USING PHYSICAL MODELS IS *NECESSARY* TO GUARANTEE STABILE ANALOG HAPTIC FEEDBACK FOR ANY USER AND HAPTIC DEVICE

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ABSTRACT

It might be easy to imagine that physical models only represent a small portion of the universe of appropriate force feedback controllers for haptic new media; however, we argue the contrary in this work, in which we apply creative physical model design to re-examine the science of feedback stability.

For example, in an idealized analog haptic feedback control system, if the feedback corresponds to a passive physical model, then the haptic control system is guaranteed to be stable, as we show. Furthermore, we argue that it is in fact *necessary* that the feedback corresponds to a passive physical model. Otherwise, there exists a passive user-haptic device transfer function that can drive the feedback control system unstable. To simplify the mathematics, we make several assumptions, which we discuss throughout the paper and reexamine in an appendix.

The work implies that besides all of the known advantages of physical models, we can argue that we should employ only them for designing haptic force feedback. For example, even though granular synthesis has traditionally been implemented using signal modeling methods, we argue that physical modeling should still be employed when controlling granular synthesis with a haptic force-feedback device.

1. INTRODUCTION

1.1 Physical Modeling

In the field of sound and music computing, there is already a strong history of physical modeling. The most basic physical modeling approach is to study the physics of a musical instrument, and then to simulate the physical equations in a computer to synthesize sound [1, 2, 3]. However, besides merely imitating pre-existing musical instruments, new virtual instruments can be designed with a computer by simulating the acoustics of hypothetical situations [4], creating a "metaphorisation of real instruments." Of particular importance is also that sounds generated using physical models tend to be physically plausible, enhancing the listener's percept due to familiarity [5, 6].

Physical models can also be employed for real-time interaction. Here the perceptual advantages can be augmented by the apparent physical reality of the simulation. For example, when interacting with a virtual acoustical object, if the user changes the interaction point and the sound changes appropriately, the immersiveness of the user's experience is enhanced, as well as the quality of the generated sound. This property is not immediately offered by techniques such as sampling, unless the musical instrument's sound is sampled at all the possible interaction points and typically also at many different excitation velocities, which can require recording and storing large amounts of data.

By employing appropriate environments for generating large-scale physical models, composers can even create entire pieces using the physical modeling paradigm. For instance, initial conditions for mass trajectories can control the evolution of a piece, or complex inner simulated dynamics can also control timbres, notes, phrases, and even whole movements [7].

1.2 Haptic Force-Feedback Interaction According to the Ergotic Function

On a philosophical level, Claude Cadoz already defined three functions according to which a user can interact with an environment (physical or virtual). The first function is the *epistemic* function, which pertains primarily to knowing, for which a user can use the eyes, ears, or kinaesthetic and tactile touch receptors. The second function is the *semiotic* function, which users employ for transmitting symbolic information by way of the voice and body language.

In contrast, when a user exchanges significant mechanical energy with the environment by way of gesticulating, he or she uses the third, *ergotic* function for interaction [8]. For instance, employing a tool to deform an object or move it is ergotic. Bowing a string or playing a drum is also ergotic. In ergotic interaction, the user not only informs and transforms the world, but the world also informs and transforms the user. This is in some sense a consequence of Newton's third law: for every force, there is an equal and opposite reaction force.

Figure 1. Hand holding haptic device.

The ergotic function can be substituted by neither the epistemic nor semiotic function. In total when the ergotic, semiotic, and epistemic functions are simulated, it is possible for a user to ultimately engage in *instrumental interaction* using a haptic device controlled by a physical model [9]. In the remainder of the paper, we will argue that we should employ physical models for programming haptic force-feedback controllers, effectively implying also that the ergotic function and instrumental interaction should be simulated using physical models. We begin with a more formal discussion of feedback control.

2. FEEDBACK CONTROL

Feedback systems with active elements have the potential to become unstable. For example, acoustic feedback from a loudspeaker into the microphone of a public address (PA) system can cause the PA system to become unstable. Typically howling sets in, where one or a few sinusoids steadily increase in volume until the PA system cannot become any louder or is reconfigured. Howling instability is unpleasant for listeners and should be avoided.

Force feedback haptic devices, such as the one shown in Figure 1, can similarly become unstable. A circuit calculates a feedback force as a function of the orientation of the device (and its history), and the feedback force is exerted on the device. If the haptic feedback control system becomes unstable, then the haptic device can begin to move about erratically [10]. The haptic device can be damaged, other objects in the vicinity of the device can be damaged, and if the device is particularly strong, the user could even be injured.

While instability can be interesting for art, it is clearly important to be able to design haptic feedback systems that are guaranteed to be stable. For simplicity of the mathematics and analysis, we initially assume

- zero feedback control delay,
- infinite control bandwidth,
- time-invariance,
- linearity, and
- zero initial conditions

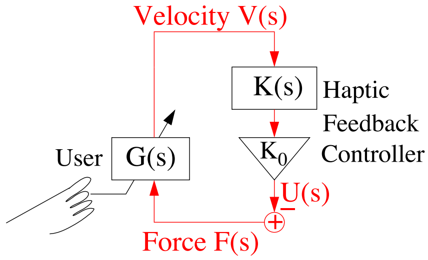

Figure 2. User $G(s)$ connected with haptic controller $K_0K(s)$ in feedback.

to arrive at a practical result, even under real conditions such as digital haptic feedback control (see Appendices A.1-A.3 and B for more discussion of the assumptions). Consequently, the analog feedback control system can be represented in the Laplace s-domain. For a time-domain function $g(t)$, the right-sided Laplace transform is

$$G(s) = \mathscr{L}\{g(t)\} = \int_{0^-}^{\infty} g(t)e^{-st}dt. \qquad (1)$$

A block diagram for the feedback control system is shown in Figure 2. $V(s)$ represents the velocity of the user's hand coupled to the haptic device,[1] and $F(s)$ represents the force exerted on the haptic device. In the absence of additional external forces, $F(s) = -U(s)$, where the minus sign is used to emphasize that the system is designed to operate using negative feedback. $K(s)$ represents the haptic feedback control filter, where the scalar loop gain K_0 has been separated out (see Figure 2). Because we employ a negative feedback configuration, we take the gain

$$K_0 \geq 0. \qquad (2)$$

3. USER-DEVICE TRANSFER FUNCTION *G(s)*

Since the user's hand and the haptic device are physical devices, *they can always be represented using a physical model* $G(s) = V(s)/F(s)$. Furthermore, since we are considering the linear, time-invariant case, the user's hand is stationary and holding the end of the haptic device with a constant grip. There is some friction at all frequencies, meaning that we assume the hand coupled to the haptic device can only dissipate energy, never create it.[2] Consequently, in the absence of feedback control, the force $F(s)$ and velocity $V(s)$ of the haptic device will never be far enough out of phase with one another to create energy. Mathematically, this implies that

$$|\angle G(s)|\Big|_{s=j2\pi f} < 90^\circ \qquad (3)$$

for all frequencies f in Hz [12]. (Mathematically speaking, $G(s)$ is *strictly positive real*. For more information, consult Appendix D.)

[1] For convenience, we employ velocities rather than positions. With the zero initial conditions assumption, one can easily convert between velocity and position by integrating and in the other direction by differentiating.

[2] Technically speaking, it could be possible for a user to intentionally destabilize some passive physical models by continually adding energy at low frequencies; however, users seem usually to be sensible enough not to do this [11].

4. *SUFFICIENCY* OF PHYSICAL MODELS FOR HAPTIC FEEDBACK CONTROL

For the moment, assume that the controller $K(s)$ is determined using a passive physical model. That is, the model consists of passive elements and no energy sources. For example, the model might consist only of masses, springs, and viscous dampers (or equivalently capacitors, inductors, and resistors) all with non-negative coefficients. Then by an analogous argument to the one in the prior section, we have that

$$|\angle K(s)|\Big|_{s=j2\pi f} \leq 90° \tag{4}$$

for all f since $K(s)$ is *positive real* (see Appendix D) [12]. Note that we allow frequencies at which there is zero damping—this could for example happen if there were no dampers/resistors and would result in an angle of 90° or $-90°$.

Next, we show that the net control system is guaranteed to be stable; however, to do so, we need to first introduce a criterion for determining the stability of the control system.

4.1 Revised Bode Stability Criterion

Since neither $K(s)$ nor $G(s)$ has any unstable poles, we can employ the Revised Bode Stability Criterion to state that the feedback system is stable if no "candidate unstable" frequency f_u exists for which:

$$|K_0 K(s) G(s)|\Big|_{s=j2\pi f_u} \geq 1 \tag{5}$$

and

$$\angle\big(K_0 K(s) G(s)\big)\Big|_{s=j2\pi f_u} = -180° - n(360°), \tag{6}$$

for any integer n [13]. In other words, the system is stable if there is no frequency at which a sinusoid traveling all the way around the loop could interfere perfectly constructively with itself (due to (6)) with the magnitude of that sinusoid increasing with every loop (due to (5)).

4.2 Proof Of Stability

Since (2), (3) and (4) hold, there is no frequency f for which (6) can hold. Thus, we have showed that the haptic feedback control system must be stable if controlled by a physical model, for any passive user-device transfer function $G(s)$. Roughly speaking, the stability is independent of the choice of haptic device and what the user is doing.

4.3 Unconditional Stability

Note that the stability is also independent of the magnitude of K_0. In other words, the control gain can be arbitrarily large! This remarkable property is known as *unconditional stability* [14, 15]. It indicates that under our ideal assumptions, the values of the physical model do not matter—the haptic control system is guaranteed always stable.

5. EXAMPLE

We now illustrate the proof of stability using a concrete example of a user touching a virtual resonator.

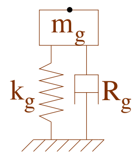

Figure 3. Simplified physical model of user-device.

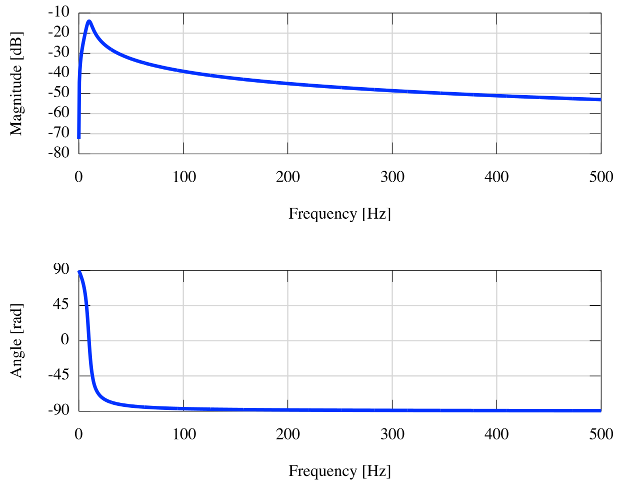

Figure 4. Magnitude and phase of user-device $G(s)|_{s=j\omega}$.

5.1 User-Device $G(s)$

Although the user can exert forces at will on the resonator, we consider these forces as inputs to the control system and not as part of the feedback loop, so we do not need to model them when examining the stability of the feedback loop. Hence, we provide a simple physical model of a user coupled to a haptic device in Figure 3. The mass m_g represents the mass of the user's hand coupled to the haptic device. The user's hand in conjunction with the haptic device presents a stiffness of k_g and viscous damping R_g. Much more complex models could be employed at this point, but it is not necessary for the illustrative purposes of this paper [16]. We can use the model to find $G(s)$:

$$G(s) = \frac{V(s)}{F(s)} = \frac{s}{m_g s^2 + R_g s + k_g}. \tag{7}$$

The model values could vary significantly, so for example we employ approximate parameters obtained by averaging results from a subject test, in which subjects gripped a haptic device with a grip force of about 9N [11]. In other words, we chose $m_g = 143$ g, $R_g = 5$ N/(m/s), and $k_g = 0.538$ N/mm. The phase response of the user coupled to the haptic device is shown in Figure 4. As required by (3), the phase response lies within the range (-90° 90°) as illustrated in Figure 4, bottom.

5.2 Controller

The physical model for the controller is shown in Figure 5. The musical resonator has mass $m_v = 4$ g and damping coefficient $R_v = 0.01$ N/(m/s), setting the exponential decay time constant to 0.8 sec. To make the resonance frequency approximately 300Hz, we choose stiffness $k_v = 14.2$ N/mm. To limit the force that the haptic

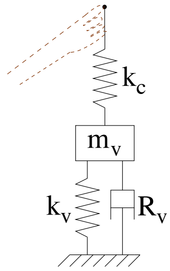

Figure 5. Physical model employed to derive controller $K_0K(s)$.

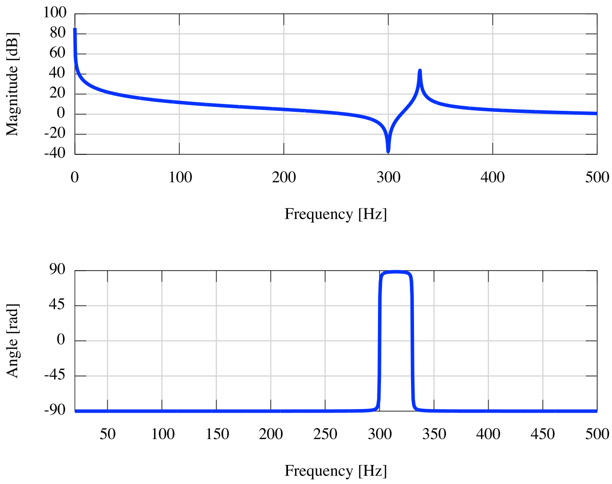

Figure 6. Magnitude and phase of controller $K_0K(s)|_{s=j\omega}$.

device must display, an additional spring is incorporated into the model $k_c = 3$ N/mm.

Solving the equations of motion and converting to the Laplace domain, we arrive at the following, from which it can be seen that k_c plays a role similar to the loop gain:

$$K_0K(s) = \frac{U(s)}{V(s)} = \frac{k_c}{s} \cdot \frac{m_v s^2 + R_v s + k_v}{m_v s^2 + R_v s + (k_v + k_c)}. \quad (8)$$

The magnitude response of the controller is shown in Figure 6 (top), which shows that the resonance frequency is increased slightly to 330Hz due to the presence of k_c. There is also an anti-resonance frequency still at approximately 300Hz. However, because the controller represents a physical model, its phase response still lies with in the range [-90° 90°] as specified by (4), even though it comes close to its allowable boundaries in Figure 6 (bottom).

Hence, as proved in Section 4.2, no "candidate unstable frequency" f_u exists satisfying (5) and (6), so the control system is guaranteed stable. This will hold for *any* passive physical model employed to specify the controller $K_0K(s)$, which is convenient for musical practice.

6. *NECESSITY* OF PHYSICAL MODELS FOR HAPTIC FEEDBACK CONTROL

Among the sound and music computing community, it appears not to be known that in the following sense, it is in fact *necessary* for the haptic controller $K(s)$ to correspond to a passive physical model. The reason for this is that the set of passive, linear physical models with collocated input and output is the same as the set of passive, linear transfer functions $K(s)$ (see [12] and Appendix D). Indeed both of these sets share the same phase relationship described by (4).

Otherwise if $K(s)$ does not correspond to a passive linear physical model, there exists a passive user-device $G(s)$ for which the feedback system can be driven unstable—in other words, a passive user and haptic device could be found for which the haptic control system would be unstable.

The proof of necessity is too long to be included in this conference paper without eliminating the examples [17]. Nevertheless in summary, it is a proof by construction that is analogous to choosing $G(s)$ such that (6) holds at some frequency f_u, and then increasing the scalar gain of $G(s)$ until (5) also holds.[3]

7. MAIN RESULT

Thus we have arrived at what we consider to be a rather remarkable result:

> if stable feedback control of a haptic device is desired for applications in new media, then we argue that designers should not start by simply employing any arbitrary feedback, rather they should design the feedback using physical models.

This has further implications particularly in sound and music computing. When employing a haptic device to control traditionally non-physical modeling sound synthesis engines, a physical modeling approach should nonetheless be employed. This is one reason why ACROE designed the CORDIS-ANIMA physical modeling language that incorporates passive physical modeling elements such as the mass, spring, friction, conditional link, etc. for simulating the ergotic function and enabling instrumental interaction [18].

For instance, if one were to implement haptic feedback control of granular synthesis [3], a good approach would be to model the grains as small masses flowing along a river. An independent external force would cause each mass to vibrate according to its own audio grain signal. Then the user could *dip* into the river using the haptic device via a conditional link, and the output audio signal would be generated by measuring the force exerted upon the haptic device. We would recommend this approach not only because of our positive experiences with physical models, but also because of the arguments in this paper.

[3] The proof involves counting the number of possible clockwise loops around the -1 point of the Nyquist plot of $K_0K(s)G(s)$. One key realization is that any counterclockwise loop would imply that either $K(s)$ or $G(s)$ were open-loop unstable, which would violate the assumptions, so there cannot be any counterclockwise loops. It is also necessary to observe that 1) a transfer function can never have a negative number of poles or zeros and 2) offsetting a transfer function by a complex constant does not change its poles.

8. CONCLUSION

Physical modeling for new media can indeed be a creative activity, and now having re-discovered this approach through a scientific stability analysis, we hope to have provided new insight for design of new media. We have attempted to present some results from the mechanical engineering literature [17] in a way that is accessible to the sound and music computing community. In re-presenting this work, we have gathered new perspective on the stability of feedback control systems and re-affirmed our enthusiasm for physical models.

Acknowledgments

We are grateful to the reviewers for their insightful comments, which we have attempted to integrate to make the paper as accessible as possible. Reviewers and readers interested in designing digital feedback controllers for plants with a specified delay may wish to consult work by Florens et al [19].

Finally, Edgar Berdahl would like to thank J. Edward Colgate for personal correspondence and Günter Niemeyer for informing him about the prior work [17].

A. NOTES ON ASSUMPTIONS

We now argue that the general results are still practically valid in light of the assumptions we made in Section 2.

A.1 User Is Time-Varying

In practice, $G(s)$ changes with time according to what the user is doing. For example, while completing certain tasks, the user changes the mobility of his or her hand [20, 21, 11]. However, this is in fact the point of the present paper—in order to ensure stable performance for *any* arbitrary user-device mobility $G(s)$, it is necessary that $K(s)$ correspond to a passive physical model. Hence, we argue that $K(s)$ may as well be designed using a passive physical model to guarantee stable performance [18].

A.2 Non-Ideal Feedback Characteristics

In practice, all feedback control systems have limited bandwidth. In addition, digital control systems exhibit additional delay in the control loop. Consequently, real controllers cannot perform significant feedback control at especially high frequencies. However, in practice, one observes that the simple theory in this paper predicts relevant aspects of practical performance, as long as one inserts a viscoelastic (optionally nonlinear) element in between the haptic device control point and the physical model, such as k_c (see Figure 5). This reduces the magnitude of the feedback control at high frequencies where practical digital control delay can be particularly problematic (see Appendix B) [10, 16].

It could also be argued that a digitized version of an ideally passive physical model may no longer be formally passive. However, we argue that the physical model should simply be discretized appropriately so that it causes an input-output delay of precisely one digital time unit. In practice then, this time unit is aligned with the converter sampling and results in no additional, unnecessary delay that could further affect the passivity [22, 23].

We relate the discussion here also to teleoperation, in which a "master" haptic device and a "slave" haptic device are linked together using force-feedback. Signal transmission delay between two separate locations can cause even more significant delay than digital sampling. In this case, the controller cannot form a good model of a simple damper and spring to link the devices together. However, this delay can be cleverly absorbed into the controller model by incorporating a vibrating string (or equivalently an electrical transmission line) into the controller model [24]. Again, we discover a solution based on physical models!

A.3 User Is Nonlinear

In practice, a real user is nonlinear. However, the user is also dissipative. If the haptic feedback controller is passive and corresponds to a physical model, then even if the physical model is nonlinear, by the conservation of energy, the energy in the feedback control system must dissipate over time in the absence of external excitation. Hence, although the *necessity* of the nonlinear case is apparently unproven,[4] conservation of energy proves *sufficiency*, and it is also very practical to design nonlinear haptic feedback controllers using nonlinear physical models [18].

A.4 Unstable Performance Could Be Desirable In Some Situations

In some situations, artists may desire to create control systems that are unstable. In fact, the E-Bow and Sustainiac are successful commercial products that drive vibrating guitar strings unstable in a controllable manner [25, 26]. Similarly, bowed strings, many wind instruments, and some drum roll techniques incorporate self-oscillations that have become accepted as sounding musical. Even the "unstable" Haptic Drum enables a performer to play arbitrarily complex drum rolls or drum rolls at superhumanly fast speeds [27].

Hence, there are some nonlinear situations where physical models will not be necessary for implementation, but they nevertheless seem to be sufficient given the very wide range of physical phenomena that could be modeled. Unstable behavior can be created using external energy source elements in physical modeling or negative dampers, or similar effects can be obtained by setting initial energetic conditions for objects. Indeed creativity causes us to rethink the science of physical modeling, so possible future work could someday involve studying necessity and sufficiency proofs for employing (nonpassive) physical models to implement unstable haptic musical simulations.

A.5 Transparency of Haptic Rendering

In this paper, we have considered only the stability of the control system according to classical passivity theory [17]. However, we have not considered how *transparently* the

[4] Personal correspondence with Ed Colgate on Jan. 17, 2011

physical model is presented to the user through the haptic feedback control system. In the classical representation, improving transparency (i.e. accuracy) requires increasing control gains, which can hamper the stability of digital control systems (see Section B). For this reason, Florens et al. have introduced a new method for deriving haptic feedback control systems, which consider the stability and transparency concurrently [19]. The result is a whole new paradigm for deriving haptic feedback controllers. The method involves modeling the coupling of the user-device to the physical model (called a *temporary hybrid system* or *THS*) and adjusting model parameters to achieve optimum dynamics [19]. We are actively carrying out further research in this domain.

B. DIGITAL DELAY

In practice, it is usually more practical to employ digital feedback instead of analog feedback, especially because computers are now so widely available and inexpensive. However, digital feedback control always causes delay in the control loop, which is due to

- analog-to-digital conversion (ADC),
- computation time,
- possible additional delay due to operating system, interrupt, and bus mechanisms on the computer, and
- digital-to-analog conversion (DAC).

Although the delay is always longer than half of one sampling interval T, this is nevertheless a convenient approximation, assuming a conventional implementation of the control loop elements [28, 23].[5] Hence, (8) becomes the following:

$$K_0K(s) = \frac{k_C}{s} \cdot e^{-sT/2} \cdot \frac{m_v s^2 + R_v s + k_v}{m_v s^2 + R_v s + (k_v + k_c)}. \quad (9)$$

The sampling interval T has no effect on the magnitude of (9) (see Figure 7, top). However, the phase response for $T = 1$ ms is shown in the thinner line of Figure 7 (bottom). There is a linear trend to further and further negative phases, which causes the phase response to dip beneath the allowable limit -90°. The phase response for $T = 0.1$ ms is shown in the thicker line of Figure 7 (bottom). It remains much closer to the limit, but for sufficiently high frequencies falls outside of the allowable range, preventing the digital controller from perfectly calculating feedback equivalent to a delayless, analog physical model.

Nevertheless, it turns out that both for $T = 1$ ms and $T = 0.1$ ms, the control system is still stable for these parameters. However, given the digital control delay, it is possible to drive the system unstable by increasing k_c, which is analogous to eliminating the "stabilizing" compliant spring k_c and attempting to bind the haptic device

[5] Converters implemented using sigma-delta modulation are usually not fast enough, so for low-latency feedback control successive approximation ADCs and resistor-ladder DACs are often used.

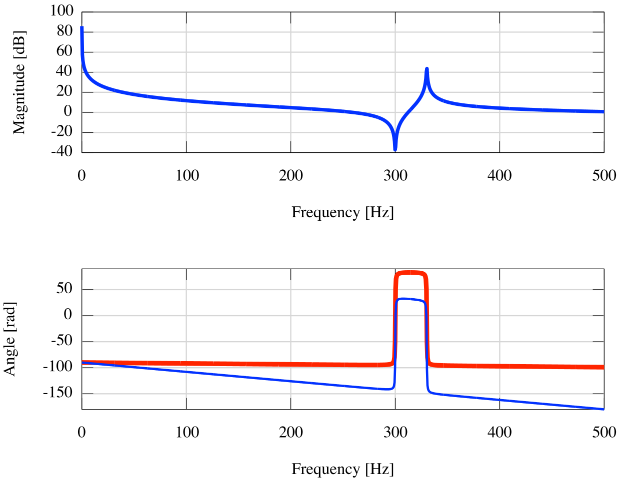

Figure 7. Magnitude and phase of digital controller $K_0K(s)|_{s=j\omega}$.

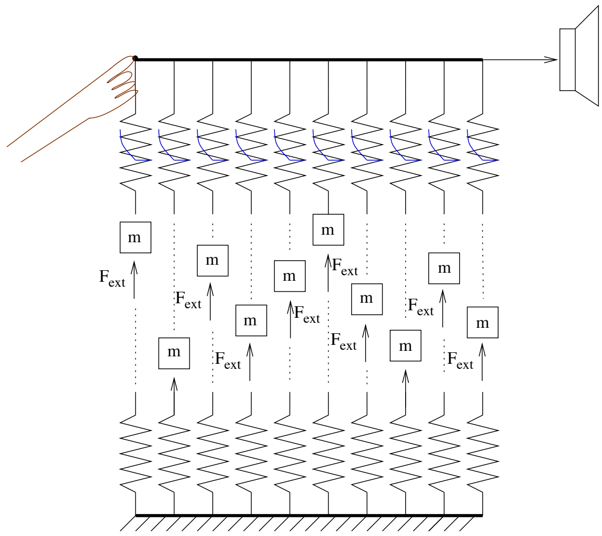

Figure 8. Physical analog model for granular synthesis using ten trapped masses.

directly to the resonator itself. However, this would be contrary to our approach of promoting stability by inserting the compliant element k_c at the interaction point. Indeed, for a more precise argument but specific to a less interesting scenario for sound and music computing, the reader should read the work by Diolati et al. [16]. Typically the shorter T can be made, the further k_c can be increased.

C. REAL-TIME USER INTERACTION WITH MORE COMPLEX EXAMPLE

To demonstrate the viability of the technology, even for digital feedback control, we briefly present an example with real-time user interaction. A physical model for a kind of granular synthesis is shown in Figure 8, in which ten "grain" masses fly back and forth vertically between a mechanical ground (below) lined with linear contact springs and squared-nonlinearity contact springs along a rigid bar (top) coupled to the user's hand by the haptic device. Despite the specificness of this model, many other physical model scenarios could be employed to implement granular-

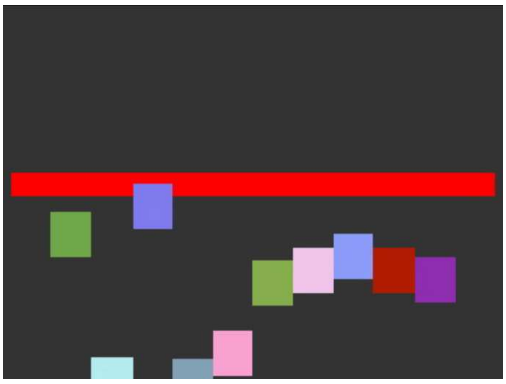

Figure 9. Snapshot from visual output of demonstration.

type synthesis. In the present example, an external force F_{ext} acts on each mass, causing it to vibrate according to input sounds. The force exerted on the haptic device is measured, highpass filtered (not shown), and passed to the audio output, as represented by the loudspeaker schematic symbol in Figure 8, top right.

For such complicated models, visual feedback is also helpful. Hence, we incorporated visual feedback in a demonstration video.[6] A snapshot of the video is shown in Figure 9. Rather than adjusting the amplitude, frequency, density, grain length, and similar signal parameters, as is typical in granular synthesis [3], these sonic characteristics are adjustable by physical means such as the position of the haptic device, force applied to the haptic device, the stiffness and damping of the hand, etc.

D. POSITIVE REAL FUNCTIONS

For the mathematically minded, we present some information on positive real functions for describing mathematically passive systems. It underscores their equivalence to passive physical models.

Positive real functions were introduced in 1931 for synthesizing transfer functions corresponding to electrical analog circuits [12]. Since then, a rational function $\tilde{K}(s)$ has usually been defined to be *positive real* if and only if

- $\tilde{K}(s)$ is real when s is real, and
- $Re\{\tilde{K}(s)\} \geq 0$ for all s such that $Re\{s\} \geq 0$.

and similarly, a rational function $\tilde{G}(s)$ has usually been defined to be *strictly positive real* if

- $\tilde{G}(s+\epsilon)$ is positive real for all real $\epsilon > 0$ [29].

However, for our purposes it is much more convenient to use the following equivalent definitions in terms of the angle along the frequency axis. We define the rational function $\tilde{K}(s)$ to be *positive real* if and only if

- $|\angle\tilde{K}(j2\pi f)| \leq 90°$ for all frequencies f,

and similarly the rational function $\tilde{G}(s)$ is *strictly positive real* if and only if

- $|\angle\tilde{G}(j2\pi f)| < 90°$ for all frequencies f [29].

[6] http://ccrma.stanford.edu/~eberdahl/CompMusic/GCG.m4v

Some further properties of positive real and strictly positive real functions are [30]:

1. $1/\tilde{K}(s)$ is positive real.
2. $1/\tilde{G}(s)$ is strictly positive real.
3. If $\tilde{K}(s)$ represents either the driving point impedance or driving point mobility of a system, meaning that the sensor and actuator must be collocated, then the system is *passive* as seen from the driving point. **In other words, if $\tilde{K}(s)$ is positive real, then it corresponds to a passive, linear physical model, and vice versa.**
4. If $\tilde{G}(s)$ represents either the driving point impedance or driving point mobility of a system, meaning that the sensor and actuator must be collocated, then the system is *dissipative* as seen from the driving point. **In other words, if $\tilde{G}(s)$ is strictly positive real, then it corresponds to a dissipative, linear physical model, and vice versa.**
5. $\tilde{K}(s)$ and $\tilde{G}(s)$ are stable.
6. $\tilde{K}(s)$ and $\tilde{G}(s)$ are minimum phase.
7. The relative degrees of $\tilde{K}(s)$ and $\tilde{G}(s)$ must be less than 2.
8. No matter what causal time-domain function $f(t)$ is used to excite the driving point, the velocity response $v(t)$ will be such that $\int_0^\infty f(t)v(t)dt \geq 0$.

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