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Active damping of a vibrating string

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ABSTRACT

This paper presents an investigation of active damping of the vertical and horizontal transverse modes of a rigidly-terminated vibrating string. A state-space model that emulates the behavior of the string is introduced, and we explain the theory behind band pass filter control and proportional-integral-derivative (PID) control as applied to a vibrating string. After describing the characteristics of various actuators and sensors, we motivate the choice of collocated electromagnetic actuators and a multi-axis piezoelectric bridge sensor. Integral control is shown experimentally to be capable of damping the string independently of the fundamental frequency. Finally, we consider the difference between damping the energy in only one transverse axis, versus simultaneously damping the energy in both the vertical and horizontal transverse axes.

1 INTRODUCTION

The study of *modal stimulation* is the study of actively controlling the vibrating structures in a musical instrument with the intent of altering its musical behavior [1]. Although it is possible to design an instrument such that many aspects are easily controllable, this study applies control engineering to a core component found in many mainstream instruments, the vibrating string. As a result, the actively-controlled instrument is accessible to many musicians; however, this approach makes the control task more challenging because the large number of resonances is not ideal from a control perspective.

We choose to limit the effects of the instrument's body and focus on the vibrating string. This forms a foundation for understanding modal stimulation because the musician interacts primarily with the string. Also, since canceling vibration is more difficult than inducing vibration, we go directly to the heart of the matter by focusing on a detailed study of actively damping the string. Furthermore, while damping the string, we strive to preserve the musical qualities of the string that have historically been optimized during the evolution of stringed instruments.

From a practical standpoint, we also desire the damping to be independent of the fundamental frequency (the lowest resonance) f_0. That is, to provide stronger command of the actively-controlled instrument by the musician, we wish for the damping parameter to be orthogonal to the string length, which is adjusted by the musician during play.

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2 PRIOR WORK

2.1 "Infinite" Sustain For Guitars

Various forms of actively-controlled musical instruments have been designed. For instance, the inverse problem of indefinitely sustaining string vibration has long been investigated, especially in the framework of electric guitars. Musicians have used acoustic feedback from power amplifiers to re-excite their electric guitar strings to produce additional sustain; however, due to the complex nature of the transfer functions involved and the nonlinear nature of the amplifiers, this approach has proven difficult to control precisely. The commercially available EBow [2] and Sustainiac [3] have mitigated this problem somewhat using locally placed sensors and actuators. In a similar manner, Weinreich and Caussé have electromechanically induced the Helmholtz "stick-slip" bowing motion in a vibrating string without using a bow [4].

2.2 Active Structural Control For Acoustic Guitars

Active control has also been applied to the body structures of instruments. For example, in order to suppress tendencies toward acoustic feedback in amplified situations, Griffin actively damped the first plate mode in an acoustic guitar [5]. Hanagud boosted the second plate mode in an inexpensive acoustic guitar to make it sound more like an expensive acoustic guitar [6]. In addition, Charles Besnainou actively tuned the Helmholtz body resonant frequency of a guitar [7].

2.3 Other Actively-Controlled Musical Instruments

Besnainou coined the term "modal stimulation" and applied this technique to a violin, a snare drum, a pipe organ, and a marimba bar [1]. For example, he changed the damping time and pitch of a marimba bar using PID control [8]. Besnainou carried out additional work on other instruments, but much of this work was not published.

More recently, Rollow wrote his PhD thesis on controlling the vibrations of an air-loaded drum head [9], and Maarten van Walstijn and Rebelo used various filtering techniques to alter the modes of a conga drum [10].

3 OVERVIEW

3.1 Physics of a Vibrating String

Transverse waves are largely responsible for determining the musical characteristics of a vibrating string. When an ideal string is actuated transversely in only the vertical *y*-axis, transverse waves arise exclusively in the *y*-axis along the string. Standing waves result when the string is terminated at two or more points. The standing waves can be any linear combination of vibrations at the harmonics, which are the resonances at nf_0 for $n = 1,2,3,\ldots$.

However, strings on musical instruments vibrate in both the vertical and horizontal axes because musicians excite the string in both axes, and non-ideal string terminations allow an exchange of energy among the horizontal and vertical axes. Furthermore, various nonlinear effects can cause additional exchanges of energy between the transverse axes and even energy exchanges between the harmonics [11].

3.2 Control Configuration

In order to control the transverse waves of a vibrating string, both the vertical and horizontal axes (or some invertible transformation thereof) must be sensed and actuated. Although there are some minor exchanges of energy between the axes, any robust control strategy should allow these exchanges to be neglected. We thus assume the axes are independent of each other so that a simplified, independent controller may be designed for each axis.

Figure 1 shows the block diagram for controlling the vertical axis. *K(s)* represents the transfer function of the controller, *Gain* represents the amount of control used, and *G(s)* represents the transfer function of the vibrating string between the actuator and sensor. Note that *G(s)* is time-varying as the musician changes the length of the string in order to adjust f_0.

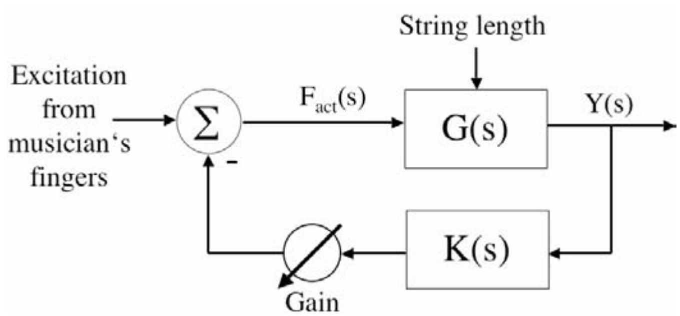

Figure 1: System block diagram for the vertical axis.

3.3 Control Strategy

Actively damping the string consists of driving the string's displacement to zero. Collocated control generally makes the problem easier, so we will assume that the string is sensed and actuated at the bridge (see section 5 for details about the actuators and sensors). Active damping at the bridge is equivalent to decreasing the resistive component of the bridge termination's effective impedance. We should, however, prevent the bridge termination impedance from becoming complex. This would result in detuning the harmonics of the string, which would conflict with our goal of preserving the musical qualities of the instrument [12].

3.4 State-Space Model

Since the lowest resonance of a vibrating string usually has the longest decay time, the simplest state-space model of the string addresses only the lowest resonance at f_0. Figure 2 shows the physical diagram of a simple system with one resonance, where *y(t)* is the displacement of the mass, and $f_{act}(t)$ is the force exerted on the mass.

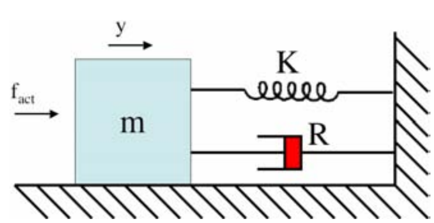

Figure 2: A simple mechanical oscillator with damping.

The behavior of the system can be easily represented in the Laplace domain:

$$(ms^2 + Rs + K)Y(s) = F_{act}(s) \tag{1}$$

By matching the decay time constant $\tau = 1/\alpha = 2m/R$ (in sec) and $\omega_0 = \sqrt{K/m}$ (in rad/sec) to those of the lowest resonance of a vibrating string, we can construct a simple model of a vibrating string. The transfer function of the linear system between the input force and the output displacement is then as follows:

$$\frac{Y(s)}{F_{act}(s)} = \frac{1}{m}\frac{1}{s^2 + 2\alpha s + \omega_0^2} \tag{2}$$

If we consider for example the $N = 10$ lowest resonances, then we can build a more accurate model. Since the modes are ideally independent of each other, the resonances may be placed in parallel:

$$G(s) = \frac{Y(s)}{F_{act}(s)} = \sum_{n=1}^{N}\frac{1}{m_n}\frac{1}{s^2 + 2\alpha_n s + (n\omega_0)^2} \tag{3}$$

The damping rates α_n for a particular string can be approximated by computing the Energy Decay Relief (EDR) of the string's impulse response and fitting straight lines through the decays of the harmonics on a log scale [13].

4 CONTROL ALGORITHMS

4.1 Band Pass Filter Control

Before investigating classical control methods, it is worth mentioning that a simple controller can be built using standard audio hardware. For example, our implementation includes a personal computer, a standard audio interface, and the signal processing environment for computer music known as Max/MSP. The system latency due to the ADC, software buffering, and DAC is roughly 6ms.

The block diagram for the controller is shown in Figure 3. The controller uses precisely-tuned band pass filters to isolate the energy in each of the harmonics. The phase of the output from each filter can be adjusted manually so that the associated mode either grows or decays over time. The gain adjustments control how fast each mode grows or decays. Due to the large number of orthogonal parameters, this control method is particularly useful for testing the controllability and the nonlinearity of a highly-resonant system. One drawback of the controller, however, is its sensitivity to the fundamental frequency f_0 and the quality factor Q of the filters. $Q = 35$ generally makes for a good compromise between sensitivity and orthogonality of the parameters.

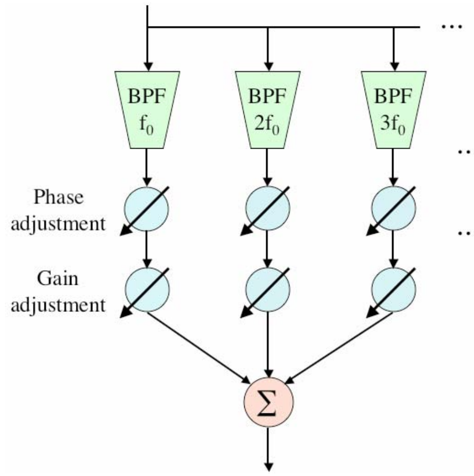

Figure 3: Block diagram for the controller $K(s)$ consisting of a bank of band pass filters.

4.2 PID Control

Classical controllers are much more appealing since the damping can be made independent of f_0. This makes them much more useful in a performance context. The analysis is simple for controlling a single resonance using the proportional, integral, and derivative functions of the velocity. For unity with the following sections, we equivalently investigate the second derivative (P_{DD}), the derivative (P_D), and the proportional (P_P) functions of the displacement $y(t)$:

$$(ms^2 + Rs + K)Y(s) = F_{act}(s) = (P_{DD}s^2 + P_D s + P_P)Y(s) \tag{4}$$

$$\left[(m - P_{DD})s^2 + (R - P_D)s + (K - P_P)\right]Y(s) = 0 \tag{5}$$

The solution is of the following form:

$$y(t) = C_0 e^{-\hat{\alpha}t} \cos(\sqrt{\hat{\omega}_0^2 - \hat{\alpha}^2}\, t + \hat{\phi}) \tag{6}$$

The undamped frequency of vibration $\hat{\omega}_0$ is affected by P_P and P_{DD}:

$$\hat{\omega}_0 = \sqrt{\frac{K - P_P}{m - P_{DD}}} \tag{7}$$

Similarly, the damping rate $\hat{\alpha}$ is affected by P_D and P_{DD}:

$$\hat{\alpha} = \frac{R - P_D}{2(m - P_{DD})} \tag{8}$$

Thus, P_D may be used to control the decay rate independently from the frequency. This is the kind of orthogonal control that we desire.

4.2.1 Derivative Control

Since a vibrating string actually has multiple resonances, it is instructive to calculate the root locus for the P_D term (see Figure 4) using the state-space model. The magenta squares show the pole locations when a moderate amount of control is applied, such that the damping time is about the same as achieved in the experimental results presented in section 5 below. As expected, the additional damping is roughly independent of the frequency of the harmonic.

We might also be concerned that the truncated order of the model would drastically change the locus' characteristics, but since each resonance of the string is accompanied by a nearby anti-resonance, each harmonic affects the locus only locally.

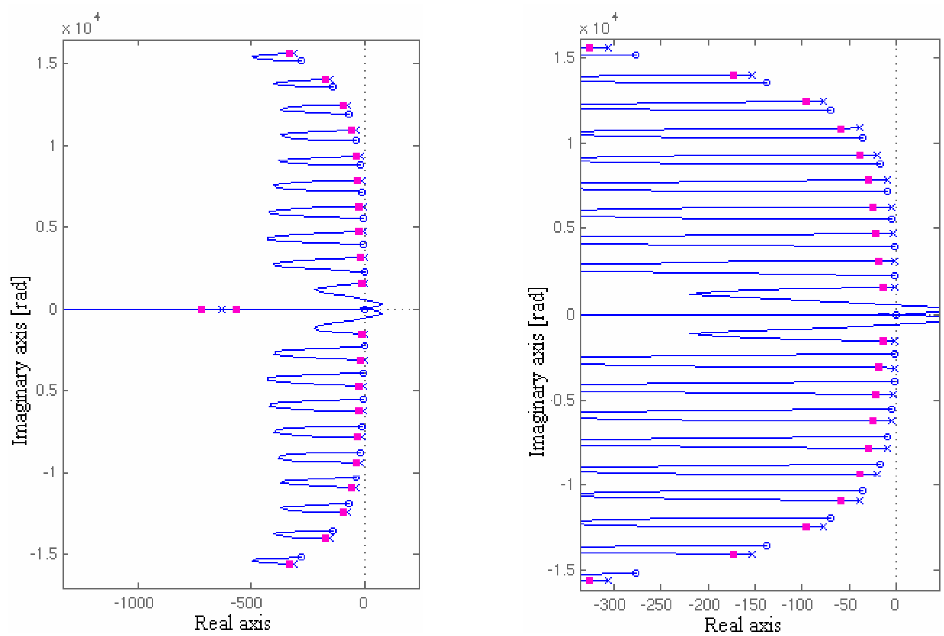

Figure 4: 180° Root locus (left: zoomed out, and right: zoomed in) for derivative control.

Because the lower harmonics of an uncontrolled string inherently have less damping, an optimum design should invest more control energy in damping the lower harmonics.

4.2.2 Integral Control

By applying integral control of the displacement (P_I), we can invest more of the control energy in damping the lower harmonics:

$$(ms^2 + 2\alpha ms + K)\tilde{Y}(s) = F_{act}(s) = \frac{1}{s} P_I \tilde{Y}(s) \tag{9}$$

A derivation of the effects of P_I can be approximated by assuming that $| P_I |$ is small enough such that the solution differs significantly from the case where $P_I = 0$ only in the damping coefficient $\tilde{\alpha}$:

$$\tilde{y}(t) = C_0 e^{-\tilde{\alpha} t} \cos(\sqrt{\omega_0^2 - \tilde{\alpha}^2}\, t + \phi) \tag{10}$$

Further simplifications can be made since $\omega_0 >> \tilde{\alpha}$ as long as the resonance is not being damped especially quickly. For example, if $\tilde{\alpha} = 1/\tilde{\tau} = 10$ sec^{-1} for a fairly fast decay time-constant of 1/10 sec, while $\omega_0 > 2\pi(248) \approx 1560$ sec^{-1}, then we certainly have $\omega_0 >> \tilde{\alpha}$. After substituting $\tilde{y}(t)$ into the time-domain version of the dynamics differential equation (9), simplifying, and solving for $\tilde{\alpha}$, the following approximation can be obtained in terms of the passive (uncontrolled) damping rate α:

$$\tilde{\alpha} \approx \frac{R + P_I / \omega_0^2}{2m} = \alpha + \frac{P_I}{2m\omega_0^2} \tag{11}$$

Thus, for moderately-sized $\tilde{\alpha}$, the increase in the damping rate decreases with $\omega_0^2 = K/m$. Integral control applied to the string model results in the 0° locus shown in Figure 5. As expected the lower harmonics are damped more than the higher harmonics.

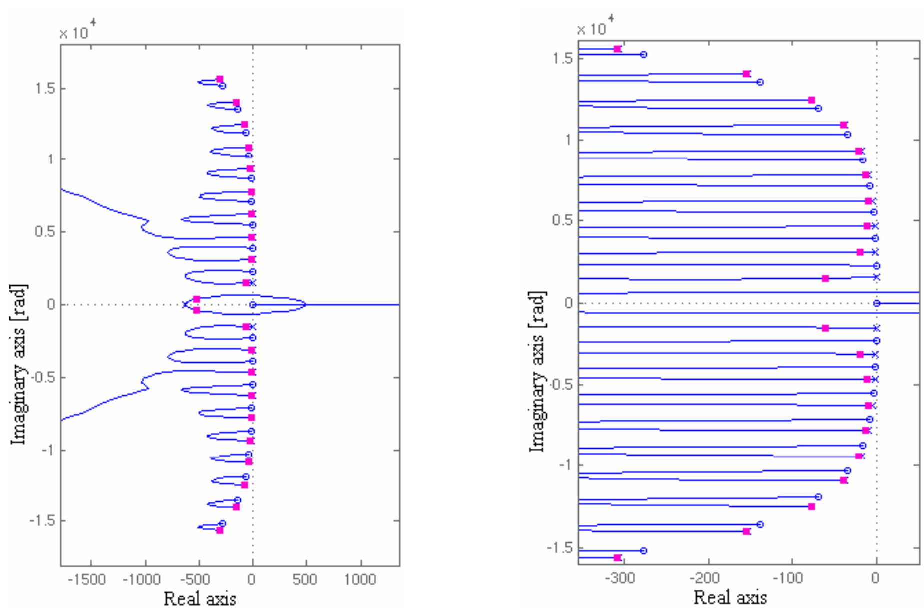

Figure 5: 0° Root locus (left: zoomed out, and right: zoomed in) for integral control.

Another advantage of integral control over derivative control is the low-pass characteristic of integration, which makes the integral type controller more robust to system delays. Theoretically, in order to damp the string as quickly and efficiently as possible, a combination of integral and derivative control would perform better than either type of control alone; however, we show that for the presented experimental system, integral control alone was suitable and robust despite the system delay.

5 EXPERIMENTAL RESULTS

We carried out several experiments to learn about the practical challenges involved in actively damping a vibrating string. In particular, we chose to use an electric guitar string due to its magnetic properties and the reasonably low tension required.

5.1 Sensors

We considered a number of different string motion sensors. Electromagnetic sensors are traditionally used in electric guitars, but they are large and behave more nonlinearly than other string sensors [15]. In contrast, piezoelectric sensors are smaller and linear, but they must usually be mounted at a string termination. Sensing both the vertical and horizontal transverse axes with piezoelectrics is more complicated, so we used a specially designed piezoelectric multi-axis string sensor [15]. In order to calibrate the multi-axis string sensor, we used a pair of optical sensors as well [11].

5.2 Actuators

We first investigated actuating the string with a piezoelectric device. However, piezoelectric bending elements have much smaller mechanical impedances than a guitar string, and piezoelectric stacks have much larger mechanical impedances than a guitar string. Surprisingly, there are no widely-available piezoelectric-based actuators that are matched to a guitar string's impedance. As a result, we chose to use electromagnetic actuators [14]. Due to the immediate canceling wave from the termination-reflection, the slight stiffness of the string, and the fact that the actuator "sees" an approximately 1cm long segment of the string, these actuators are inefficient when mounted too closely to the termination; however, they behave too nonlinearly when mounted too far from the termination, so a compromise was required.

5.3 Actuator/Sensor Placement

Since collocated control is generally more robust, we chose to mount the sensors and actuators as closely together as possible at the bridge (see Figure 6). In this manner, the musician may change the length of the string by moving the opposite string termination (not shown). A pair of electromagnetic actuators was placed 4cm to the left of the string termination (see Figure 6a) for a good compromise between efficiency and linearity. A pair of optical sensors was mounted 1cm from the string termination (see Figure 6b) in order to record the motion of the string and calibrate the piezoelectric multi-axis string sensor, which terminated the string (see Figure 6c). The piezoelectric sensor was used to close the control loop as it was less prone to noise than the optical sensors. All sensors and actuators were mounted such that the horizontal and vertical axes could be sensed and actuated approximately independently.

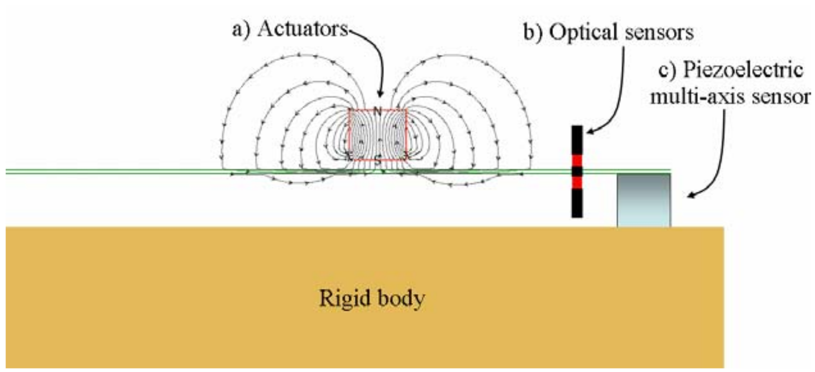

Figure 6: Diagram of one end of the string showing the actuator and sensor placement at the bridge.

5.4 Control Configuration

In order to reduce the effects of the body, we mounted the string on an 8.9cm x 8.9cm (3.5” x 3.5”) block of hardwood ash, and the string terminations were made as rigid as possible. The string was an E electric guitar string (1.1mm in diameter) with a tin-plated steel core, which was wound with a nickel-plated steel wire. We powered the actuators using a 100W/channel Marantz audio power amplifier and, because the current-output of the amplifier behaved roughly like an integrator above 200Hz, we shortened the string to approximately 24cm. The fundamental frequency then became $f_0 = 248\,\text{Hz}$.

Derivative control was found to be unsuitable for the experiment due to system delay—the actuator and sensor were not exactly collocated. In particular, the actuator was placed 4cm from the string termination, and so was near a node for the 6^{th}, 12^{th}, 18^{th},… harmonics. As a result, the system could only be considered collocated for the lowest four or five harmonics. Integral control was applied to greatly reduce the damping time of the string.

5.5 Dual-Axis Damping

Since the active instrument was designed to be played by a musician, the string was plucked by hand in the tests. Figure 7 depicts four actively-damped plucks followed by one pluck without control. The top waveform in Figure 7 shows the horizontal displacement of the string at the bridge, while the bottom waveform shows the vertical displacement.

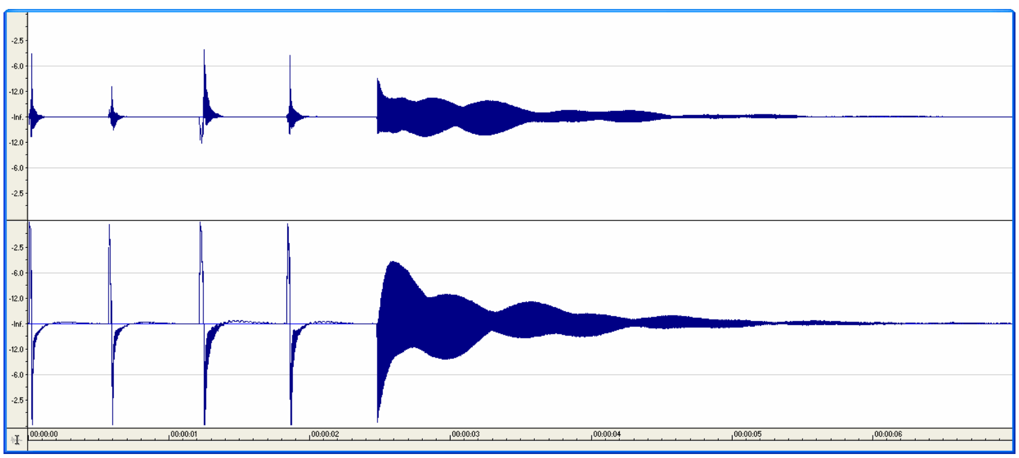

Figure 7: Waveforms displaying four actively damped string plucks, then one pluck without control. (top: horizontal axis, bottom: vertical axis)

Spectrograms of the same waveforms are displayed in Figure 8. The decay times of the lowest four modes were greatly reduced, while the decay times of the higher modes remained approximately constant. The small differences in the plucks were due to variations caused by the use of a human hand, but the plucks were nearly the same magnitude.

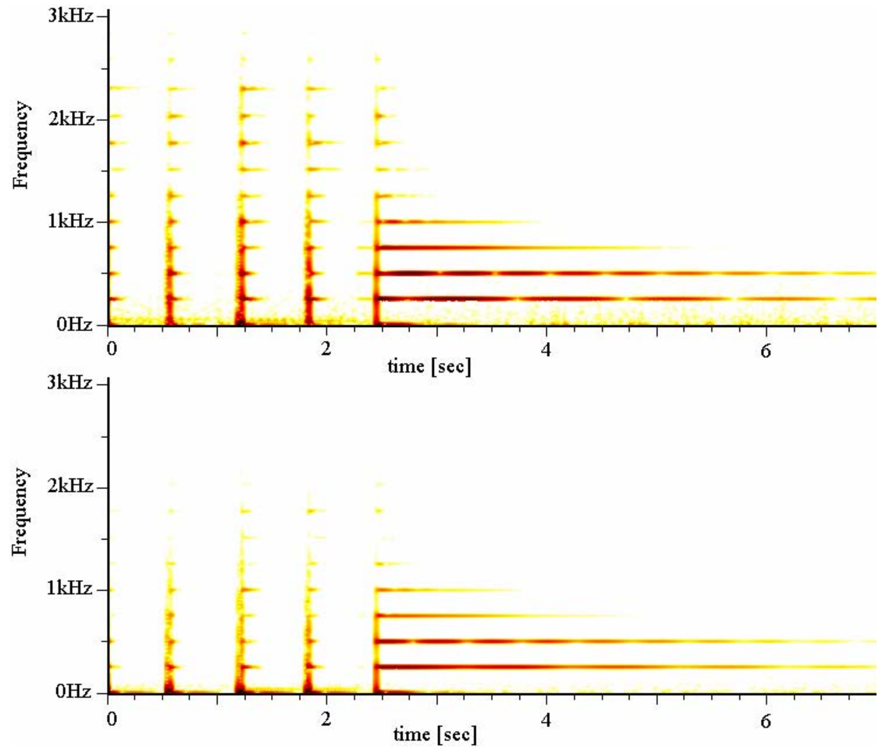

Figure 8: Spectrograms displaying four actively damped string plucks, then one pluck without control. (top: horizontal axis, bottom: vertical axis)

The string behavior was consistent and independent of fundamental frequency. Active damping required less than 1 Watt of power according to the meters on the Marantz amplifier. By varying the loop gain, it was of course possible to vary the damping over a large range of intermediate values.

5.6 Single-Axis Damping

In practice the sensors and actuators cannot be perfectly aligned, so there must be some exchange of energy between the axes. It follows that single-axis damping is a good test of sensor/actuator alignment.

Figure 9 shows the waveforms of the string displacement resulting from an experiment where the string was plucked by hand repeatedly at nearly the same magnitude: at the beginning (the left side of the waveforms), control was applied to only the vertical axis. Then, as time progressed, control was added to the horizontal axis. Near the midpoint, the amounts of control were equal, after which time the control applied to the vertical axis was slowly reduced to zero.

It can be seen that the vertical actuator and sensor were well aligned—at the beginning of the waveforms, the vertical axis remains largely damped even when the horizontal axis is not. On the other hand, the horizontal actuator and sensor were not perfectly aligned as the horizontal axis was only well-damped when both the vertical and horizontal axes were damped.

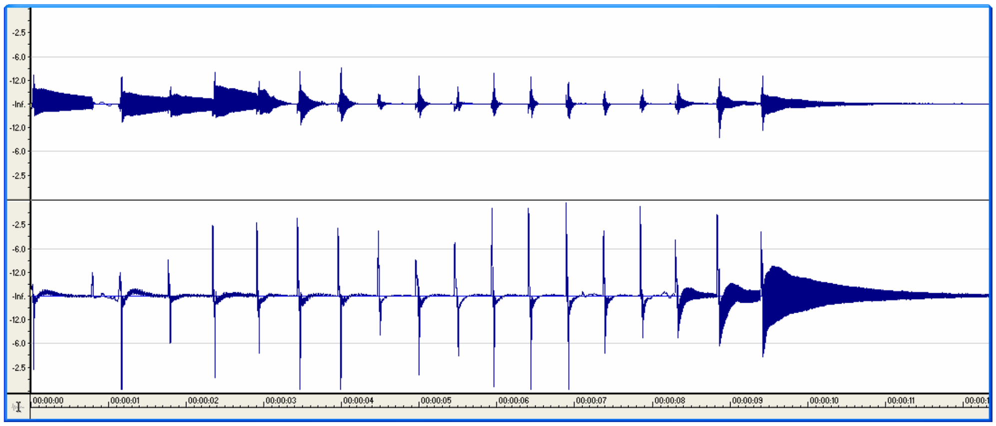

Figure 9: Waveforms displaying actively damped string plucks as the controlled axes are varied. (top: horizontal axis, bottom: vertical axis)

6 SUMMARY AND FUTURE WORK

We reviewed the prior work on actively-controlled musical instruments and developed a state-space model of the vibrating string. Then, we discussed band pass filter control, which could control the modes independently but was dependent on the fundamental frequency f_0 of the string. In contrast, PID control of the string's displacement did not depend on f_0 and could be used to damp the string quickly. Finally, we discussed the advantages and drawbacks of various string actuators and sensors and described an experimental setup. We presented waveforms of actively damped string vibrations and demonstrated that the sensors and actuators must be carefully aligned in order to obtain completely independent control of the vertical and horizontal transverse axes.

Future work will involve implementing various sound effects for the electric guitar directly on a guitar string. For example, the tremolo effect consists of periodically varying the amplitude of an input signal, and so the active damping parameter could be varied periodically to achieve this effect.

7 WAVEFORMS

The waveforms corresponding to the model, the magenta poles displayed in the root loci, and the experiments are available on the internet. They can be downloaded from the website: http://ccrma.stanford.edu/~eberdahl/Projects/ActiveDamping/index.html

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