A Lagrangian method for Stokes-Oldroyd-B flow

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Abstract

I will discuss a new numerical method for modeling viscoelastic fluid flow. A viscoelastic fluid has a time dependent response to stress that gives it behavior in between a viscous fluid (like water) and an elastic solid. A common set of viscoelastic fluids are polymer solutions composed of long chain molecules in a viscous solvent. Energy can be stored, at least for a short time, when the polymers stretch and compress due to shear flows in the solution. A simple model of a polymer solution is given by the Oldroyd-B equations, which are the traditional Navier-Stokes equations of fluid flow coupled to an equation for the time evolution of the average polymer stress in the fluid. I will consider the case in which the fluid flow is slow and on a small spatial scale, so that the Navier-Stokes equations may be replaced by the Stokes equations. I will then present a Lagrangian formulation of the Stokes-Oldroyd-B equations, in which the position of specific fluid particles are tracked over time. I and my collaborators have derived a regularized version of these equations that smoothes the singularities in the Lagrangian formulation, which we use to model the motion of swimming microorganisms in a viscoelastic fluid.