CS2262 Spring 2007

Assignment 5: Taylor Series and Root Finding

1. Draw an arbitrary graph of a function, and pick a point $x = x_0$. Draw on the graph the linear Taylor polynomial, $p_1(x)$ at $x = x_0$.

2. (a) An *even* function $f(x)$ is one where $f(-x) = f(x)$ for all x of interest. An *odd* function $f(x)$ is one where $f(-x) = -f(x)$ for all x of interest. Which of the following functions are even or odd? (i) $\sin(x)$, (ii) x, (iii), x^2, (iv) e^x, (v) $\cos(x)$.

   (b) Let the polynomial $p(x)$ be an *even* function. Show that this implies that the coefficients of the polynomial $p(x)$ are zero for all terms of odd degree.

   (c) Let the polynomial $p(x)$ be an *odd* function. Show that this implies that the coefficients of the polynomial $p(x)$ are zero for all terms of even degree.

3. Find linear and quadratic Taylor polynomial approximations to $f(x) = (x)^{1/3}$ about the point $x = 8$. Bound the error in each of your approximations on the interval $8 \leq x \leq 8 + \delta$ with $\delta > 0$. Obtain the actual numerical bound on the interval [8,8.1]/

4. (a) Bound the error in the approximation

   $$\sin(x) \approx x$$

   for $-\pi/4 \leq x \leq \pi/4$

   (b) Since this is a good approximation for small values of x, also consider the 'percentage error'

   $$\frac{\sin(x) - x}{\sin(x)} \approx \frac{\sin(x) - x}{x}$$

   Bound the absolute value of the latter quantity for $-\delta \leq x \leq \delta$. Pick δ to make the absolute value of the percentage error less than 1%.

5. How large should the degree $2n - 1$ be chosen in a Taylor polynomial expansion of $\sin(x)$ to have

   $$|\sin(x) - p_{2n-1}(x)| \leq 0.001$$

   for all $-\pi/2 \leq x \leq \pi/2$? Check your result by evaluating the resulting $p_{2n-1}(x)$ at $x = \pi/2$.

6. Use a Taylor polynomial with remainder term to evaluate the limit

   $$\lim_{x \to 0} \frac{\log(1 + x^2)}{2x}$$

7. Evaluate

   $$I = \int_0^1 \frac{e^x - 1}{x} dx$$

   within an accuracy of 10^{-6}

8. Write a MATLAB program that can solve for roots of an arbitrary function (which would be provided in a separate MATLAB subroutine), where the user can specify at the MATLAB command line the method to use (either bisection, newton's method, or the secant method), the error tolerance, the maximum number of iterations, and depending on the method chose the starting conditions. The program should provide a graph of the estimated error in the solution, a graph of the function itself, and should provide the user with information about the final solution, including the value, the number of iterations, and the convergence rate. Make sure that your program checks for errors.

9. Use your program to solve for the indicated roots to a tolerance of $\epsilon = 10^{-6}$ using each of the implemented methods. (To allow for easy grading, add another user option to choose the equation to solve, for each question submit the parameters you used to solve for the root, and the value of the root found.)

   (a) The real root of $x^3 - x^2 - x - 1 = 0$
   (b) The smallest positive root of $\cos x = 1/2 + \sin x$
   (c) The root of $x = e^{-x}$

Due March 1st 2007
Email completed assignments to `cs2262_assignments@cct.lsu.edu`