- On the final spin from the coalescence of two black holes, arXiv:0712.3541 [gr-qc]

RED STICK

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References

- Cactus: http://www.cactuscode.org/
- Carpet: http://www.carpetcode.org/