Estimating the state of a one-dimensional waveguide

Edgar Berdahl and Julius O. Smith III

Department of Electrical Engineering
Center for Computer Research in Music and Acoustics (CCRMA)
Stanford University
Stanford, CA, 94305

Signal Processing in Acoustics
Monday Afternoon at 4:45PM, June 4th, 2007

—

Special thanks to the Wallenberg Global Learning Network for supporting the REALSIMPLE project

Outline

Overview

- Estimating the state of a 1D waveguide has applications in reflectometry, measuring termination impedances, and control.

Overview

- Estimating the state of a 1D waveguide has applications in reflectometry, measuring termination impedances, and control.
- We parameterize the waveguide state using right-going and left-going traveling waves.

Overview

- Estimating the state of a 1D waveguide has applications in reflectometry, measuring termination impedances, and control.
- We parameterize the waveguide state using right-going and left-going traveling waves.
- Estimating the traveling wave components at a single point is relatively simple.

Overview

- Estimating the state of a 1D waveguide has applications in reflectometry, measuring termination impedances, and control.
- We parameterize the waveguide state using right-going and left-going traveling waves.
- Estimating the traveling wave components at a single point is relatively simple.
- For many waveguides, the history of a simple estimator may serve as time-delayed or time-advanced estimates for the remainder of the waveguide's state.

Overview

- Estimating the state of a 1D waveguide has applications in reflectometry, measuring termination impedances, and control.
- We parameterize the waveguide state using right-going and left-going traveling waves.
- Estimating the traveling wave components at a single point is relatively simple.
- For many waveguides, the history of a simple estimator may serve as time-delayed or time-advanced estimates for the remainder of the waveguide's state.
- Goal: Maximize the bandwidth above 200Hz of the right-going traveling wave estimate $\hat{y}_r$ given

Overview

- Estimating the state of a 1D waveguide has applications in reflectometry, measuring termination impedances, and control.
- We parameterize the waveguide state using right-going and left-going traveling waves.
- Estimating the traveling wave components at a single point is relatively simple.
- For many waveguides, the history of a simple estimator may serve as time-delayed or time-advanced estimates for the remainder of the waveguide's state.
- Goal: Maximize the bandwidth above 200Hz of the right-going traveling wave estimate $\hat{y}_r$ given
    1. that the left-going traveling wave y_l is suppressed by at least 50dB.

Overview

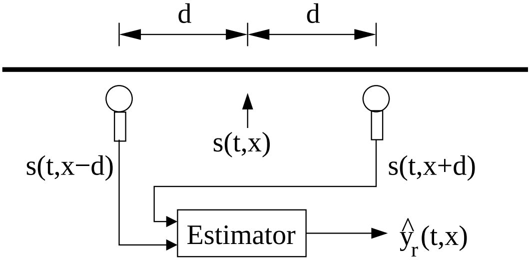
- Estimating the state of a 1D waveguide has applications in reflectometry, measuring termination impedances, and control.
- We parameterize the waveguide state using right-going and left-going traveling waves.
- Estimating the traveling wave components at a single point is relatively simple.
- For many waveguides, the history of a simple estimator may serve as time-delayed or time-advanced estimates for the remainder of the waveguide's state.
- Goal: Maximize the bandwidth above 200Hz of the right-going traveling wave estimate $\hat{y}_r$ given
  1. that the left-going traveling wave y_l is suppressed by at least 50dB.
  2. that the sensor noise power is not increased by more than 12dB at any frequency.

Overview

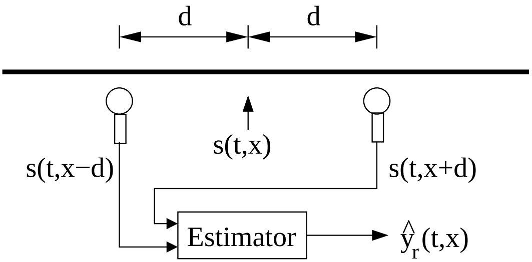
- Estimating the state of a 1D waveguide has applications in reflectometry, measuring termination impedances, and control.
- We parameterize the waveguide state using right-going and left-going traveling waves.
- Estimating the traveling wave components at a single point is relatively simple.
- For many waveguides, the history of a simple estimator may serve as time-delayed or time-advanced estimates for the remainder of the waveguide's state.
- Goal: Maximize the bandwidth above 200Hz of the right-going traveling wave estimate $\hat{y}_r$ given
    1. that the left-going traveling wave y_l is suppressed by at least 50dB.
    2. that the sensor noise power is not increased by more than 12dB at any frequency.
    3. that no pair of sensors be placed fewer than 4cm apart.

Outline

Traveling Wave Decomposition

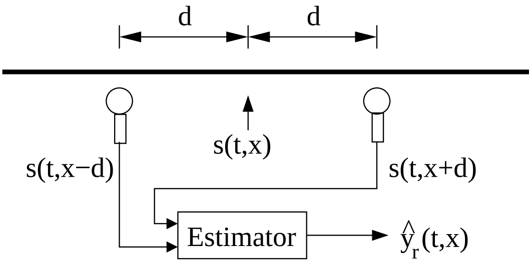


Traveling Wave Decomposition



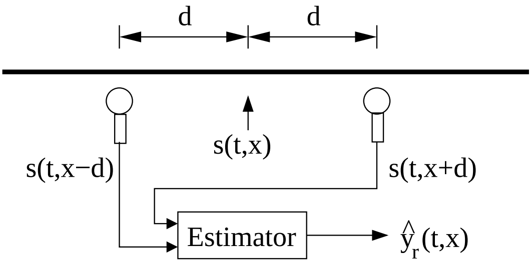
- c is the wave speed.

Traveling Wave Decomposition



- c is the wave speed.
- The traveling-wave solution for a lossless 1D waveguide is $s(t,x) = y_r(t - x/c, 0) + y_l(t + x/c, 0)$.

Traveling Wave Decomposition



- c is the wave speed.
- The traveling-wave solution for a lossless 1D waveguide is $s(t,x) = y_r(t - x/c, 0) + y_l(t + x/c, 0)$.
- The form of the traveling-wave solution for a lossless 1D waveguide suggests that we could use integration to separate y_r and y_l.

Traveling Wave Decomposition

- $s(t,x) = y_r(t - x/c, 0) + y_l(t + x/c, 0)$

Traveling Wave Decomposition

- $s(t,x) = y_r(t - x/c, 0) + y_l(t + x/c, 0)$
- We define

$$\tilde{s}(t,x) \triangleq -c \int \frac{\partial}{\partial x} s(t,x) dt = -c \int -\frac{1}{c} \dot{y}_r(t - x/c, 0) + \frac{1}{c} \dot{y}_l(t + x/c, 0) dt$$

Traveling Wave Decomposition

- $s(t,x) = y_r(t - x/c, 0) + y_l(t + x/c, 0)$
- We define

$$\tilde{s}(t,x) \triangleq -c\int \frac{\partial}{\partial x} s(t,x)dt = -c\int -\frac{1}{c}\dot{y}_r(t - x/c, 0) + \frac{1}{c}\dot{y}_l(t + x/c, 0)dt$$

- $\tilde{s}(t,x) = y_r(t - x/c, 0) - y_l(t + x/c, 0)$

Traveling Wave Decomposition

- $s(t,x) = y_r(t - x/c, 0) + y_l(t + x/c, 0)$
- We define

$$\tilde{s}(t,x) \triangleq -c \int \frac{\partial}{\partial x} s(t,x) dt = -c \int -\frac{1}{c}\dot{y}_r(t-x/c,0) + \frac{1}{c}\dot{y}_l(t+x/c,0) dt$$

- $\tilde{s}(t,x) = y_r(t - x/c, 0) - y_l(t + x/c, 0)$
- $y_r(t - x/c, 0) = \frac{1}{2}(\tilde{s}(t,x) + s(t,x))$

Traveling Wave Decomposition

- $s(t,x) = y_r(t - x/c, 0) + y_l(t + x/c, 0)$
- We define

$$\tilde{s}(t,x) \triangleq -c \int \frac{\partial}{\partial x} s(t,x) dt = -c \int -\frac{1}{c}\dot{y}_r(t - x/c, 0) + \frac{1}{c}\dot{y}_l(t + x/c, 0) dt$$

- $\tilde{s}(t,x) = y_r(t - x/c, 0) - y_l(t + x/c, 0)$
- $y_r(t - x/c, 0) = \frac{1}{2}(\tilde{s}(t,x) + s(t,x))$
- We estimate $\frac{\partial}{\partial x} s(t,x)$ with a finite difference approximation.

Traveling Wave Decomposition

- $s(t,x) = y_r(t - x/c, 0) + y_l(t + x/c, 0)$
- We define

$$\tilde{s}(t,x) \triangleq -c\int \frac{\partial}{\partial x} s(t,x)dt = -c\int -\frac{1}{c}\dot{y}_r(t-x/c,0)+\frac{1}{c}\dot{y}_l(t+x/c,0)dt$$

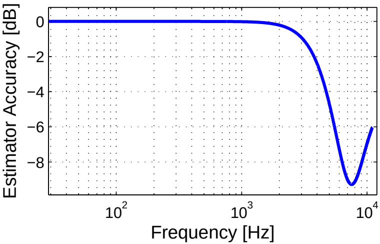
- $\tilde{s}(t,x) = y_r(t - x/c, 0) - y_l(t + x/c, 0)$
- $y_r(t - x/c, 0) = \frac{1}{2}(\tilde{s}(t,x) + s(t,x))$
- We estimate $\frac{\partial}{\partial x} s(t,x)$ with a finite difference approximation.
- The required signal processing consists only of applying gains, integrating, summing, and differencing.

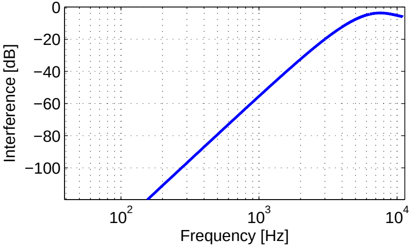
Decomposition Method Example

- Consider $c = 400$m/s and $M = 5$ sensors with spacings -8cm, 4cm, 0cm, 4cm, and 8cm.

Decomposition Method Example

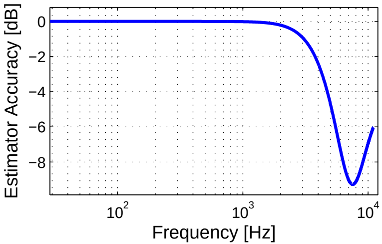
- Consider $c = 400$m/s and $M = 5$ sensors with spacings -8cm, 4cm, 0cm, 4cm, and 8cm.

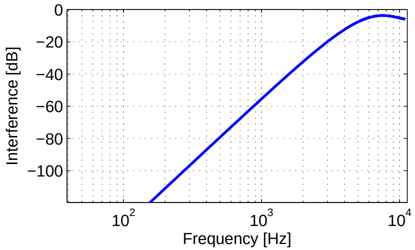




Decomposition Method Example

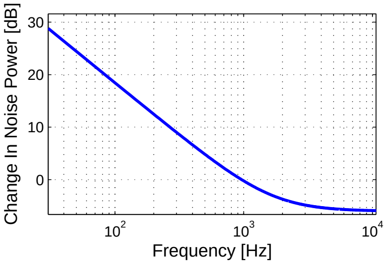
- Consider $c = 400$m/s and $M = 5$ sensors with spacings -8cm, 4cm, 0cm, 4cm, and 8cm.





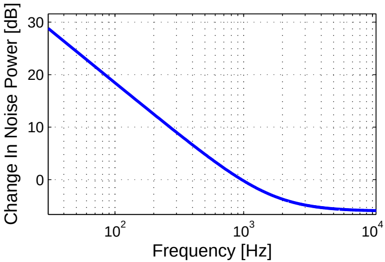
- The estimator is accurate and the interference from the left-going traveling wave is more than 50dB down for $f \in [\ 0kHz \quad 1.2kHz\ ]$.

Decomposition Method Example



- The sensor noise power increases by more than 12dB below 200Hz.

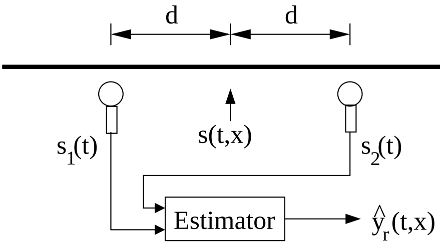
Decomposition Method Example



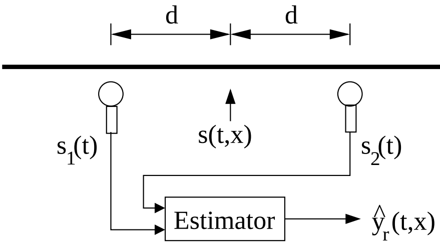
- The sensor noise power increases by more than 12dB below 200Hz.
- The usable band is $f \in [\ 0.2kHz \quad 1.2kHz\ ]$.

Outline

Can We Completely Cancel The Left-Going Wave?

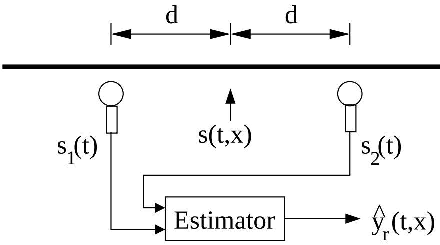


Can We Completely Cancel The Left-Going Wave?



- We know that
  $y_r(t, x + d) = y_r(t - 2d/c, x - d)$

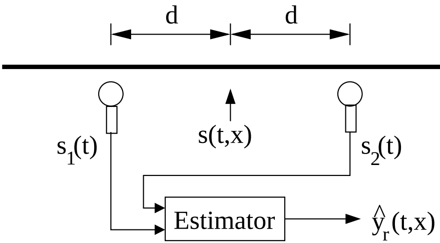
Can We Completely Cancel The Left-Going Wave?



- We know that

$$y_r(t, x + d) = y_r(t - 2d/c, x - d)$$
$$y_l(t, x - d) = y_l(t - 2d/c, x + d)$$

Can We Completely Cancel The Left-Going Wave?



- We know that
  $y_r(t, x + d) = y_r(t - 2d/c, x - d)$
  $y_l(t, x - d) = y_l(t - 2d/c, x + d)$

- $\hat{y}_r(t, x - d) = s_1(t) - s_2(t - 2d/c) + \hat{y}_r(t - 4d/c, x - d)$

Causal Wiener Filter Estimator

- Let $y_r(t - x/c, 0)$ and $y_l(t + x/c, 0)$ be uncorrelated, white, and have power σ_S^2, and let each sensor suffer additive white noise with power σ_N^2.

Causal Wiener Filter Estimator

- Let $y_r(t - x/c, 0)$ and $y_l(t + x/c, 0)$ be uncorrelated, white, and have power σ_S^2, and let each sensor suffer additive white noise with power σ_N^2.
- $\widehat{y_r}(t, x - d) = \frac{(1+\eta^2)\sigma_S^2}{2\sigma_S^2+\sigma_N^2}(s_1(t) - \eta s_2(t - 2d/c)) + \eta^2 \widehat{y_r}(t - 4d/c, x - d)$

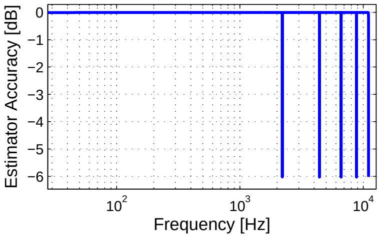
Causal Wiener Filter Estimator

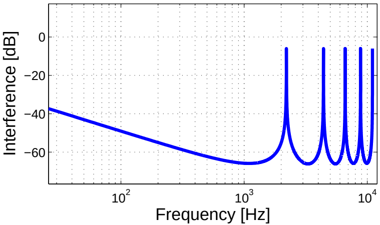
- Let $y_r(t - x/c, 0)$ and $y_l(t + x/c, 0)$ be uncorrelated, white, and have power σ_S^2, and let each sensor suffer additive white noise with power σ_N^2.
- $\widehat{y_r}(t, x - d) = \frac{(1+\eta^2)\sigma_S^2}{2\sigma_S^2+\sigma_N^2}(s_1(t) - \eta s_2(t - 2d/c)) + \eta^2 \widehat{y_r}(t - 4d/c, x - d)$
- For $\sigma_N^2 << \sigma_S^2$, $\eta \approx 1 - \sqrt{\sigma_N^2/\sigma_S^2}$.

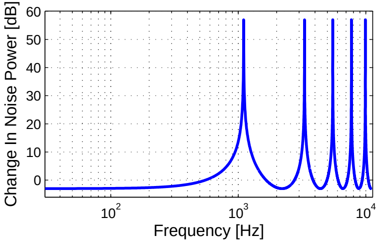
Causal Wiener Filter Estimator

- Let $y_r(t - x/c, 0)$ and $y_l(t + x/c, 0)$ be uncorrelated, white, and have power σ_S^2, and let each sensor suffer additive white noise with power σ_N^2.
- $\widehat{y_r}(t, x - d) = \frac{(1+\eta^2)\sigma_S^2}{2\sigma_S^2+\sigma_N^2}(s_1(t) - \eta s_2(t - 2d/c)) + \eta^2 \widehat{y_r}(t - 4d/c, x - d)$
- For $\sigma_N^2 << \sigma_S^2$, $\eta \approx 1 - \sqrt{\sigma_N^2/\sigma_S^2}$.
- For $\sigma_N^2 = 0$ (i.e. $\eta = 1$), the solution is the same as before in the noiseless case. $\sigma_N^2 > 0$ stabilizes the estimator poles.

Delay Method Example ($\sigma_S^2/\sigma_N^2 = 60dB$)



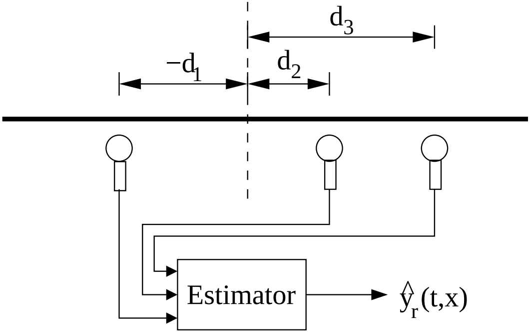




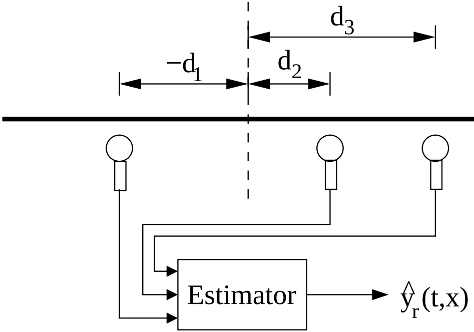
Interference is more than 50dB down and the increase in sensor noise power is less than 12dB for $f \in [\ 0.2kHz \quad 1kHz\ ]$.

Outline

Apply FIR Filter To Each Sensor Signal

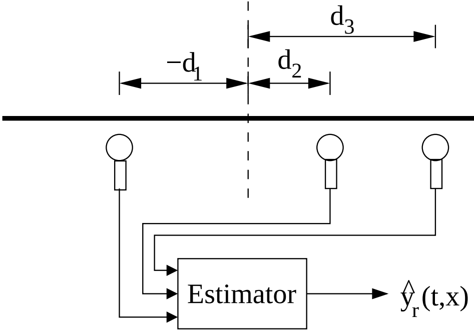


Apply FIR Filter To Each Sensor Signal



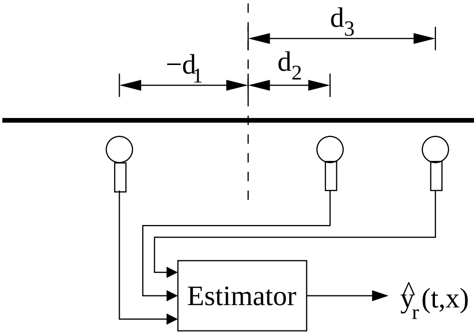
- $\hat{y}_r(n,x) = \sum_{m=1}^{M} w_m(n) * s_m(n)$

Apply FIR Filter To Each Sensor Signal



- $\hat{y}_r(n, x) = \sum_{m=1}^{M} w_m(n) * s_m(n)$
- $f_S = 22\text{kHz}$ and $L = 100$ taps

Apply FIR Filter To Each Sensor Signal



- $\hat{y}_r(n,x) = \sum_{m=1}^{M} w_m(n) * s_m(n)$
- $f_S = 22$kHz and $L = 100$ taps
- Let $w = [w_1^T w_2^T \ldots w_M^T]^T \in \mathbb{R}^{LM\times 1}$, where each of the distinct FIR filters has length L.

Beamforming Using Regularized Least-Squares

- Goals:
  1. Estimate the right-going wave ($C_R w \approx \mathbf{1}$)

Beamforming Using Regularized Least-Squares

- Goals:
  1. Estimate the right-going wave ($C_R w \approx \mathbf{1}$)
  2. Reject the left-going wave ($C_L w \approx 0$)

Beamforming Using Regularized Least-Squares

- Goals:
  1. Estimate the right-going wave ($C_R w \approx \mathbf{1}$)
  2. Reject the left-going wave ($C_L w \approx 0$)
- Let R_{NN} is the covariance matrix of the sensor noise.

Beamforming Using Regularized Least-Squares

- Goals:
  1. Estimate the right-going wave ($C_R w \approx \mathbf{1}$)
  2. Reject the left-going wave ($C_L w \approx 0$)
- Let R_{NN} is the covariance matrix of the sensor noise.
- We minimize $f(w) = w^T R_{NN} w$

Beamforming Using Regularized Least-Squares

- Goals:
  1. Estimate the right-going wave ($C_R w \approx \mathbf{1}$)
  2. Reject the left-going wave ($C_L w \approx 0$)
- Let R_{NN} is the covariance matrix of the sensor noise.
- We minimize $f(w) = w^T R_{NN} w + \lambda(C_R w - 1)^T (C_R w - \mathbf{1})$

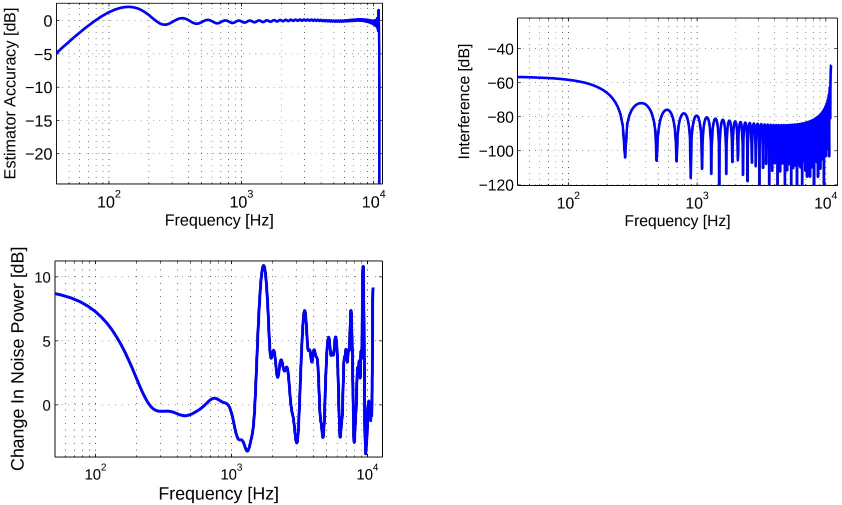
Beamforming Using Regularized Least-Squares

- Goals:
  1. Estimate the right-going wave ($C_R w \approx \mathbf{1}$)
  2. Reject the left-going wave ($C_L w \approx 0$)
- Let R_{NN} is the covariance matrix of the sensor noise.
- We minimize $f(w) = w^T R_{NN} w + \lambda(C_R w - 1)^T (C_R w - \mathbf{1}) + \mu(C_L w)^T (C_L w)$.

Beamforming Using Regularized Least-Squares

- Goals:
  1. Estimate the right-going wave ($C_R w \approx \mathbf{1}$)
  2. Reject the left-going wave ($C_L w \approx 0$)
- Let R_{NN} is the covariance matrix of the sensor noise.
- We minimize $f(w) = w^T R_{NN} w + \lambda(C_R w - 1)^T (C_R w - \mathbf{1}) + \mu(C_L w)^T (C_L w)$.
- Optimal $w^* = (R_{NN} + \lambda C_R^T C_R + \mu C_L^T C_L)^{-1} \lambda C_R^T \mathbf{1}$

Beamforming Results



The estimator has 0.8dB of ripple for $f \in [\ 0.2kHz \quad 10.2kHz\ ]$

Outline

Comparison Of Methods

- Parameters for the different methods were adjusted such that
  1. the interference rejection was at least 50dB
  2. the peak increase in noise power was about 12dB
  3. no pair of sensors was placed fewer than 4cm apart

Method	Bandwidth[1]	Sensor Placement
Decomposition	1kHz	-8cm, -4cm, 0cm, 4cm, 8cm
Delay	0.8kHz	-5cm and 5cm
Beamforming	10kHz	-4cm, 8cm, and 18cm

[1]Bandwidth is measured from 200Hz upward

Thanks

Questions?

- J. Guerard and X. Boutillon,
Real Time Acoustic Wave Separation in a Tube,
Proceedings of the International Symposium on Musical Acoustics, 451, Edinburgh, Scotland, August 19-22, 1997.

- B. Van Veen and K. Buckley,
Beamforming: A Versatile Approach to Spatial Filtering,
IEEE ASSP Magazine, Vol 5, pp. 4-24, 1998.

- M. van Walstijn and M. Campbell,
Large-Bandwidth Measurement of Acoustic Input Impedance of Tubular Objects,
Proceedings of the Institute of Acoustics, Spring Conference, Salford, UK, 2002.