Applications of Passivity Theory to the Active Control of Acoustic Musical Instruments

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Outline

Introduction

Passivity For Linear Systems

PID Control

Other Passive Linear Controllers

Nonlinear PID Control

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- The resulting instrument is like a haptic musical instrument whose interface is the whole acoustical medium.
- We apply the technology to a vibrating string, but the controllers are applicable to any passive musical instrument.

System Block Diagram

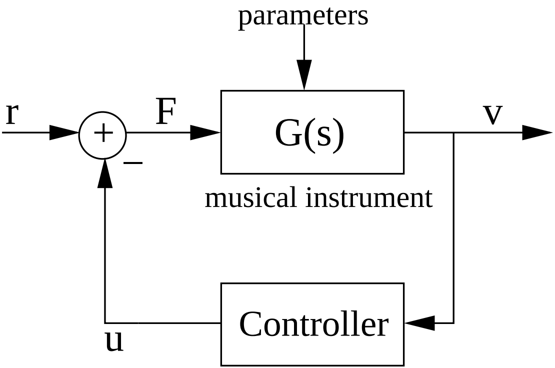

Figure: System block diagram for active feedback control

- We would like the controller to be robust to changes in $G(s)$.

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- Note: The bilinear transform preserves s-domain and z-domain sense positive realness.

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- This property is known as unconditional stability.

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- and the envelope of the impulse response decays exponentially with time constant $\tau = 2m/R$.

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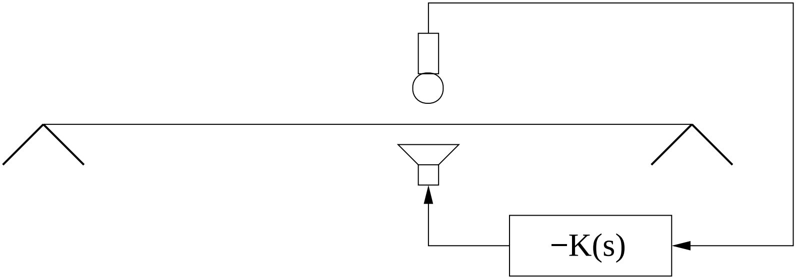
PID Control Mechanical Equivalent



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  5. Filter alternating between $\pm\pi/2$ radians

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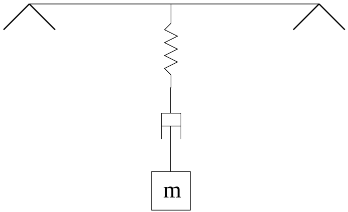
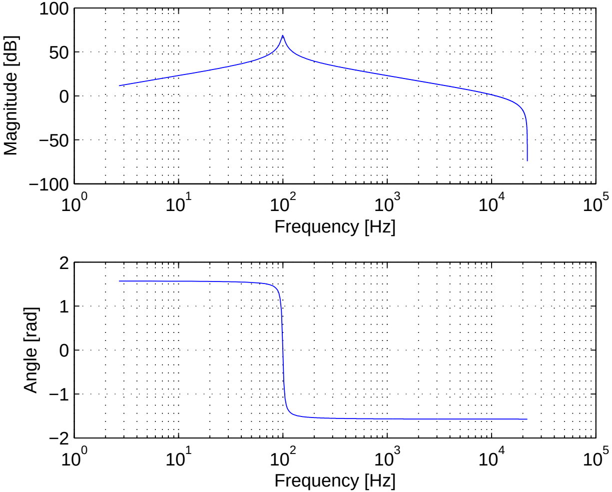


Bandpass Filter



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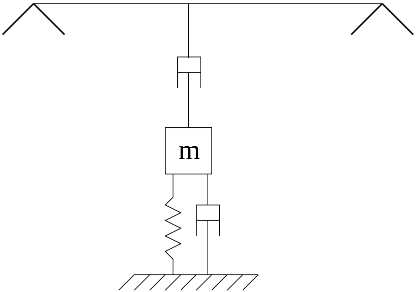

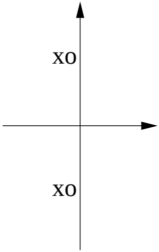

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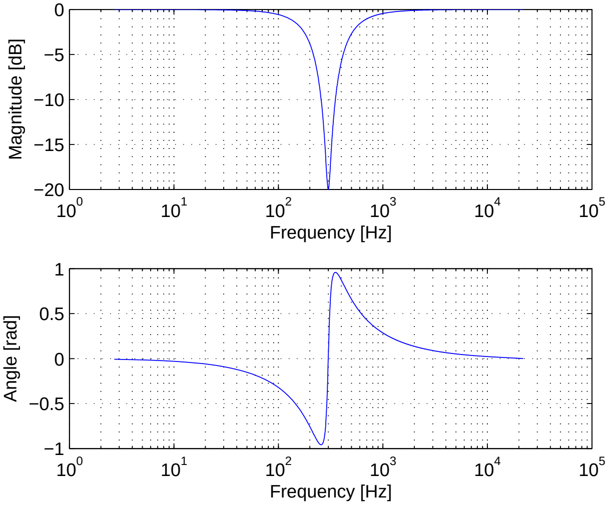
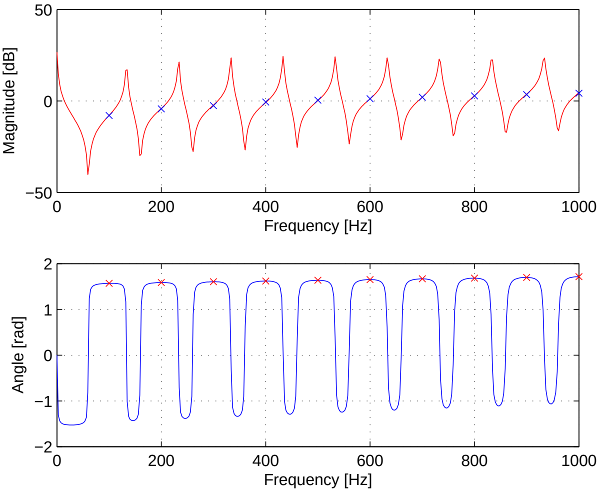
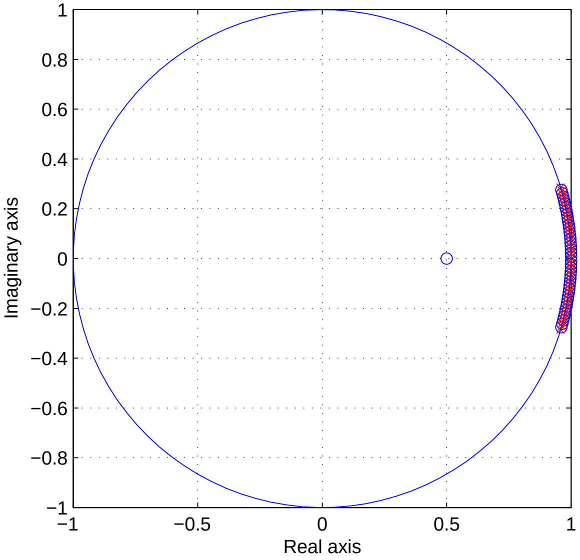


Alternating Filter

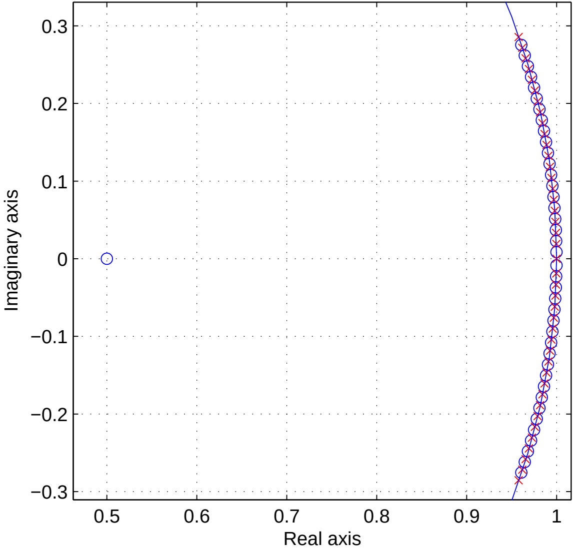
- The frequency response shown below is such that partials at n100Hz (shown by x's) will be pushed flat.



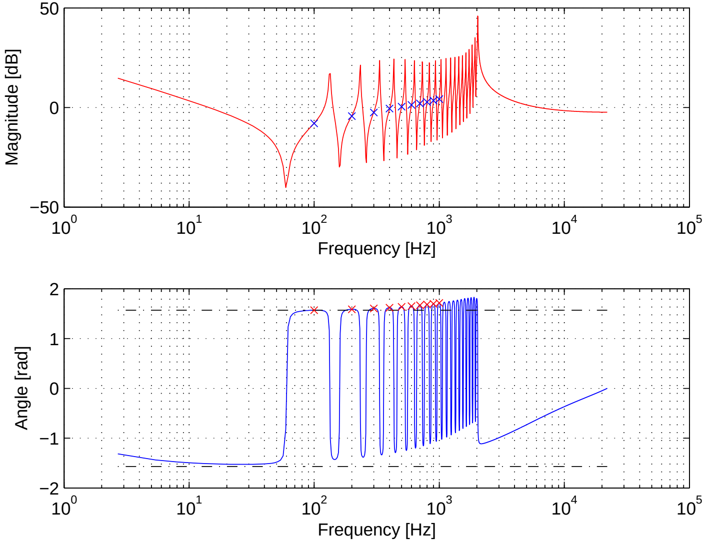
Alternating Filter Implementation



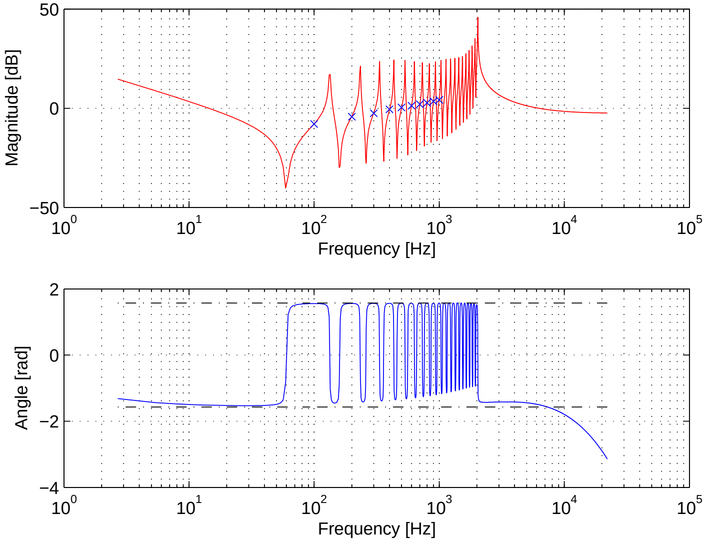
Alternating Filter Implementation (Zoomed)



Wideband Idealized Frequency Response



Wideband Frequency Response Including Delay



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- There are many methods for analyzing the behavior of second-order nonlinear systems.

Linear Dashpot

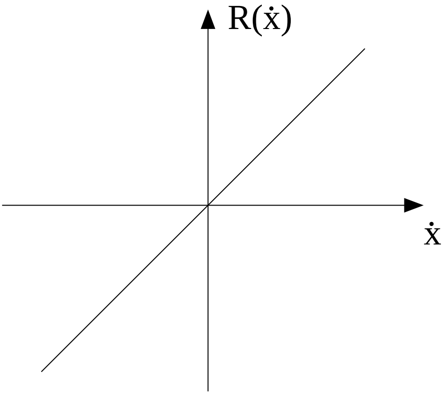

Figure: Linear Dashpot

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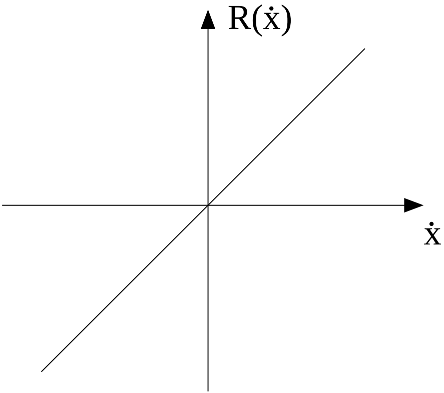
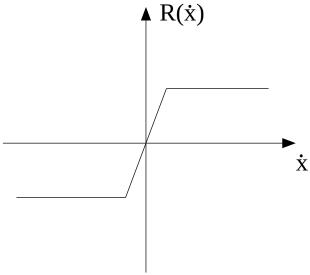


Figure: Linear Dashpot

- $R(\dot{x}, x) = R\dot{x}$ for some constant R

Saturating Dashpot



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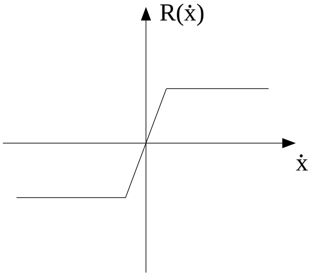

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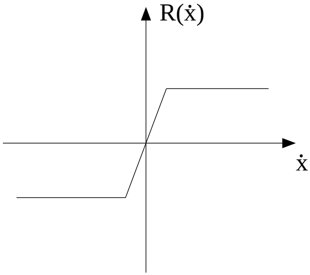

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- Damping is *strictly passive* if $\dot{x}R(\dot{x},x) > 0$ for all x and for all $\dot{x} \neq 0$ (i.e. there is no deadband).

Spring

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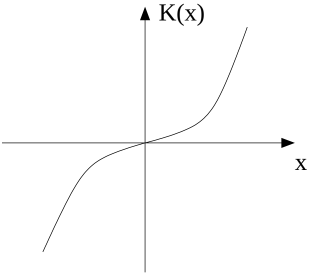

Figure: Stiffening Spring

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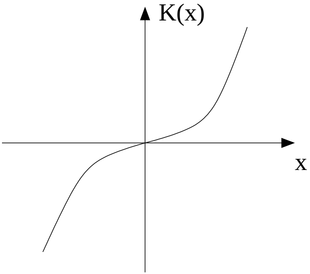

Figure: Stiffening Spring

- The spring is *strictly locally passive* if $xK(x) > 0 \; \forall x \neq 0$.

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$$\dot{V} = -\frac{R(\dot{x}, x)\dot{x}}{m} \leq 0 \tag{8}$$

Nonlinear Dashpot for Bow at Rest

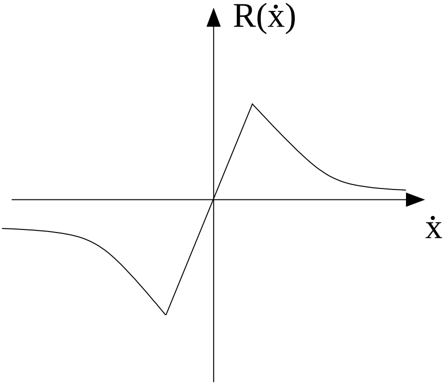
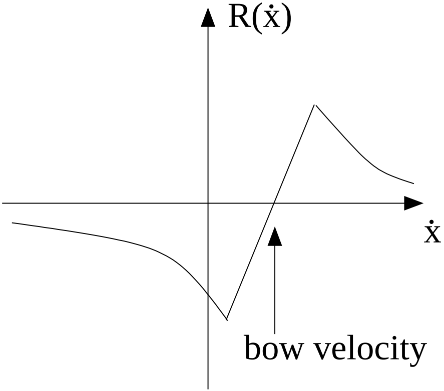


Figure: Bowing Nonlinearity

Nonlinear Dashpot For Moving Bow



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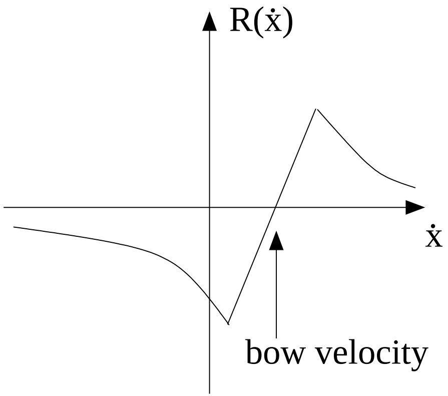

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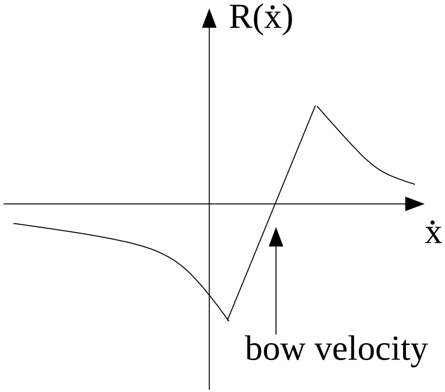

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- Now the dashpot is *NOT* passive.
- The negative damping region adds energy so that the bowed instrument can self-oscillate.

Thanks!

- Sound examples are on the website *http://ccrma.stanford.edu/˜eberdahl/Projects/PassiveControl*

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- Questions?

Bibliography

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