CS2262 Spring 2007

Assignment 7: Interpolation

1. Given the data points (0,2) and (1,1) find the following (a) the straight line interpolating this data. (b) the function $f(x) = a + be^x$ interpolating this data. (c) the function $f(x) = a/(b + x)$ interpolating this data. In each of the three cases graph the interpolating function.

2. (a) Find the function $P(x) = a + b\cos(\pi x) + c\sin(\pi x)$ which interpolates the data

x	0	0.5	1
y	2	5	4

   (b) Find the quadratic polynomial interpolating this data.

   (c) Graph the interpolating function for (a) and (b)

3. The following table was obtained in solving a differential equation. Using linear interpolation between adjacent nodes, x_i, produce a continuous graph of this data in the interval $0 \le x \le 6$.

x_i	0.0	1.0	2.0	3.0	4.0	5.0	6.0
y_i	2.0000	2.1592	3.1697	5.4332	9.1411	14.406	21.303

4. Using Lagrange's formula for the quadratic interpolating polynomial, find the polynomial $P_2(x)$ that interpolates the following data, In each case simplify the expression for the polynomial as much as possible and graph the points and interpolating polynomial, and comment on your results.

   (a) $\{(0,1),(1,2),(2,3)\}$

   (b) $\{(0,1),(1,1),(2,1)\}$

5. Write out the complete formula for $P_3(x)$, including all four of the polynomials $L_0(x)$, $L_1(x)$, $L_2(x)$ and $L_3(x)$. Explain why $L_0(x) + L_1(x) + L_2(x) + L_3(x) = 1$.

6. Write a MATLAB program to do either linear or quadatic interpolation and check its accuracy. Input, two and three node points, and then generate the data values using the exponential function (e.g. e^x). For a variety of values of x both inside and outside the range of the node values compute $P_1(x)$, $P_2(x)$, e^x, and the errors $E_1(x) = e^x - P_1(x)$, $E_2(x) = e^x - P_2(x)$. Plot the values of the errors to see how they vary with x.

7. Let $x_0 = 0.85$, $x_1 = 0.87$, $x_2 = 0.89$. Using the values of e^{x_0}, e^{x_1}, e^{x_2} and $f(x) = e^x$, calculate $f[x_0, x_1]$, $f[x_1, x_2]$, and $f[x_0, x_1, x_2]$. Check the accuracy of the approximation

$$f[x_0, x_1] \approx f'\left(\frac{x_0 + x_1}{2}\right)$$

8. Produce a MATLAB program to check the accuracy of higher-order interpolation by using Newton's divided difference formula. For some $f(x)$ and some node points $x_0, x_1, \ldots, x_6$, use the function `divdif` provided in Atkinson & Han to produce the divided differences

$$D_i = f[x_0, \ldots, x_i], \qquad i = 0, 1, \ldots, 6$$

   Then evaluate $P_6(x)$ for a variety of values of x, and compare them to the true values of $f(x)$. Check the program by reproducing some of the results in Tables 4.1 and 4.2 of the textbook for $f(x) = \cos(x)$. Then repeat the process with a nonpolynomial function $f(x)$ of your choice.

Due March 16th 2007
Email completed assignments to cs2262_assignments@cct.lsu.edu