

Curricula 2262

1. General Skills

- Ability to calculate error terms, deduce convergence rate, and general error properties, given the expression for an error.
- Ability to provide pseudo-code for any numerical method
- Ability to graphically represent numerical methods and their application
- Definition of error, sources of different kinds of error and their contribution to numerical solution of real world scientific problems.
- Definition of convergence, and ability to determine the rate of convergence from numerical errors from an iterative or other method.
- Knowledge of different ways that a problem may be ill-behaved.
- Mathematics: equation of a line, basic integration and differentiation, chain rule for differentiation, integration by parts, form of basic functions (e^x , $\ln x$, $\cos x$, $\sin x$), definition of a tridiagonal system, solution of quadratic equations, mean value theorem, matrix determinants.

2. Computer Arithmetic

- Floating-point numbers, scientific notation, single precision and double precision IEEE floating-point formats, binary numbers, hexadecimal numbers, conversion between formats, accuracy of floating point representation.
- Rounding and chopping of numbers, loss of significant figures, noise in evaluating functions, underflow and overflow, summation of numbers, loop errors.

3. Taylor Polynomials

- Derivation of general Taylor polynomials of arbitrary order about a particular point of approximation, Taylor's remainder theorem, calculation of and use of error bounds, calculating Taylor polynomials for simple expressions, Taylor's series.

4. Root Finding

- Derivation of algorithm for bisection method, error bounds and rate of convergence for bisection method.
- Derivation of algorithm for false position and secant method, error bounds and rate of convergence for both methods.
- Derivation of algorithm for Newton's method. General properties of its application and error.
- General method of fixed point iteration. Ability to express and apply the contraction mapping theorem and its corollary.
- Aitken error estimation and extrapolation.
- Ill-behaving root finding, general understanding of multiple roots and stability of roots.
- Graphical representation and interpretation of all root finding methods.

5. Interpolation & Approximation

- Definition of polynomial interpolation. Given a polynomial, you should be able to say whether or not it interpolates a set of data. Difference between interpolating polynomials and approximating polynomials.
- Detailed derivation and application of linear polynomial interpolation.
- Definition of piecewise interpolating polynomials.
- Form of, and use of the general Lagrange polynomial interpolation formula.
- Properties of errors in polynomial interpolation.
- Motivation for and use of the near-minimax approximation
- Definition and use of divided differences, including their use in formulae for interpolation
- Definition of and motivation for cubic splines. Ability to calculate natural cubic splines for simple cases when provided with all defining equations. Given a function, you should be able to say whether or not it is a natural cubic spline for some set of data.

6. Integration & Differentiation

- Derivation and use of trapezoidal rule, use of Simpsons rule, midpoint rule, graphical interpretation, properties of errors for methods.
- Deriving number of node points to use for a particular error tolerance.
- Corrected trapezoidal/Simpsons rule
- Derivation and use of Richardson extrapolation formula
- Gaussian integration, derivation for cases $n=1$ and $n=2$, change of variable for general ranges.
- Numerical derivatives, derivation of forward and backward difference formulae, method for differentiation using interpolation, method of undetermined coefficients.

7. Linear Systems

- Examples of linear systems, e.g. polynomial interpolation, difference between direct and iterative methods.
- Matrices: order, notation, transpose, arithmetic and algebra, row operations, inverse, singular matrices, determinant, norms
- Solvability theorem.
- Direct methods: Gaussian elimination, pivoting, LU decomposition, operations count, calculating matrix inverse, tri-diagonal systems
- Iterative methods: General scheme, Jacobi, Gauss-Seidel, convergence criteria and behaviour of general iterative solvers, role of diagonally dominant matrices, role of matrix eigenvalues, residual correction for a general scheme, pseudo-code algorithms for Jacobi and Gauss-Seidel

8. Ordinary Differential Equations

- Examples of ODEs, e.g. harmonic oscillator.
- Definition of order, linear, nonlinear and examples.
- Derivation of Eulers method using Taylors series and discussion of error term.