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Frontiers of Scientific Computing Lecture Series

Mimetic discretizations and what they can do for you

Dr. Pavel Bochev, Sandia National Laboratories

Principal Member of the Technical Staff, Computational Mathematics and Algorithms Department

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Abstract:
Recent advances in compatible discretizations enabled impressive gains in computational science and affirmed the key role of homological principles in numerical PDEs. Thanks to homological ideas and tools, we now have a much better understanding of why some discretization methods work so well and why other methods fail spectacularly. More importantly, homological ideas can be used to develop stable and physically consistent discretizations, such as mimetic methods, which replace PDEs by algebraic equations that inherit their fundamental structural properties. We provide a common framework for mimetic methods using algebraic topology to guide our analysis. The key concept in our approach is the natural inner product on co-chains. This inner product is sufficient to generate a combinatorial Hodge theory on co-chains but avoids complications attendant in the construction of robust discrete Hodge-star operators. In particular, using a reduction and a reconstruction maps between differential forms and co-chains we define mutually consistent sets of natural and derived discrete operations that preserve the invariants of the De Rham homology groups and obey a discrete Stokes theorem. By choosing a specific reconstruction operator we obtain well-known mixed FE, mimetic FD and covolume methods and explain when they are equivalent. The second half of the talk will discuss several applications of the mimetic framework. We will start with a new interpretation of a certain class of compatible least-squares methods, as discrete realizations of a Hodge-star operator, obtained from weakly enforced material laws. Among other things, we will show that least-squares, Galerkin and mixed Galerkin methods, for a class of second order elliptic problems, can be derived from a common constrained optimization problem. Our second example will use the mimetic framework to reformulate the discrete Maxwell's equations into a system that is dominated by discrete Hodge-Laplace operators. As a result, the reformulated system can be solved by standard “black-box” AMG solvers for the Poisson equation. Time permitting, we will conclude with an example that explains how mimetic discretizations can be used to remap divergence free fields without advection algorithms. This talk is based on joint work with M. Gunzburger (CSIT, Florida State University), M. Shashkov and M. Hyman (Theoretical Division, Los Alamos National Laboratory).

Speaker's Bio:
Dr. Bochev is a Principal Member of the Technical Staff at Sandia National Laboratories in Albuquerque where he works in the Computational Mathematics and Algorithms Department. His research interests are in the area of applied mathematics and numerical analysis with particular focus on finite element methods for partial differential equations. Dr. Bochev earned his Magister of Mathematics degree from the University of Sofia, Bulgaria in 1987. After three years as a research associate at the Bulgarian Academy of Sciences he came to the US to pursue a graduate degree at Virginia Tech. Dr. Bochev completed his PhD under the supervision of Prof. M. Gunzburger in 1994. His thesis was awarded the SIAM Student paper prize for 1994. Before joining Sandia in 2001, Dr. Bochev was Assistant and then Associate Professor of Mathematics at the University of Texas at Arlington. There, his research focus was on finite element methods of least-squares type and their applications to computational fluid dynamics, including optimal control of fluid flows and mesh generation. Since coming to Sandia, Dr. Bochev’s research has been in the area of compatible and stabilized discretizations of multiphysics problems and multiscale methods for atomistic to continuum coupling. Some of his recent work deals with application of algebraic topology and differential geometry in the design of stable, mimetic discrete models of PDEs, formulation of new stabilized methods for incompressible flow problems, and analysis of stabilized methods for problems with multiple time scales arising in chemically reacting flows. Dr. Bochev has authored and co-authored over 50 research papers, one book and several book chapters, and has given numerous plenary and invited lectures in the US and abroad. He serves on the editorial boards of the SIAM Journal on Numerical Analysis, Discrete and Continuous Dynamical Systems – Series B, and the International Journal of Numerical Analysis and Modeling.

This lecture has a reception.

Center for Computation & Technology
2003 Digital Media Center • Telephone: +1 225/578-5890 • Fax: +1 225/578-8957