CS2262 Spring 2007

Assignment 6: Fixed Point Iteration and Interpolation

1. To be completed **before Tuesday 6th March Class:** Work through Atkinson & Han Pages 117 —128.

2. (a) Calculate the first six iterates in the iteration

$$x_{n+1} = 1 + 0.3\sin(x_n)$$

with $x_0 = 1$.Choose other initial guesses x_0 and repeat this calculation.

(b) Find an interval $[a, b]$ satisfying the property

$$a \leq x \leq b \qquad \text{implies that} \qquad a \leq g(x) \leq b$$

where $g(x)$ is such that $x_{n+1} = g(x_n)$.
Hint: Let

$$a = min_{[-\infty<x<\infty]} g(x), \qquad b = max_{[-\infty<x<\infty]} g(x)$$

(c) Prepare a table of results from the fixed point iteration method with columns n, x_n, $\alpha - x_n$, r_n. Where n is the iteration number ($n \geq 0$), α is the fixed point, and r_n is the ratio of the error ($r_n = (\alpha - x_n)/(\alpha - x_{n-1})$).

3. For the functions $g(x)$ shown in Figures 23 and 24, draw on successive iterations of the fixed point iteration method starting from an initial estimate $x = x + 0$.

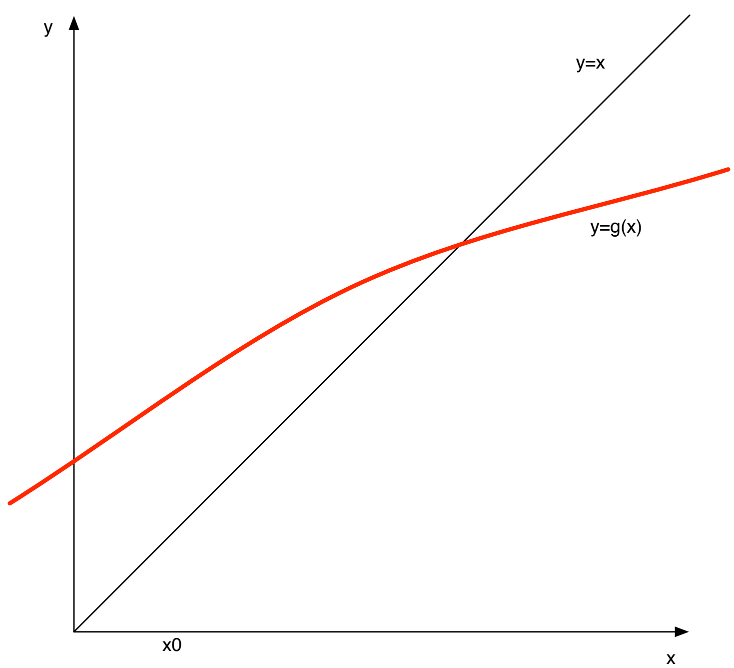

Figure 23: Draw on iterations of the fixed point iteration method starting from $x = x_0$

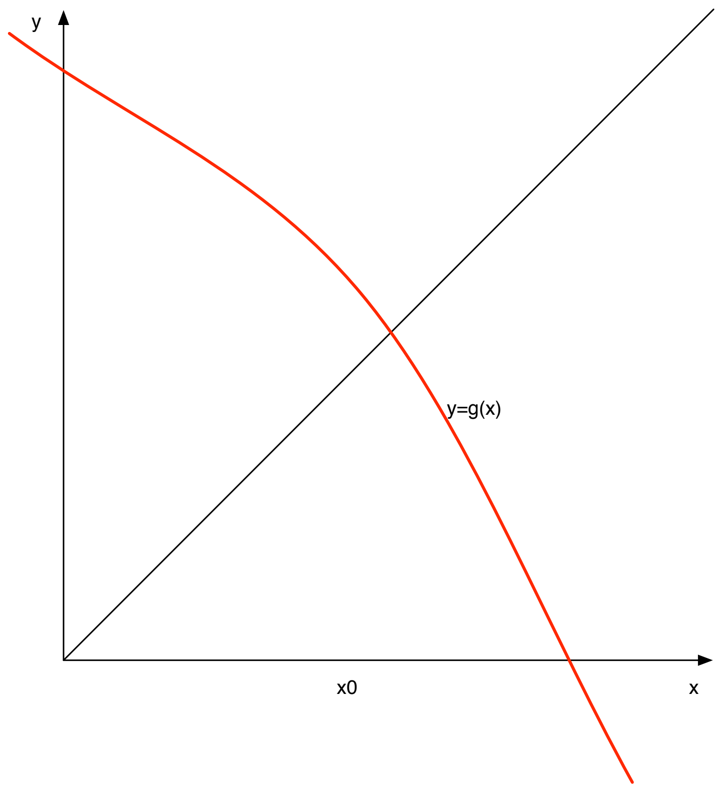

Figure 24: Draw on iterations of the fixed point iteration method starting from $x = x_0$

4. What are the solutions α, if any, of the equation $x = \sqrt{1+x}$? Does the iteration $x_{n+1} = \sqrt{1+x_n}$ converge to any of these solutions (assuming x_0 is chosen sufficiently close to α)?

5. Consider the rootfinding problem $f(x) = 0$ with root α, with $f'(x) \neq 0$. Convert it to the fixed-point problem

$$x = x + cf(x) \equiv g(x)$$

with c a nonzero constant. How should c be chosen to ensure rapid convergence of

$$x_{n+1} = x_n + cf(x_n)$$

to α (provided that x_0 is chosen sufficiently close to α)? Apply your way of choosing c to the rootfinding problem $x^3 - 5 = 0$.

6. Use Aitken's error estimation formula (3.53) to estimate the error $\alpha - x_2$ in the following iterations:

   (a)
$$x_{n+1} = e^{-x_n}, \qquad x_0 = 0.57$$

   (b)
$$x_{n+1} = \frac{0.5}{1 + x^4{}_n}, \qquad x_0 = 0.48$$

   (c)
$$x_{n+1} = 1 + 0.5\sin(x_n), \qquad x_0 = 1.5$$

7. For slowing convergent series, the Aitken extrapolation formula can greatly accelerate the convergence. Use the following algorithm:

$$\begin{aligned} x_1 &= g(x_0) \\ x_2 &= g(x_1) \\ x_3 &= \text{Aitken extrapolate of } x_0,\ x_1, \text{ and } x_2 \\ x_4 &= g(x_3) \\ x_5 &= g(x_4) \\ x_6 &= \text{Aitken extrapolate of } x_3,\ x_4, \text{ and } x_5 \end{aligned}$$

Continue this process in the same manner. Apply it to the following iterations

(a)

$$x_{n+1} = 2e^{-x_n}, \qquad x_0 = 0.8$$

(b)

$$x_{n+1} = \frac{0.9}{1 + x_n^4}, \qquad x_0 = 0.75$$

(c)

$$x_{n+1} = 6.28 + \sin(x_n), \qquad x_0 = 6$$

Due March 8th 2007
Email completed assignments to `cs2262_assignments@cct.lsu.edu`