Frontiers of Scientific Computing Lecture Series

A New Class of Discontinous Petrov-Galerkin (DPG) Finite Element (FE) Methods with Application to Challenging Singular Perturbation Problems

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Abstract:
The hp-adaptive finite elements combine elements of varying size h and polynomial order p to deliver approximation properties superior to any other discretization methods. The best approximation error converges exponentially fast to zero as a function of number of degrees-of-freedom. The hp methods are thus a natural candidate for singularly perturbed problems experiencing internal or boundary layers like in compressible gas dynamics. This is the good news. The bad news is that only a small number of variational formulations is stable for hp-discretizations. By the hp-stability we mean a situation where the discretization error can be bounded by the best approximation error times a constant that is independent of both h and p. To this class belong classical elliptic problems (linear and non-linear), and a large class of wave propagation problems whose discretization is based on hp spaces reproducing the classical exact grad-curl-div sequence. Examples include acoustics, Maxwell, elastodynamics, poroelasticity and various coupled and multiphysics problems. We will present a new paradigm for constructing discretization schemes for virtually arbitrary systems of linear PDE's that remain stable for arbitrary hp meshes, extending thus dramatically the applicability of hp approximations. For a start, we focus on a challenging model problem - convection dominated diffusion. The presented methodology incorporates the following features: The problem of interest is formulated as a system of first order PDE's in the distributional (weak) form, i.e. all derivatives are moved to test functions. We use the DG setting, i.e. the integration by parts is done over individual elements. As a consequence, the unknowns include not only field variables within elements but also fluxes on interelement boundaries. We do not use the concept of a numerical flux but, instead, treat the fluxes as independent, additional unknowns. For each trial function corresponding to either field or flux variable, we determine a corresponding optimal test function by solving an auxiliary local problem on one element. The use of optimal test functions guarantees attaining the supremum in the famous inf-sup condition from Babuska-Brezzi theory. The local problems for determining optimal test functions are solved approximately with an enhanced approximation (a locally hp-refined mesh). By selecting right norms for test functions, we can obtain amazing stability properties uniform not only with respect to discretization parameters but also with respect to the diffusion constant (perturbation parameter) , i.e. the resulting discretization is robust. The presentation will cover a general abstract theory illustrated with numerical examples for 1D and 2D "confusion" problems. We have been able to solve in a fully automatic mode problems with diffusion constant eps = 10^{-11} in 1D and eps = 10^{-7} in 2D using hp-adaptivity. Time permitting, I may show also examples of application of the method to wave propagation (control of dispersion error), beams and shells (locking, resolution of boundary layers) and 1D Burgers and Navier-Stokes equations.

Speaker's Bio:
Leszek F. Demkowicz is Assistant Director of the Institute for Computational Engineering and Sciences (ICES) and a Professor in the Department of Aerospace Engineering and Engineering Mechanics, at the University of Texas at Austin. He has a M.S. in mathematics from Jagiellonian University, and M.S. (Hons.), Ph.D. and Sc.D. degrees in mechanics from Cracow University University of Technology, Poland. Prior to joining the University of Texas faculty, he held various positions at Cracow University University of Technology. In years 1990-1993, he was the head of Section of Applied Mathematics and Director of Computer Center. Dr. Demkowicz authored a monograph on adaptive methods (in Polish), co-authored with Prof. J.T. Oden a textbook on Functional Analysis (CRS Press, 1996, second edition 2010), and co-edited two books. Dr. Demkowicz has also authored over 140 journal articles, conference proceedings, book chapters and technical reports in the general area of computational mechanics and mathematics. He is Associate Editor of four international journals. Dr. Demkowicz was the founding member of Polish Association for Computational Mechanics and served as its first President. He is a fellow of both U.S. and International Associations for Computational Mechanics and a member of several other professional organizations. He graduated eleven Ph.D. and numerous M.S. students. His work and scientific interests span across numerical analysis, adaptive finite element methods, and wave propagation problems, including acoustics, elastodynamics and electromagnetics. Among other applications, Dr. Demkowicz and his group developed original numerical methods for structural vibrations, analysis of acoustics of human ear, dynamic modeling of gears, analysis of optical waveguides, calculation of Radar Crossections, borehole electromagnetics and acoustics. He has given numerous invited talks on the subjects. His work has been sponsored by NSF, Navy, Air Force, DOE, Schlumberger, Baker-Hughes and Boeing. His research on high accuracy adaptive methods has recently been summarized in a two volume book - ``Computing with hp Elements'' (Chapman & Hall, Vol. I - 2006, Vol. II - 2007). For his work on hp methods, he was recently awarded Zienkiewicz Medal by PACM and Computational Sciences Award by USACM (both in 2009).

Refreshments will be served.
This lecture has a reception.