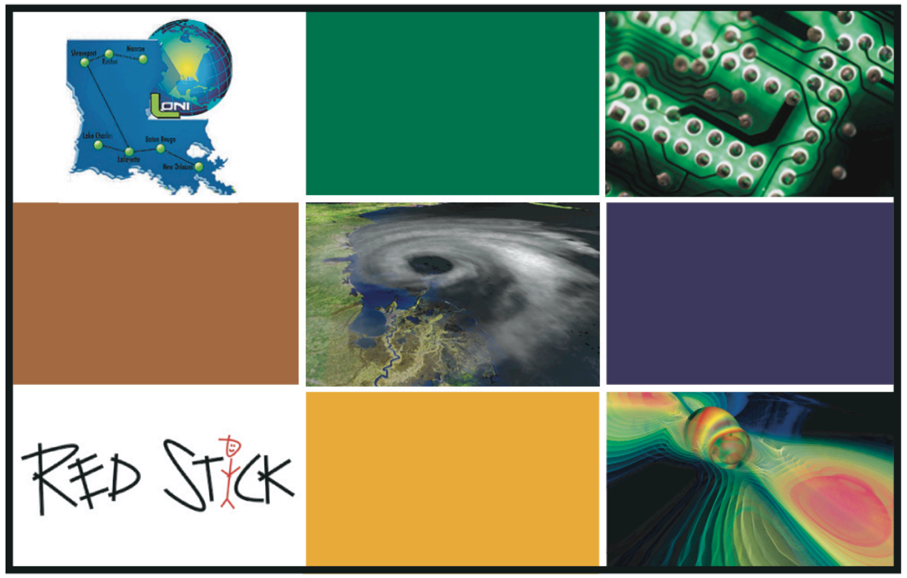
Modelling Black Hole Binary Mergers

Erik Schnetter
Oxford, MS, February 2008

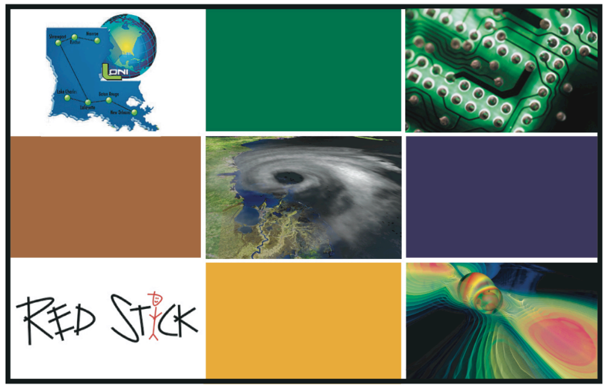
LSU
CENTER FOR COMPUTATION
& TECHNOLOGY



LSU
CENTER FOR COMPUTATION
& TECHNOLOGY

Outline

- Introduction: Binary black hole systems
- Peeking behind the scenes:
  Numerical and computational infrastructure
- Modelling merger events as black box:
  Predicting the final state
- Future development

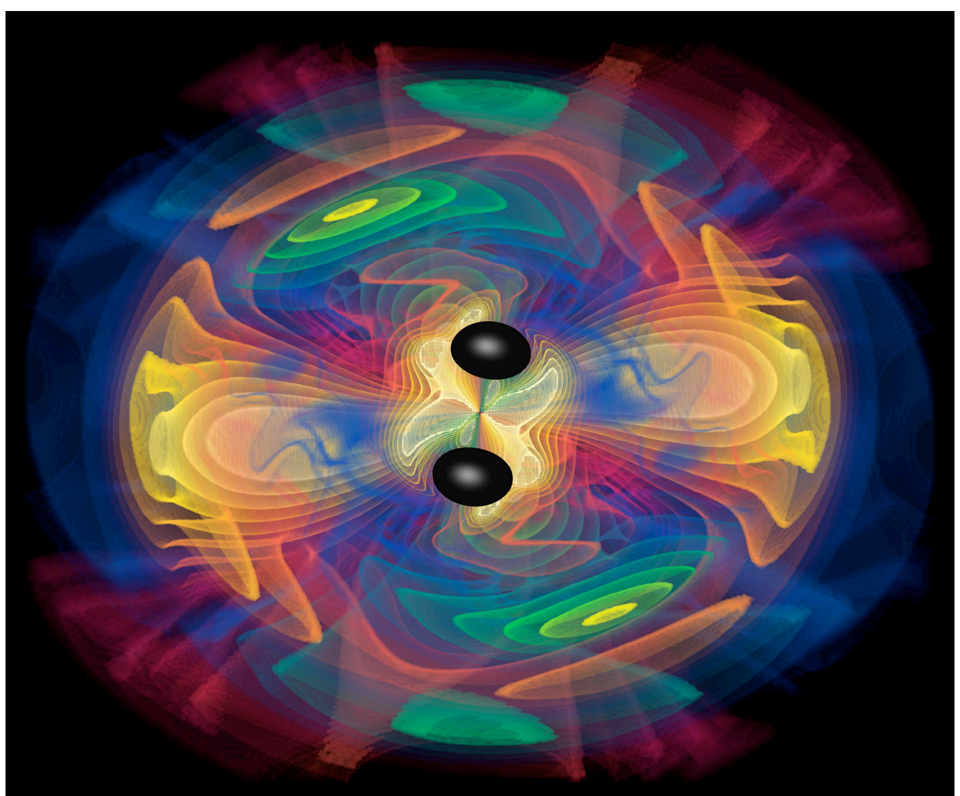


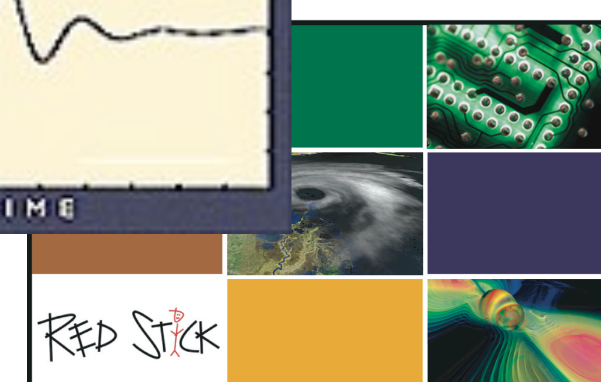
LSU
CENTER FOR COMPUTATION
& TECHNOLOGY

Gravitational Wave Physics

Observations

Models





Predictions

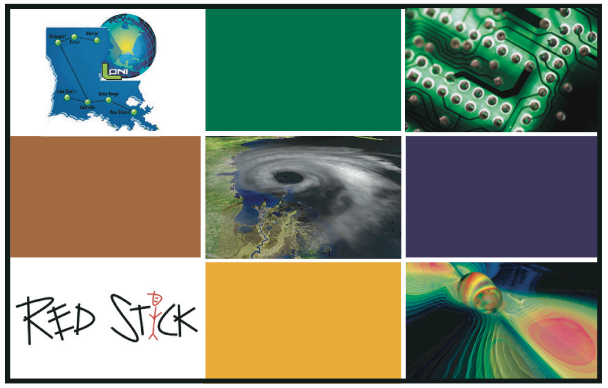
CENTER FOR COMPUTATION
& TECHNOLOGY

Binary Black Holes

- Very "simple" system – only Einstein equations, no matter, no magnetic fields, no radiation
- Interesting to relativists: two-body problem, determined by very few parameters
- Interesting to astrophysicists: fine source of gravitational radiation for LIGO and LISA; also: first step towards binaries with matter

Single Black Holes

- Theoretically discovered 1915 (K. Schwarzschild)
- They exist, there is ample (indirect) evidence
- There is a *horizon* and a *singularity*
- BH can be highly dynamic, e.g. when two black holes merge into one
- Horizon location cannot be determined experimentally
- *Not* "cosmic vacuum cleaners"



Formulations of the Einstein Equations

- Einstein equations are 10 coupled (non-linear) wave equations
- Einstein equations *need to be rewritten* to be a well posed IVP: it took the community many years to solve this
- Today: two main formulations, *BSSN* and *harmonic*
- Also: need to choose good gauge conditions

History of Binary Black Hole Simulations

- Earlier: BSSN system used for matter evolution
- Alcubierre et al. (2003): stable gauge conditions (for BSSN)
- Brügmann et al. (2004): first orbit
- Pretorius (2005): first merger
- Campanelli et al. (2005), Baker et al. (2005): much simpler method to handle singularities ("moving punctures")

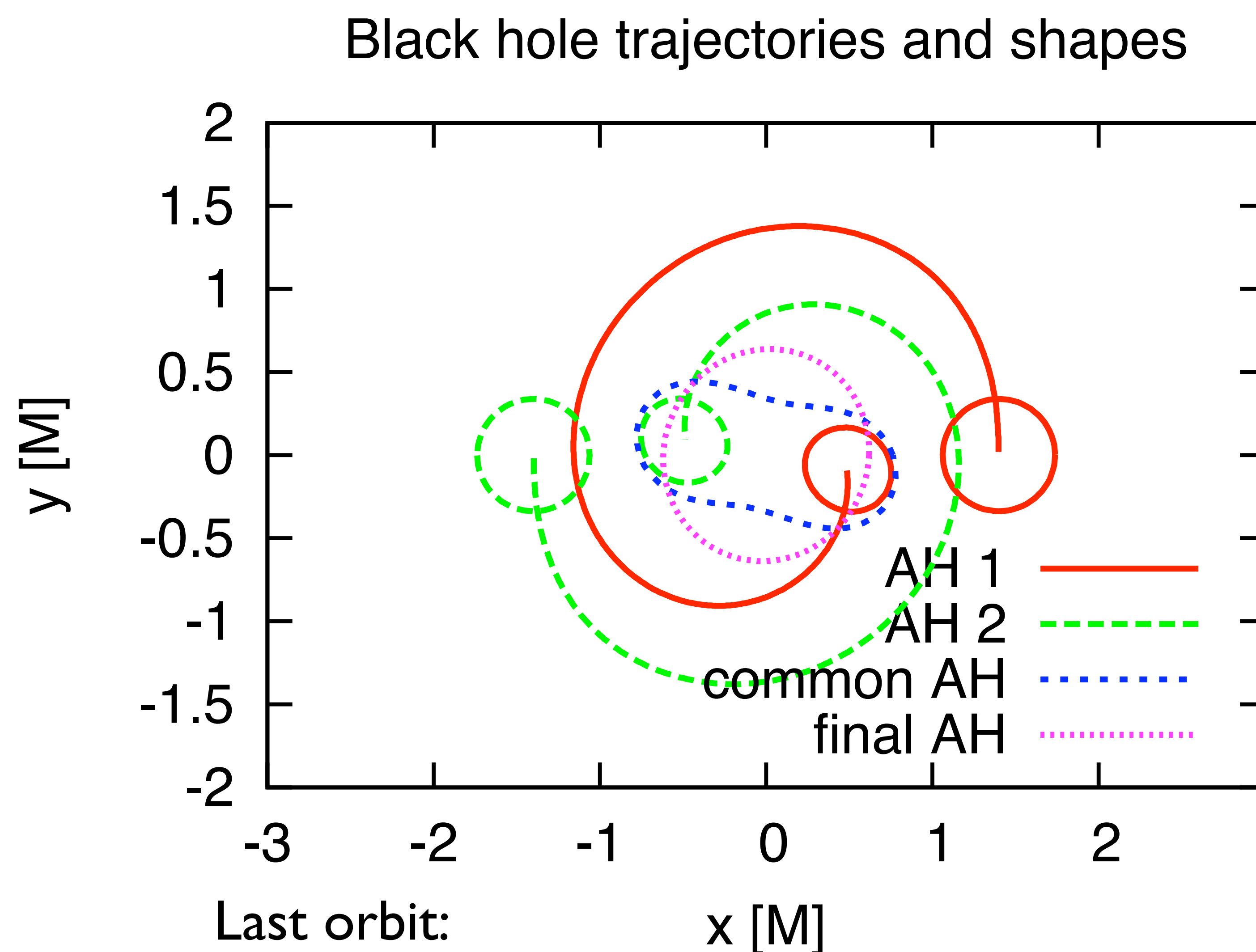
CENTER FOR COMPUTATION & TECHNOLOGY

Phenomenology of Binary Black Hole Mergers

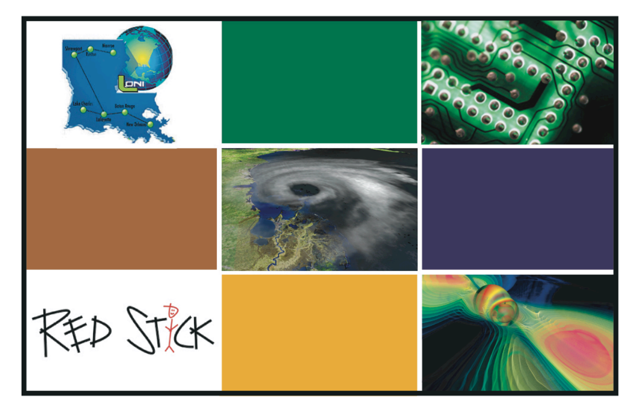
- Three phases:

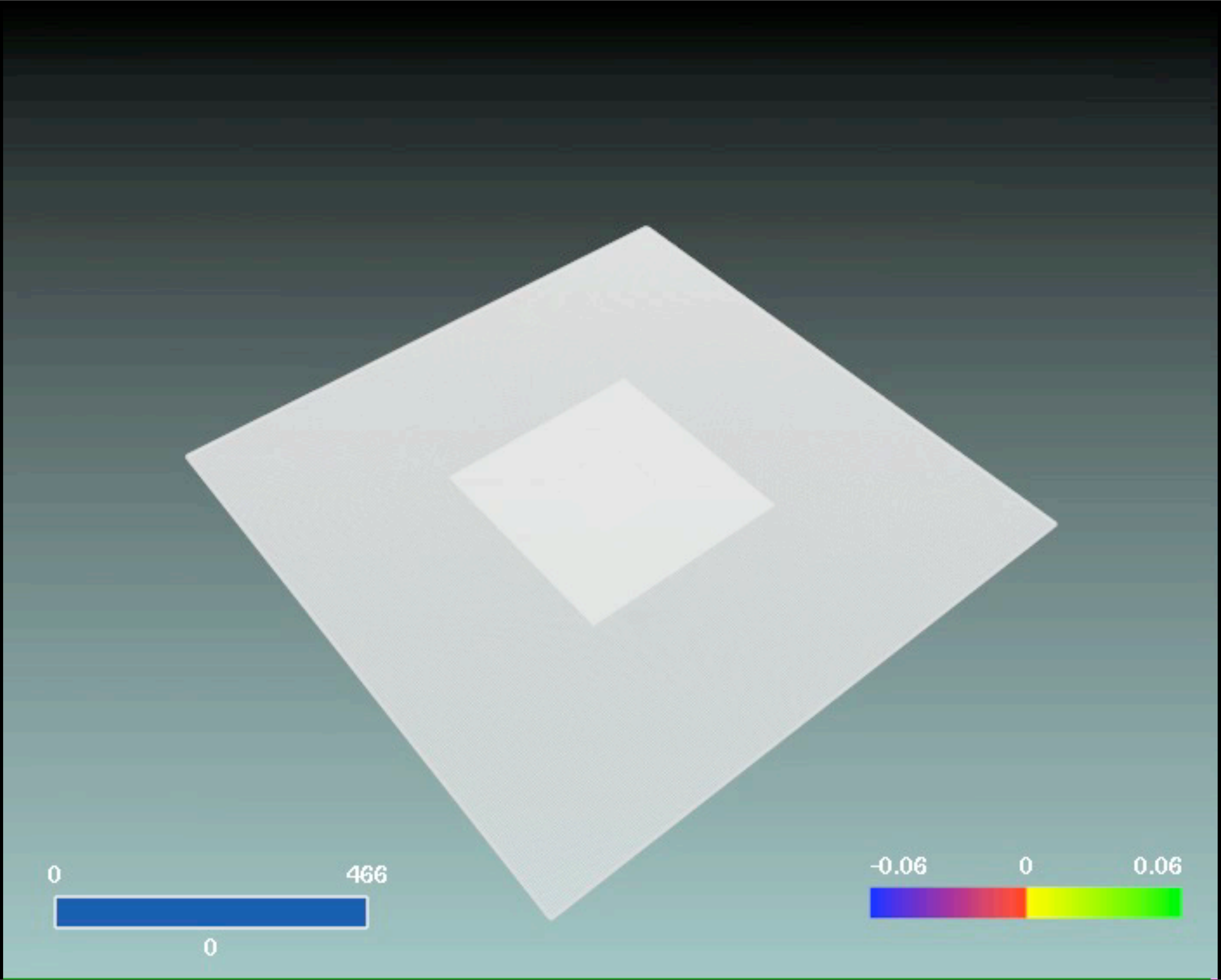
1. Inspiral (red/green), here: slightly eccentric
2. Merger: Common horizon forms (blue)
3. Ring-down; final state (magenta)

- Not shown: possible recoil (“kick”), if system is not symmetric

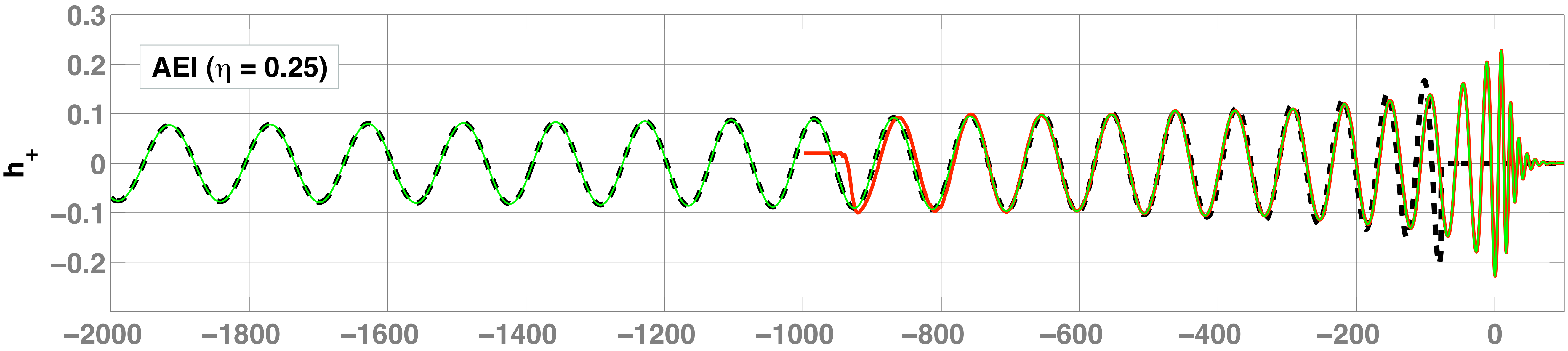


Last orbit: Cook-Pfeiffer initial data




0
466
0
-0.06
0
0.06

Typical BBH Waveform

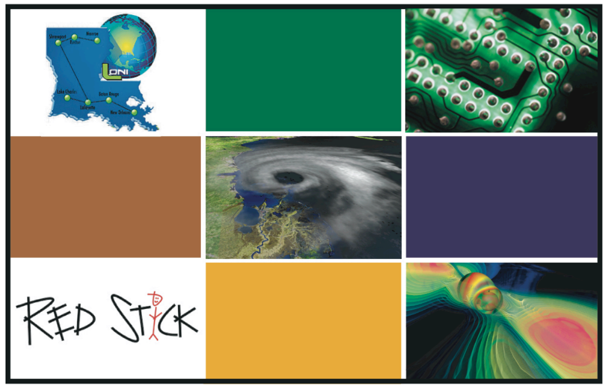


Ajith et al., arXiv:0710.2335 [gr-qc]

black: post-Newtonian approximation

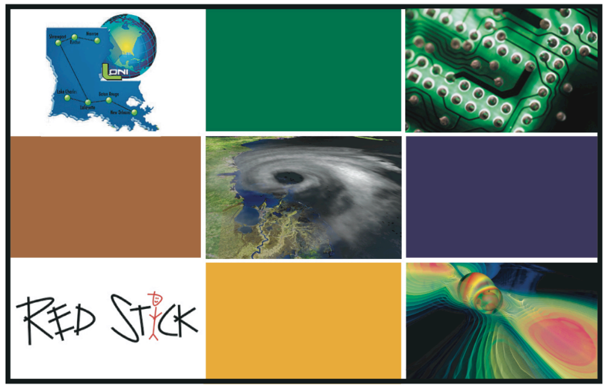
red: numerical solution

green: combination (best prediction)



Behind the Scenes: Numerical Infrastructure

- Initial data solver for quasi-circular configuration
- CCATIE: Efficient 4th order BSSN code for time evolution
- Horizon finding, measuring horizon quantities
- Wave extraction
- Built on the Cactus framework,
  much is public, part of the *Einstein Toolkit*



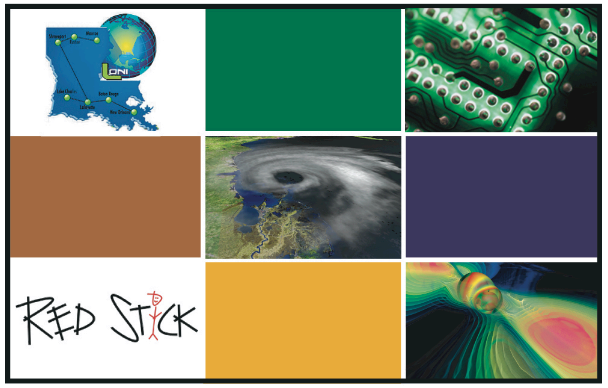
Automatic Code Generation

- BSSN equations contain thousands of terms, very tedious to write down
- Changes to formulation/gauge condition difficult to implement
- Solution: generate code automatically from a Mathematica script, using abstract index notation
- Important:
  correctness, efficiency, flexibility

LSU
CENTER FOR COMPUTATION & TECHNOLOGY

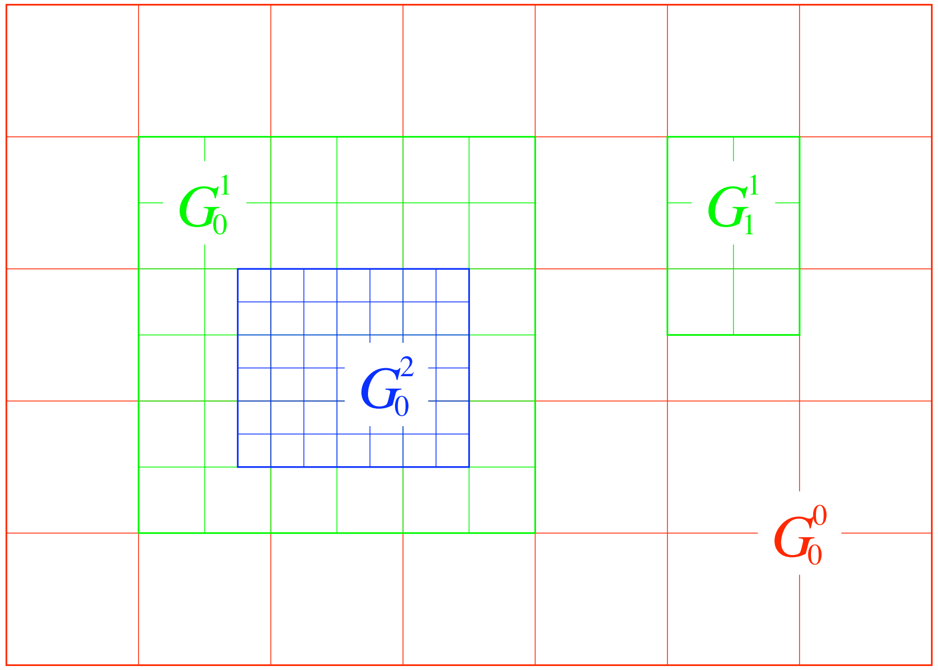
Behind the Scenes: Computational Infrastructure

- Large scale differences and moving objects require adaptive mesh refinement (AMR) [typical: L=1000, h=0.02, using 9 refinement levels]
- Long time evolutions and desired accuracy require high order methods (4th order or higher)
- Same infrastructure (Cactus, Carpet) also used for GRMHD simulations
- Computation time/efficiency still an issue



CENTER FOR COMPUTATION
& TECHNOLOGY

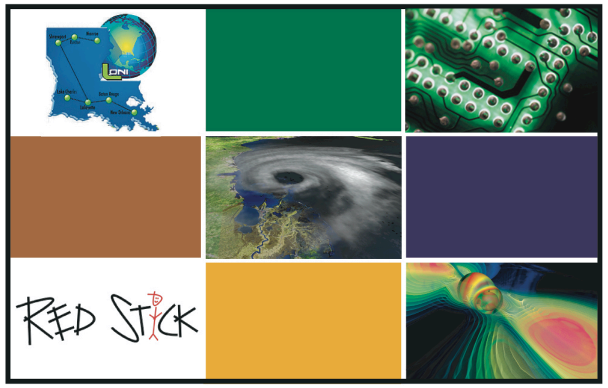
Carpet: Mesh Refinement



- Berger-Oliger adaptive mesh refinement (AMR) with subcycling in time
- Using *buffer zones* for stable AMR boundaries
- Domain decomposition parallelisation (typically 3 ghost zones – expensive!)
- AMR tracks physics features explicitly, refining e.g. around black holes

Efficiency and Scalability

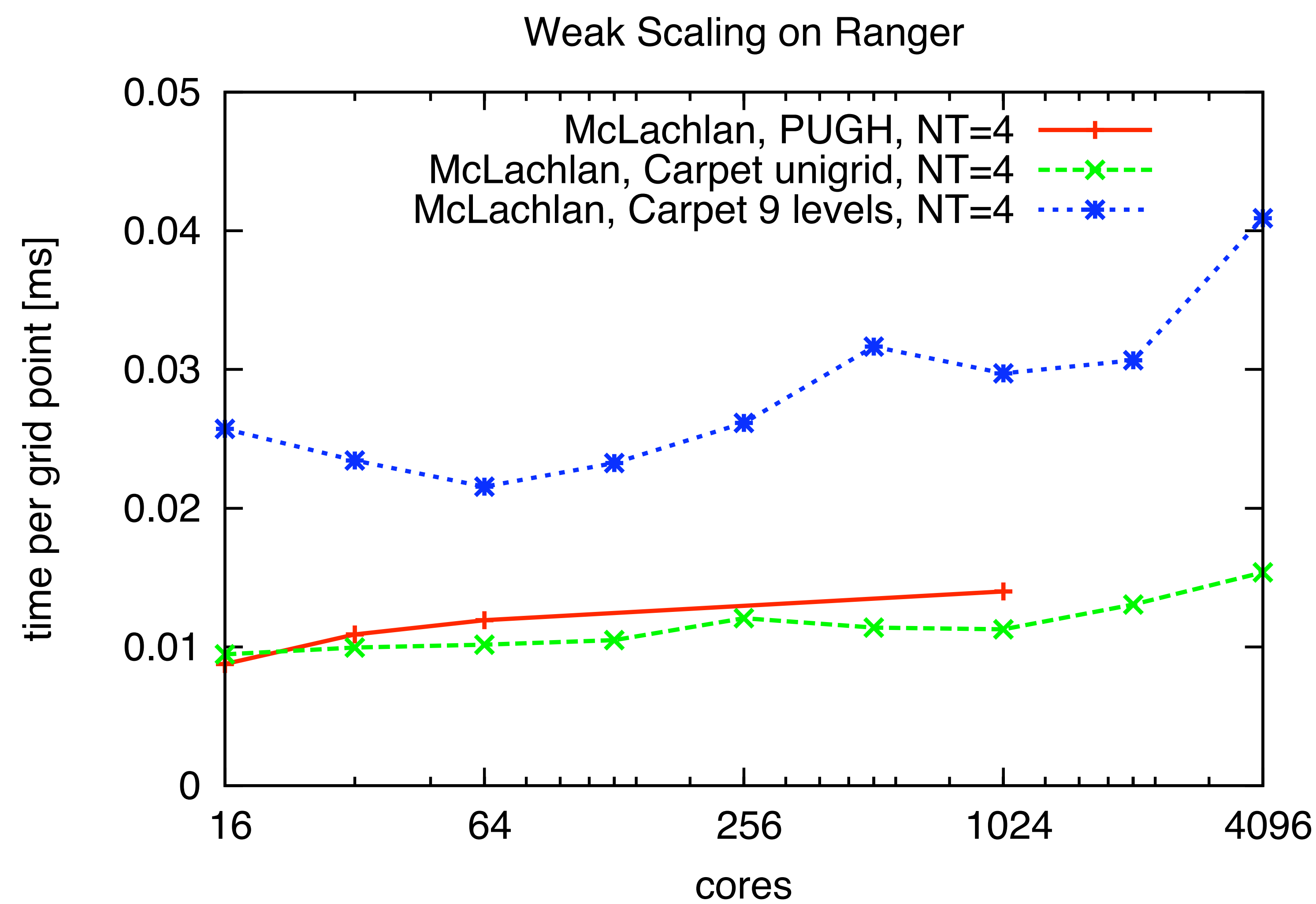
- Many variables (~25 evolved, ~250 in total, many ghost zones (higher order differencing):
  - requires much memory
  - can have only few evolved grid points per processor – inefficient
- Currently, a very simple AMR testcase requires already 8 GByte memory

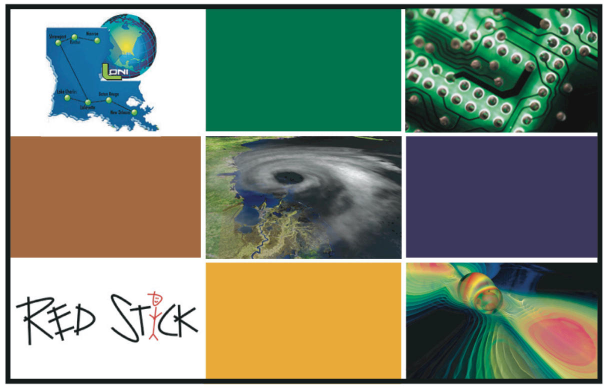


AMR Parallel Scaling on Ranger (TACC)

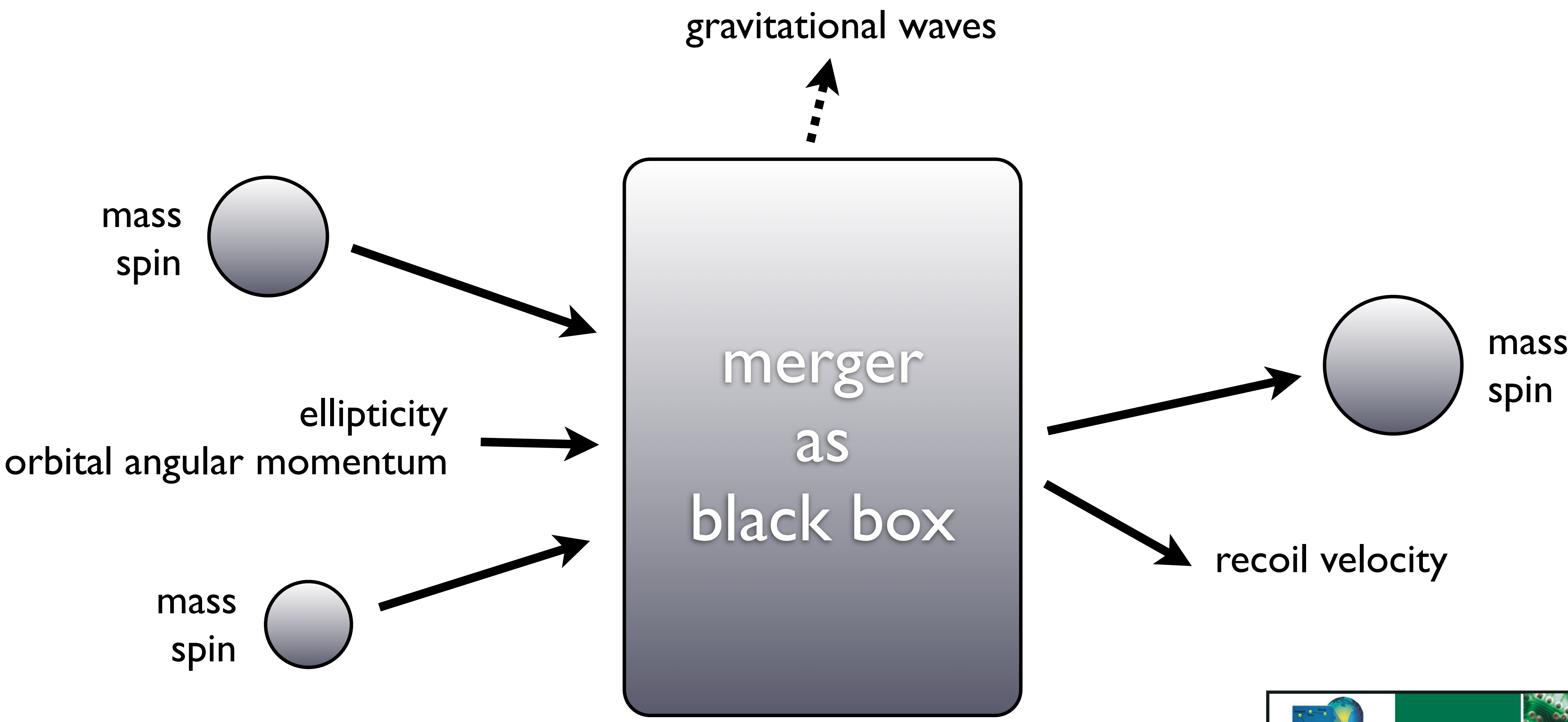
CENTER FOR COMPUTATION & TECHNOLOGY

- Ranger is a new supercomputer at TACC (Texas) with 60,000 cores
- Using OpenMP to reduce parallelisation overhead
- McLachlan: New BSSN code for experimenting with performance





Binary Black Hole Mergers as Black Box



Merger determined by nine parameters

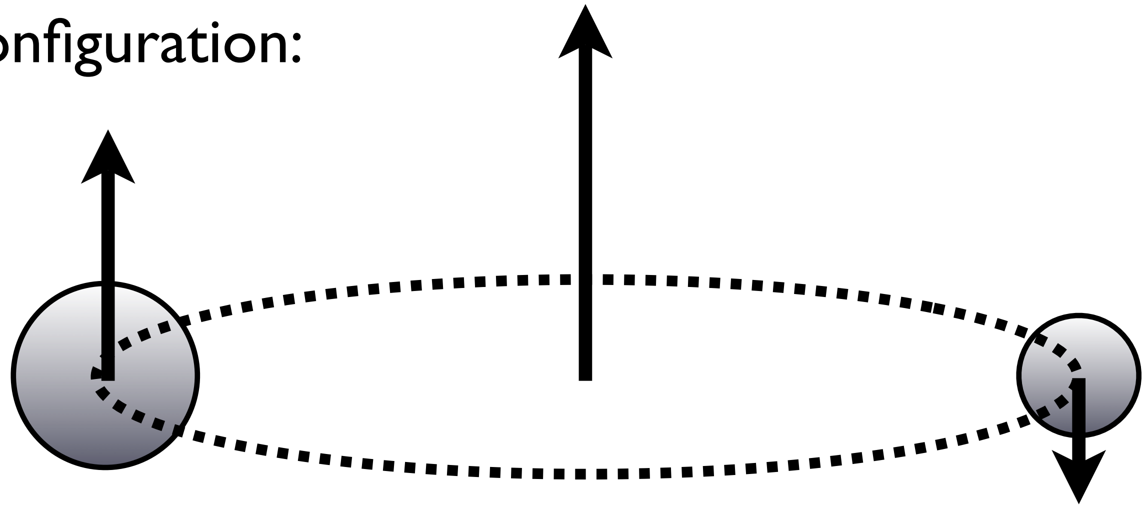
Goal: Find Analytic Description for Final State

- Full numerical treatment for all parameter values is too expensive – use fitting functions instead
- Previous work: Campanelli et al. 2007 (large recoils), González et al. 2007 (non-spinning)
- Take analytic approximations for special cases (e.g. extreme mass ratios) into account
- Initially, restrict parameter space

Assumptions

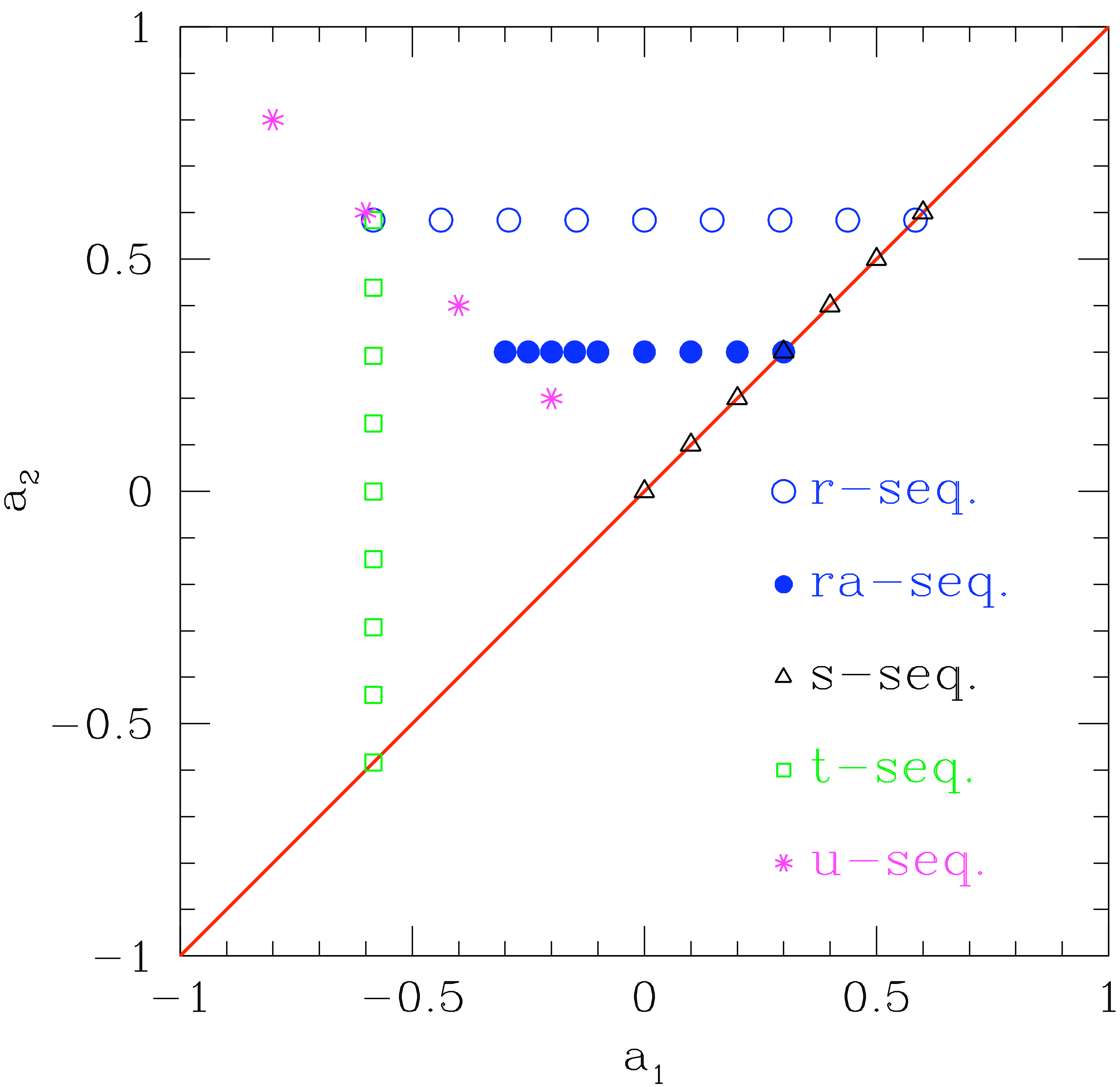
- Masses equal or unequal
- Circular orbits
- Spins aligned and/or anti-aligned with orbital angular momentum

Binary Configuration:



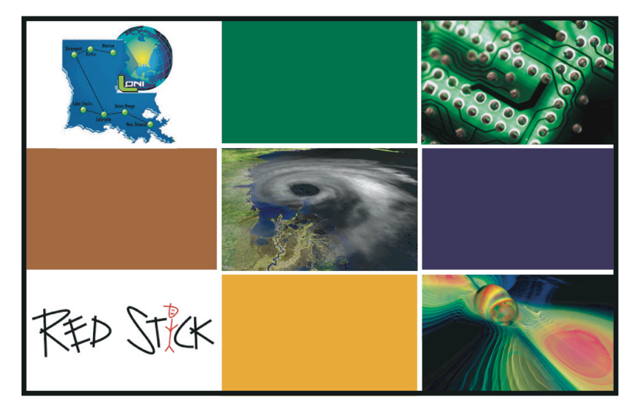
LSU
CENTER FOR COMPUTATION
& TECHNOLOGY

Equal Mass Case: Spin Diagrams



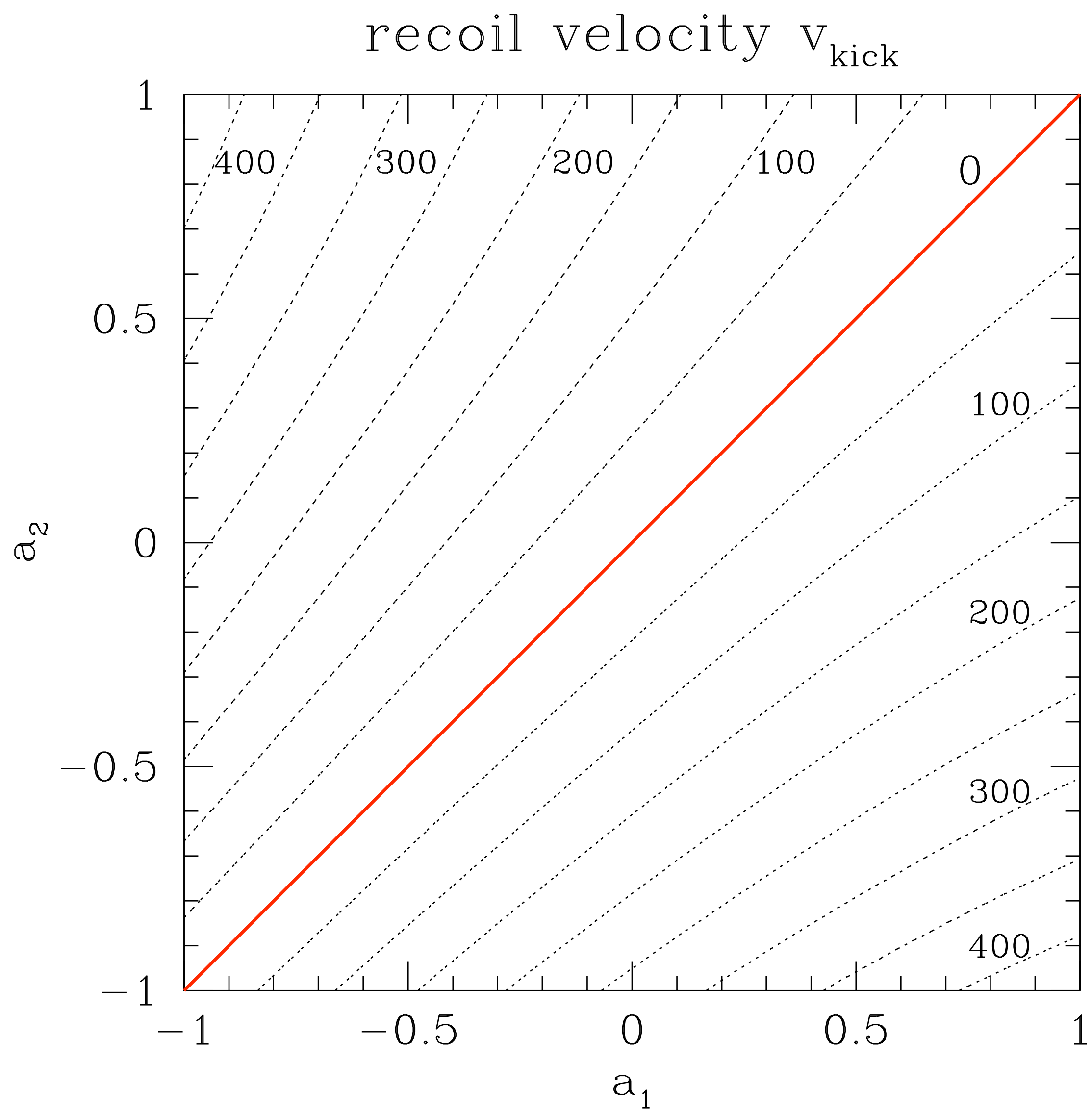
	$\pm x/M$	$\pm p/M$	m_1/M	m_2/M	a_1	a_2	$\widetilde{M}_{\rm ADM}$	$\widetilde{J}_{\rm ADM}$	$\lvert v_{\rm kick}\rvert$	$\lvert v^{\rm fit}_{\rm kick}\rvert$	err. (%)	$a_{\rm fin}$	$a^{\rm fit}_{\rm fin}$	err. (%)
$r0$	3.0205	0.1366	0.4011	0.4009	-0.584	0.584	0.9856	0.825	261.75	258.09	1.40	0.6891	0.6883	0.12
$r1$	3.1264	0.1319	0.4380	0.4016	-0.438	0.584	0.9855	0.861	221.38	219.04	1.06	0.7109	0.7105	0.06
$r2$	3.2198	0.1281	0.4615	0.4022	-0.292	0.584	0.9856	0.898	186.18	181.93	2.28	0.7314	0.7322	0.11
$r3$	3.3190	0.1243	0.4749	0.4028	-0.146	0.584	0.9857	0.935	144.02	146.75	1.90	0.7516	0.7536	0.27
$r4$	3.4100	0.1210	0.4796	0.4034	0.000	0.584	0.9859	0.971	106.11	113.52	6.98	0.7740	0.7747	0.08
$r5$	3.5063	0.1176	0.4761	0.4040	0.146	0.584	0.9862	1.007	81.42	82.23	1.00	0.7948	0.7953	0.06
$r6$	3.5988	0.1146	0.4638	0.4044	0.292	0.584	0.9864	1.044	45.90	52.88	15.21	0.8150	0.8156	0.07
$r7$	3.6841	0.1120	0.4412	0.4048	0.438	0.584	0.9867	1.081	20.59	25.47	23.70	0.8364	0.8355	0.11
$r8$	3.7705	0.1094	0.4052	0.4052	0.584	0.584	0.9872	1.117	0.00	0.00	0.00	0.8550	0.855	0.00
$ra0$	2.9654	0.1391	0.4585	0.4584	-0.300	0.300	0.9845	0.8250	131.34	132.58	0.95	0.6894	0.6883	0.16
$ra1$	3.0046	0.1373	0.4645	0.4587	-0.250	0.300	0.9846	0.8376	118.10	120.28	1.85	0.6971	0.6959	0.17
$ra2$	3.0438	0.1355	0.4692	0.4591	-0.200	0.300	0.9847	0.8499	106.33	108.21	1.77	0.7047	0.7035	0.17
$ra3$	3.0816	0.1339	0.4730	0.4594	-0.150	0.300	0.9848	0.8628	94.98	96.36	1.46	0.7120	0.7111	0.13
$ra4$	3.1215	0.1321	0.4757	0.4597	-0.100	0.300	0.9849	0.8747	84.74	84.75	0.01	0.7192	0.7185	0.09
$ra6$	3.1988	0.1290	0.4782	0.4602	0.000	0.300	0.9850	0.9003	63.43	62.19	1.95	0.7331	0.7334	0.04
$ra8$	3.2705	0.1261	0.4768	0.4608	0.100	0.300	0.9852	0.9248	41.29	40.55	1.79	0.7471	0.7481	0.13
$ra10$	3.3434	0.1234	0.4714	0.4612	0.200	0.300	0.9853	0.9502	19.11	19.82	3.72	0.7618	0.7626	0.11
$ra12$	3.4120	0.1209	0.4617	0.4617	0.300	0.300	0.9855	0.9750	0.00	0.00	0.00	0.7772	0.7769	0.03
$s0$	2.9447	0.1401	0.4761	0.4761	0.000	0.000	0.9844	0.8251	0.00	0.00	0.00	0.6892	0.6883	0.13
$s1$	3.1106	0.1326	0.4756	0.4756	0.100	0.100	0.9848	0.8749	0.00	0.00	0.00	0.7192	0.7185	0.09
$s2$	3.2718	0.1261	0.4709	0.4709	0.200	0.200	0.9851	0.9251	0.00	0.00	0.00	0.7471	0.7481	0.13
$s3$	3.4098	0.1210	0.4617	0.4617	0.300	0.300	0.9855	0.9751	0.00	0.00	0.00	0.7772	0.7769	0.03
$s4$	3.5521	0.1161	0.4476	0.4476	0.400	0.400	0.9859	1.0250	0.00	0.00	0.00	0.8077	0.8051	0.33
$s5$	3.6721	0.1123	0.4276	0.4276	0.500	0.500	0.9865	1.0748	0.00	0.00	0.00	0.8340	0.8325	0.18
$s6$	3.7896	0.1088	0.4002	0.4002	0.600	0.600	0.9874	1.1246	0.00	0.00	0.00	0.8583	0.8592	0.11
$t0$	4.1910	0.1074	0.4066	0.4064	-0.584	0.584	0.9889	0.9002	259.49	258.09	0.54	0.6868	0.6883	0.22
$t1$	4.0812	0.1103	0.4062	0.4426	-0.584	0.438	0.9884	0.8638	238.37	232.62	2.41	0.6640	0.6658	0.27
$t2$	3.9767	0.1131	0.4057	0.4652	-0.584	0.292	0.9881	0.8265	200.25	205.21	2.48	0.6400	0.6429	0.45
$t3$	3.8632	0.1165	0.4053	0.4775	-0.584	0.146	0.9879	0.7906	174.58	175.86	0.73	0.6180	0.6196	0.26
$t4$	3.7387	0.1204	0.4047	0.4810	-0.584	0.000	0.9878	0.7543	142.62	144.57	1.37	0.5965	0.5959	0.09
$t5$	3.6102	0.1246	0.4041	0.4761	-0.584	-0.146	0.9876	0.7172	106.36	111.34	4.68	0.5738	0.5719	0.33
$t6$	3.4765	0.1294	0.4033	0.4625	-0.584	-0.292	0.9874	0.6807	71.35	76.17	6.75	0.5493	0.5475	0.32
$t7$	3.3391	0.1348	0.4025	0.4387	-0.584	-0.438	0.9873	0.6447	35.36	39.05	10.45	0.5233	0.5227	0.11
$t8$	3.1712	0.1419	0.4015	0.4015	-0.584	-0.584	0.9875	0.6080	0.00	0.00	0.00	0.4955	0.4976	0.42
$u1$	2.9500	0.1398	0.4683	0.4685	-0.200	0.200	0.9845	0.8248	87.34	88.39	1.20	0.6893	0.6883	0.15
$u2$	2.9800	0.1384	0.4436	0.4438	-0.400	0.400	0.9846	0.8249	175.39	176.78	0.79	0.6895	0.6883	0.17
$u3$	3.0500	0.1355	0.3951	0.3953	-0.600	0.600	0.9847	0.8266	266.39	265.16	0.46	0.6884	0.6883	0.01
$u4$	3.1500	0.1310	0.2968	0.2970	-0.800	0.800	0.9850	0.8253	356.87	353.55	0.93	0.6884	0.6883	0.01

38 parameter sets calculated;
several simulations each



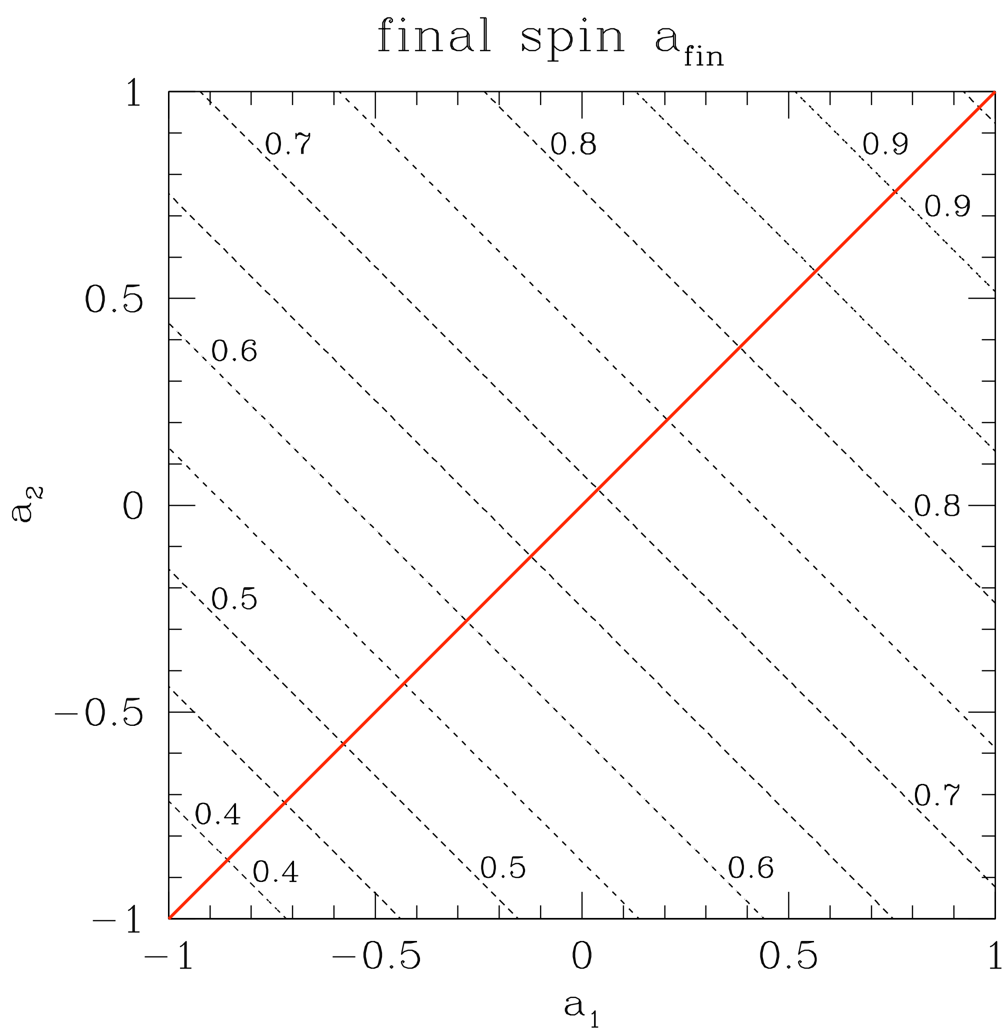
Fitting Functions (Recoil)

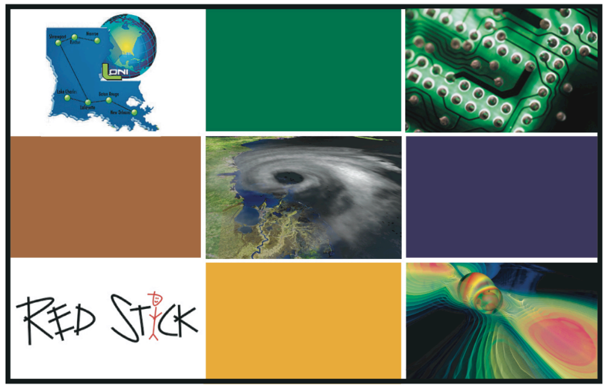
- Make generic quadratic ansatz, then fit coefficients
- Maximum recoil about 440 km/s for anti-aligned spins
- Quadratic behaviour: improvement over linear PN predictions
- Very good agreement with other numerical studies



Fitting Functions (Final Spin)

- Again, results consistent with brute-force fitting functions
- Possible final spins approximately in [0.35 ... 0.96]
- Quadratic term possibly zero
- No local maxima as suggested by Effective One-Body approximation (Damour 2001)

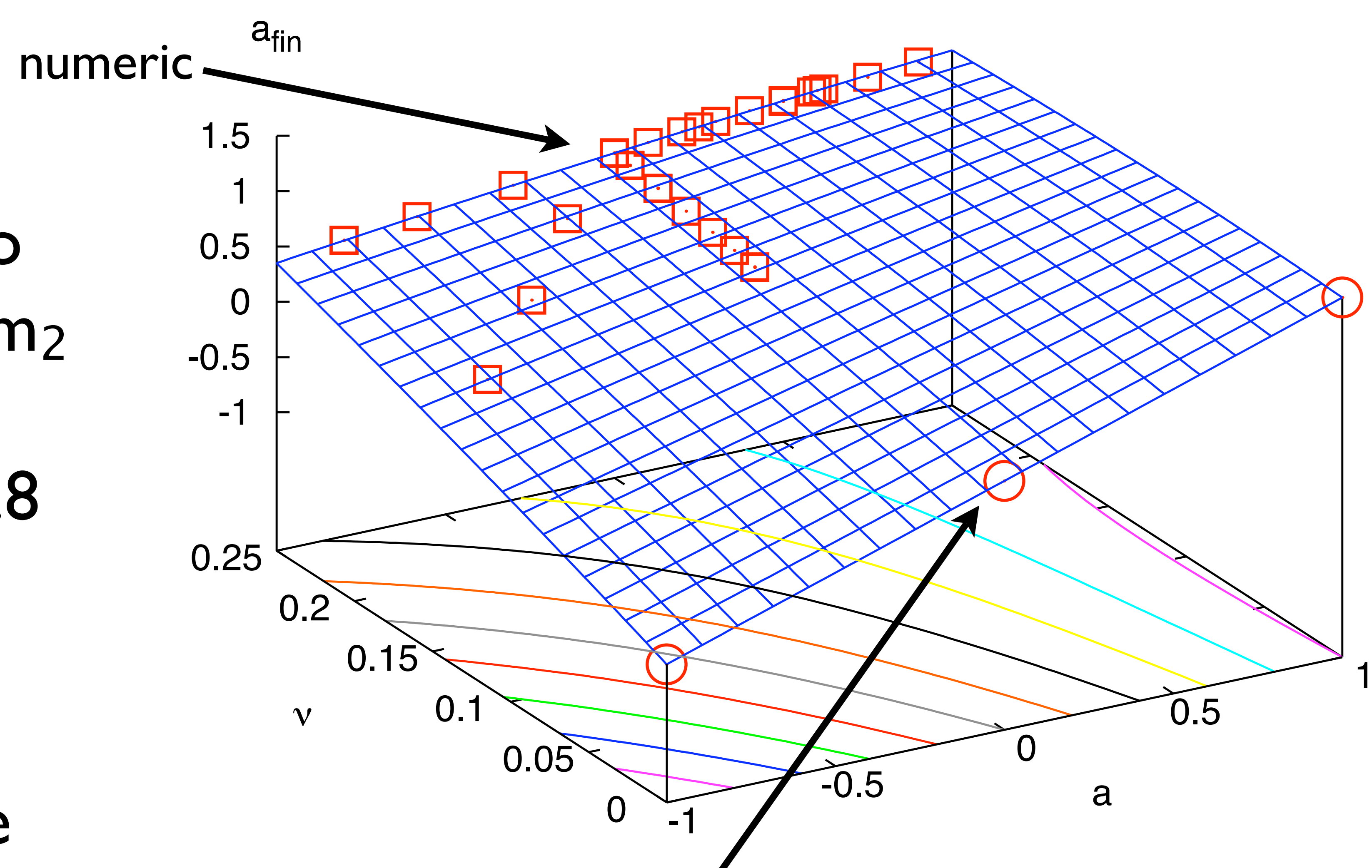


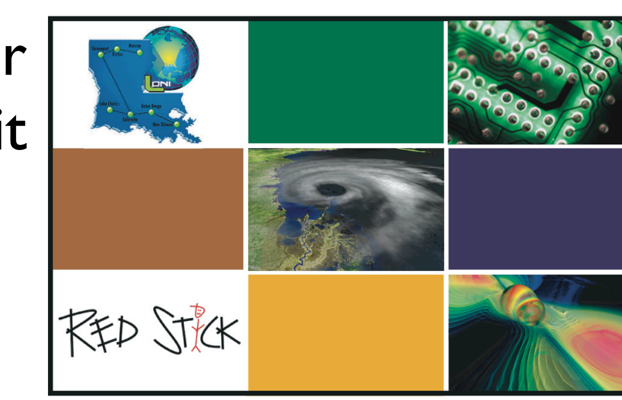


LSU
CENTER FOR COMPUTATION
& TECHNOLOGY

Unequal Mass, Equal Spin Case

- ν: symmetric mass ratio
  $\nu = q/(1+q)^2, \quad q = m_1/m_2$
- $\nu <\sim 0.16$ and $|a| >\sim 0.8$ difficult to access for numerical relativity
- Use analytic knowledge about extreme mass ratio limit ($\nu=0$)

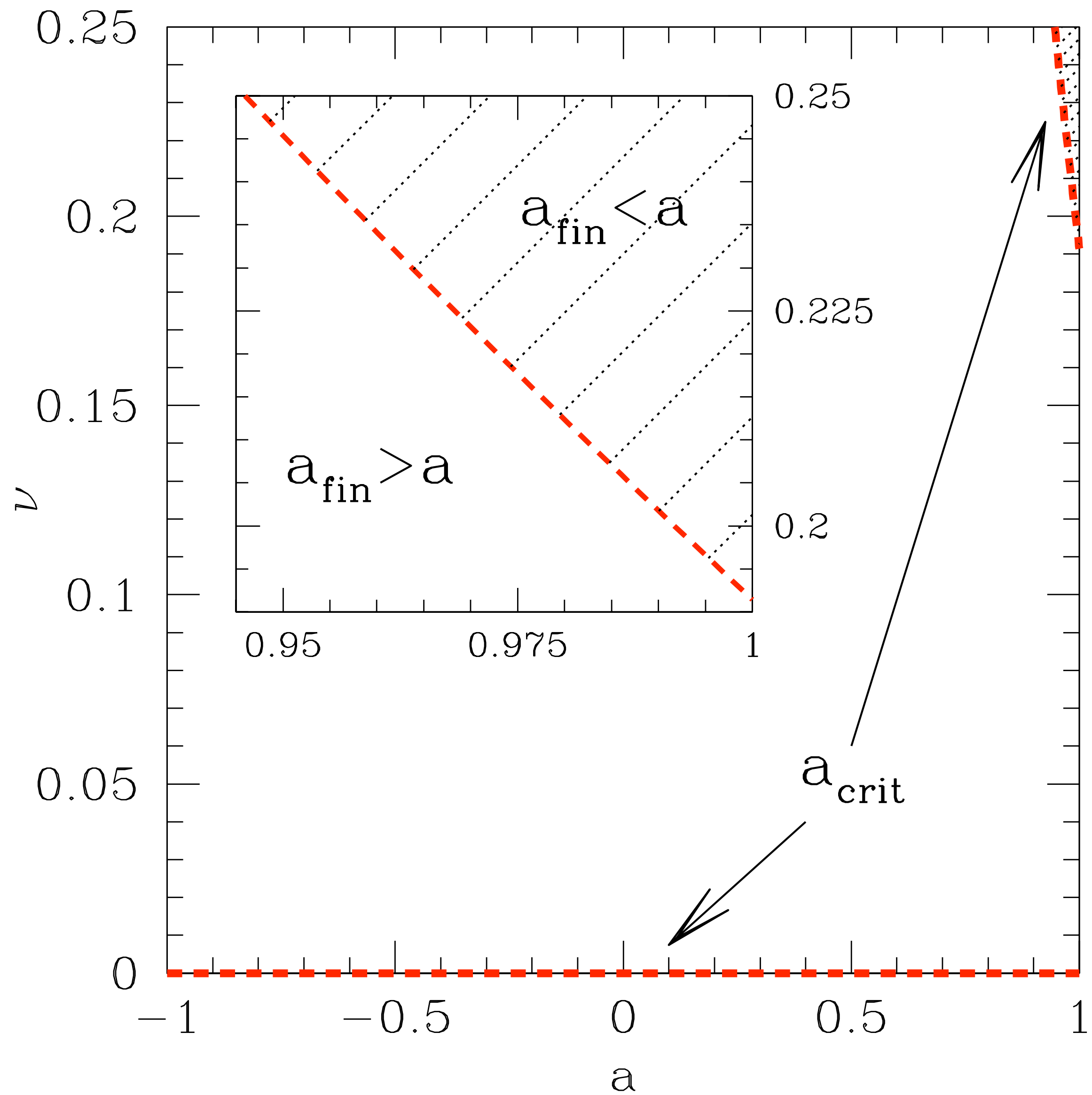




LSU CENTER FOR COMPUTATION & TECHNOLOGY

Spin-Up or Spin-Down?

- Just ask for *a*[fin] = *a* in fitting function
- Almost all equal-spin configurations are *spun up*
- Note: fitting function is unphysical near |*a*| = 1, probably due to extrapolation

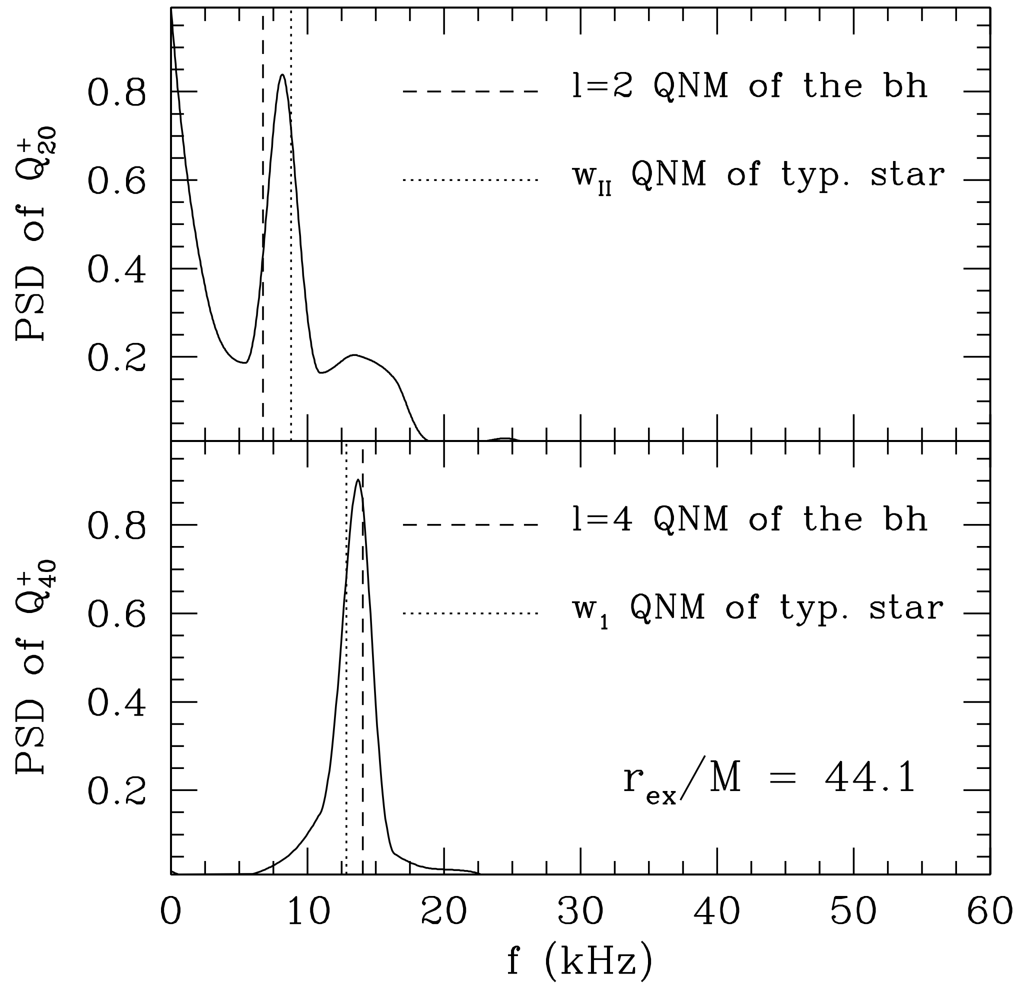


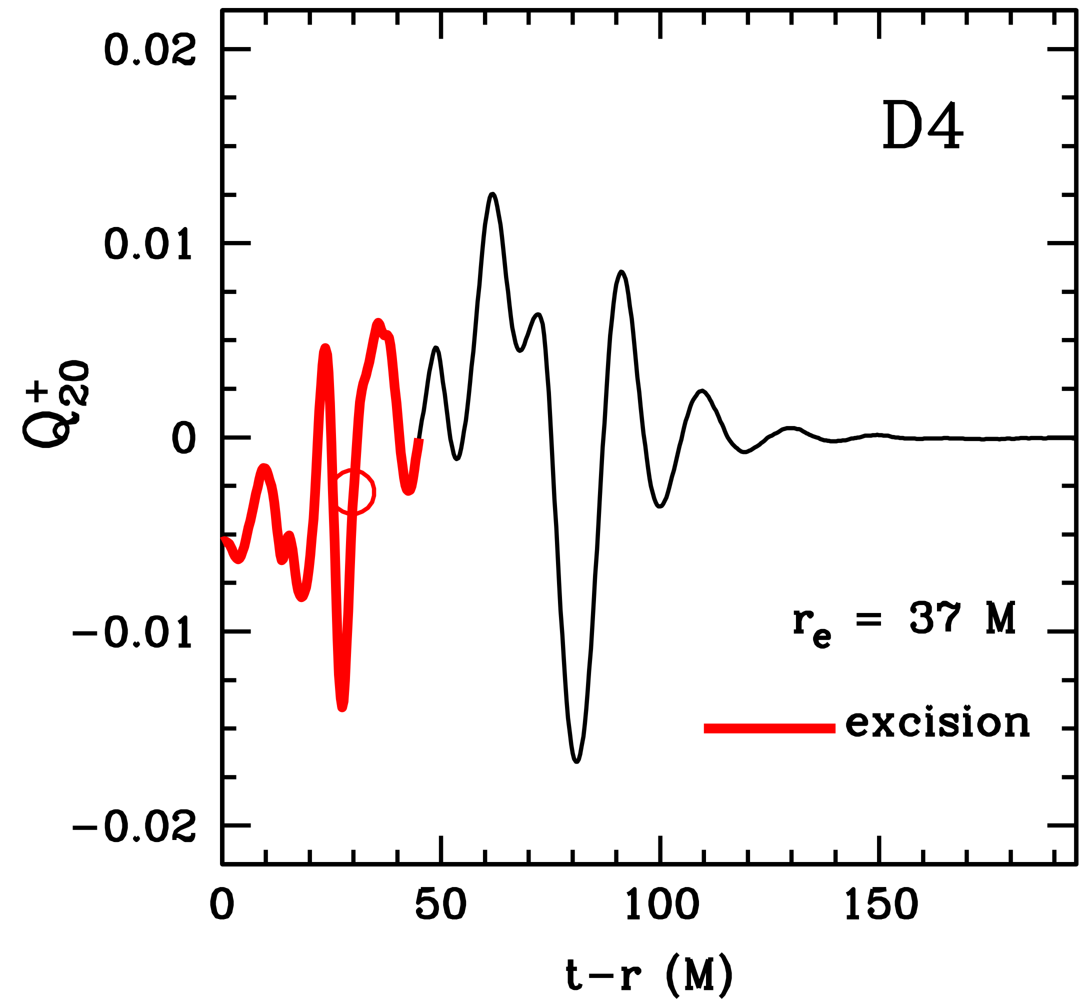
Summary: Black Box Binaries

- Consider binary black hole mergers as black box
- Model final state using generic fitting function depending on initial state – cheaper and more accurate than other approximations
- Could e.g. be used for N-body simulations
- Excellent agreement between fitting function and numerical results for unequal (aligned) spins and for unequal masses
- Work underway to generalise this to arbitrary configurations

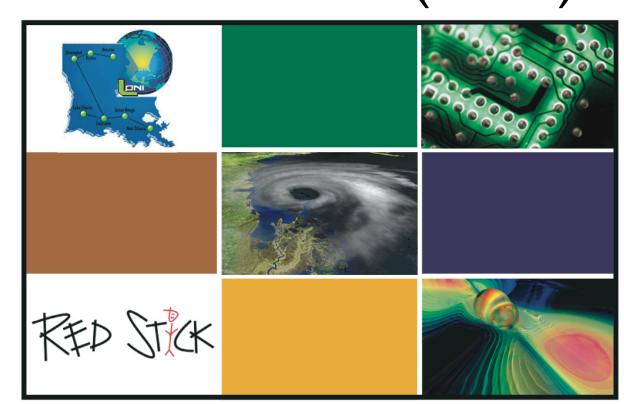
LSU
CENTER FOR COMPUTATION
& TECHNOLOGY

Beyond Black Holes



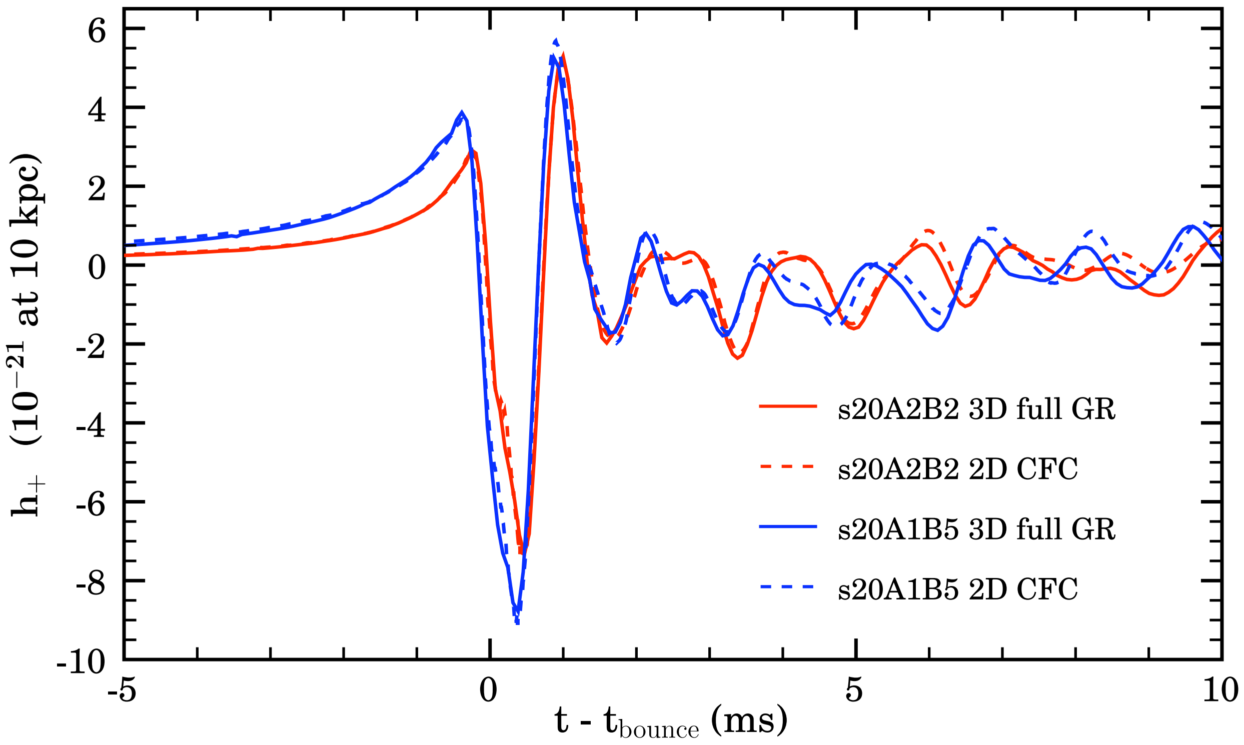


Baiotti et al. (2005),
Baiotti et al. (2006)

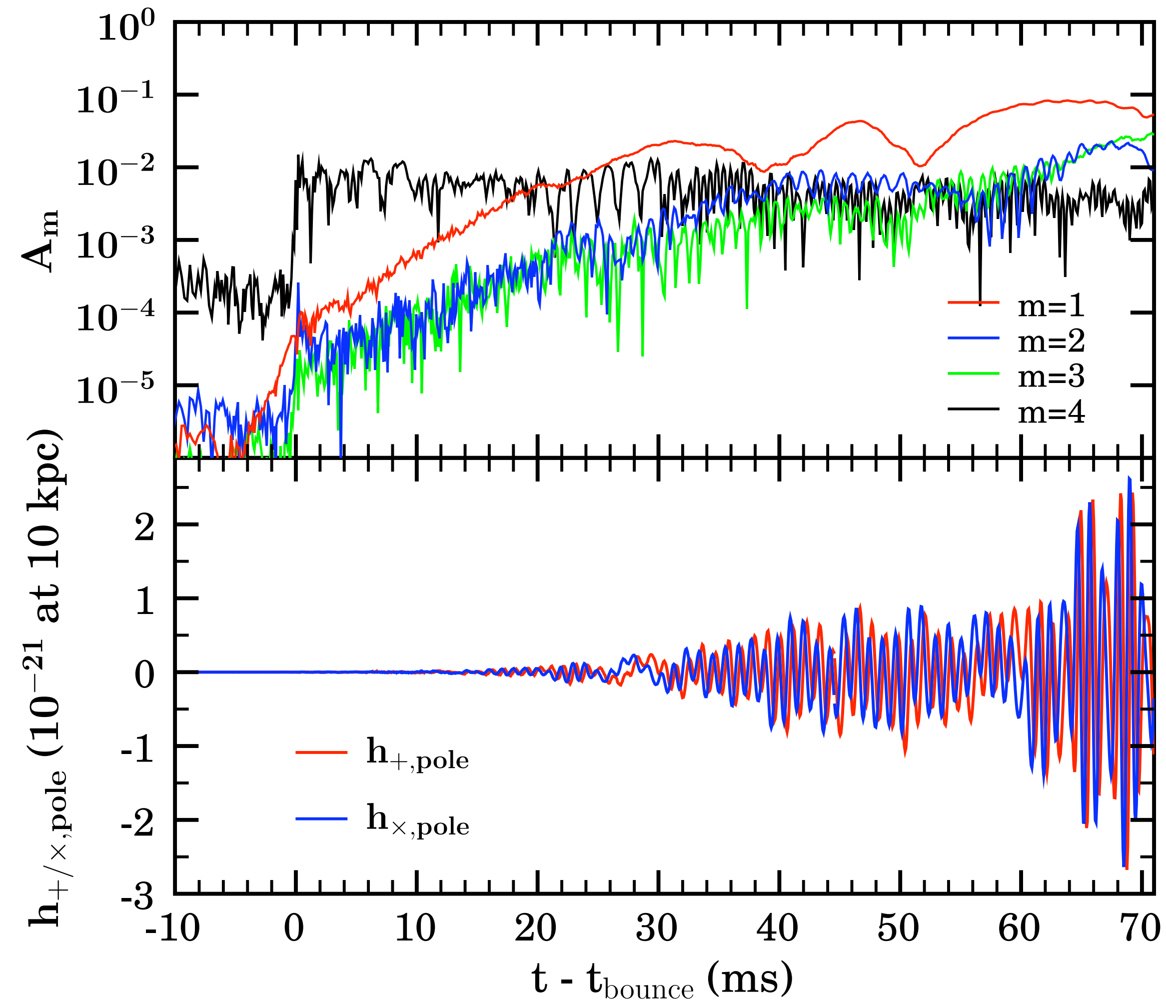


- Collapse of a rotating neutron star to a black hole
- Polytropic EOS, fast rotating (model D4), dynamically unstable configuration

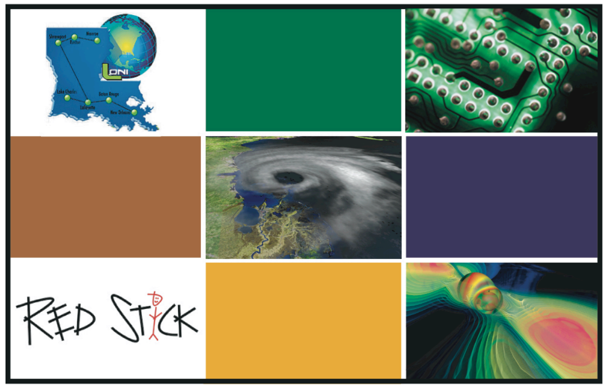
Beyond Black Holes



Ott et al. (2007)

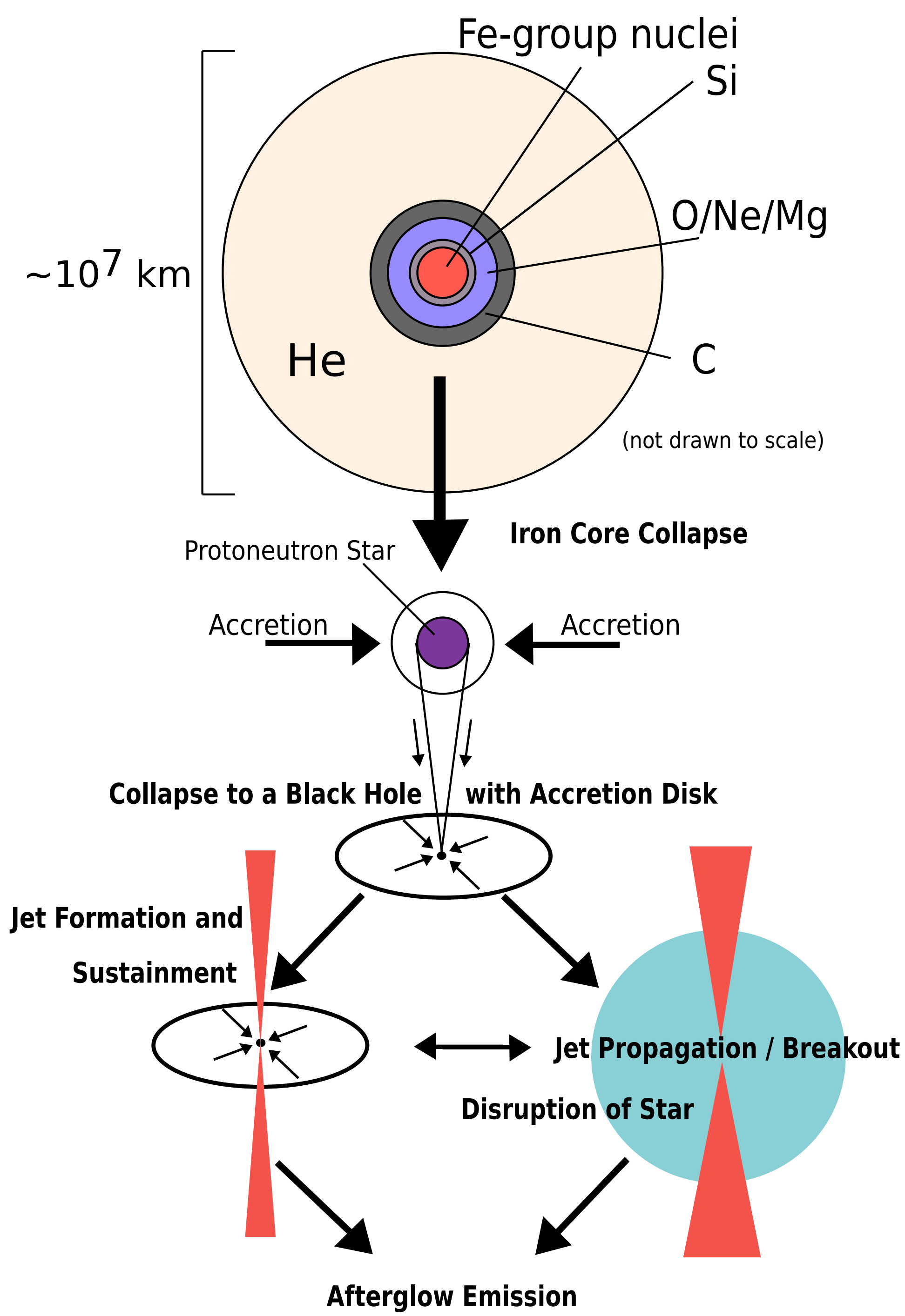


- Collapsing stellar iron cores (in supernovae)
- Beginning to take neutrino radiation into account

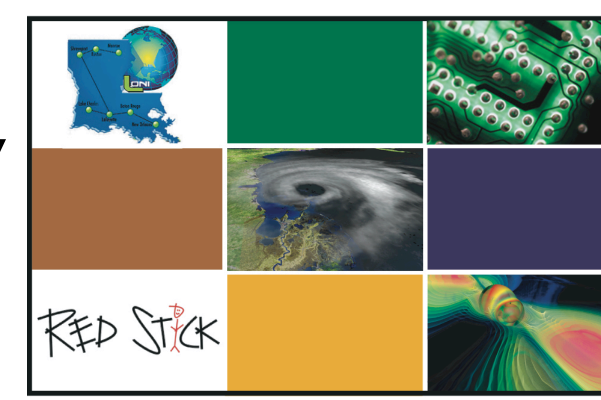


CENTER FOR COMPUTATION
& TECHNOLOGY

Future: Gamma Ray Bursts?

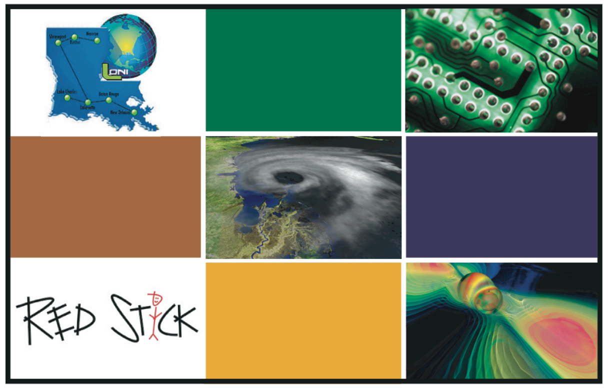


- GRB are intense, narrowly-beamed flashes of high-energy photons; most energetic events in universe
- Mechanism still a riddle; gravitational waves likely to be detected by LIGO in coming years
- GRB combine many different fields of physics (GR, MHD, neutrinos, radiation, nuclear physics, ...)
- Modelling requires very powerful computers



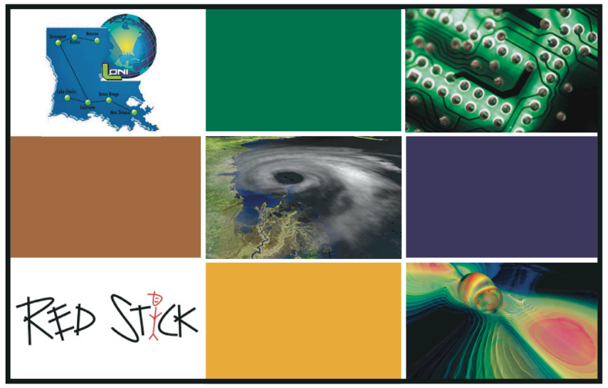
Summary

- Can now model general relativistic systems on computers; young and exciting field
- Not just black holes, also neutron stars, stellar core collapse, mixed binaries, exotic objects (strange stars, boson stars, ...)
- Requires careful use of large supercomputers – like performing experiments there
- Expected to benefit greatly from interaction with LIGO, LISA collaboration



References

- Spin Diagrams for Equal-Mass Black-Hole Binaries with Aligned Spins, arXiv:0708.3999 [gr-qc]
- The final spin from the coalescence of aligned-spin black-hole binaries, arXiv:0710.3345 [gr-qc]
- On the final spin from the coalescence of two black holes, arXiv:0712.3541 [gr-qc]



References

- Cactus: http://www.cactuscode.org/
- Carpet: http://www.carpetcode.org/