

# CS2262 Spring 2007

## Assignment 9: Numerical Integration

1. For both the integrals

a)

$$\int_0^1 x^{5/2} dx \quad (= 2/7)$$

b)

$$\int_0^5 \frac{dx}{1 + (x - \pi)^2} \quad (= \tan^{-1}(5 - \pi) + \tan^{-1}(\pi))$$

Graph the integrand. Use the program for the Trapezoidal rule given in the textbook to prepare a table of values of  $T_n(f)$  for  $n = 2, 4, 8, \dots, 512$ . Include in the table the errors and the ratios by which the errors decrease.

2. Repeat (1) using Simpsons rules giving values for  $S_n(f)$

3. a) As another approximation to  $I(f) = \int_a^b f(x)dx$ , replace  $f(x)$  by the constant  $f[(a + b)/2]$  on the entire interval  $a \leq x \leq b$ . Show that this leads to the numerical integration formula

$$M_1(f) = (b - a)f\left(\frac{a + b}{2}\right)$$

Graphically illustrate this approximation.

b) In analogy with the derivation of the trapezoidal rule and Simpson's rule, generalize a) to the numerical integration formula

$$M_n(f) = h[f(x_1) + f(x_2) + \dots + f(x_n)]$$

where  $h = (b - a)/n$  and

$$x_j = a + (j - 1/2)h, \quad j = 1, \dots, n.$$

This is called the *midpoint rule*.

4. Repeat (1) using the midpoint rule.

5. Using the ratios by which the errors decrease in (1), (2), and (4) estimate the rate of convergence for each method/function duo.

6. Use the error formula for the Trapezoidal rule,

$$E_n^T(f) = \frac{-h^2(b - a)}{12} f''(c_n)$$

to bound the error in 1a) above. Compare your answer to the numerical errors you obtained.

7. Using the asymptotic error formula for the trapezoidal rule, estimate the number  $n$  of subdivisions to evaluate the following integrals to the given accuracy  $\epsilon$ .

a)

$$\int_1^3 \log(x) dx, \quad \epsilon = 10^{-8}$$

b)

$$\int_0^2 e^{-x^2} dx, \quad \epsilon = 10^{-10}$$

8. Use Gaussian quadrature applying  $I_2$ ,  $I_3$  and  $I_4$  to the integrals in Question 1. (You will need to use values from the table on p223 of the textbook). Compare the errors with those using previous methods in this assignment.

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Due March 30th 2007

Email completed assignments to [cs2262\\_assignments@cct.lsu.edu](mailto:cs2262_assignments@cct.lsu.edu)