Special Guest Lectures

In the Search of 2D Boussinesq Solutions: A Numerical Approach

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Abstract:
Boussinesq's equation (BE) was the first model for surface waves in shallow fluid layer that accounts for both nonlinearity and dispersion. The balance between the steepening effect of the nonlinearity and the flattening effect of the dispersion maintains the shape of the wave. This is a new paradigm in physics and can be properly termed `Boussinesq Paradigm'. In a coordinate frame moving with the center of the propagating wave, BE reduces to Korteweg-de Vries which is widely studied in 1D. Little is known about the shape of Boussinesq solitary waves in 2D. In the works of present author the problem was attacked by means of perturbation methods. The difficulties for a direct difference solution stem from the unboundedness of the region and the bifurcation nature of the problem. In this work, a difference scheme on a nonuniform grid is constructed for the stationary propagating localized waves of the 2D Boussinesq equation in an infinite region. Using an argument stemming form an perturbation expansion for small wave phase speeds, the asymptotic decay of the wave profile is identified as second order algebraic. For algebraically decaying solution a new kind of non-local boundary condition is derived, which allows to rigorously project the asymptotic boundary condition at the boundary of a finite-size computational box. The difference approximation of this condition together with the bifurcation condition complete the algorithm. Numerous numerical validation are performed and it is shown that the results comply with the second-order estimate for the truncation error even at the boundary lines of the grid. Results are obtained for different values of the so-called `rotational inertia' and for different subcritical phase speeds. The solution for the shape of the solitary wave is obtained for wide range of the governing parameters. It is found that the limits of existence of the 2D solution roughly correspond to the similar limits on the phase speed that ensure the existence of subcritical 1D stationary propagating waves of the Boussinesq equation. Note: 338 Johnston is Access Grid viewing. Live Presentation is at Abdalla Hall at ULL..

Speaker's Bio:
My research interests are in Applied Mathematics and Mathematical Physics: operator-splitting methods for Navier-Stokes and higher-order diffusion equations; implicit schemes for generalized wave equations and nonlinear evolution equations; spectral methods; method of variational imbedding for inverse problems; nonlinear dynamical systems; solitons; dissipative structures; method of random point approximation; nonlinear acoustics; material invariant electrodynamics; unified field theory.