Visualization of Multi Patch Data in Astrophysics

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1. Visualization

In research, numerical simulations are used to approximate analytical solutions to complex equations. When one introduces mathematical models and tries to solve them numerically, large amounts of data are generated.
Visualization is used to better understand different aspects of the simulations, to identify and locate problems in the numerical codes, and to present the results of the simulations to people outside the field. It is often easier to present and interpret these results in a simple graphical representation.

2. Multiple Patches

Spherical domains are usually preferred for physical simulations of stars and black holes. The angular resolution does not depend on the radius. The system of coordinates has to be adapted to the geometry of the physical object - for example black holes and stars are almost spherical.
In addition, in numerical simulations of black holes singularities are a particular challenge. The excision method removes these from the computational domain, introducing an inner boundary. In order for this to be smooth, it is easier to use spherical domains. A standard spherical polar coordinate system is problematic due to the coordinate singularities at the north and south pole; for example at the north pole, θ - the polar angle - is equal to zero and ϕ - the azimuthal angle - can take any value.
To deal with these problems we use an inflated cube coordinate system, as suggested by Thornburg[1] in 1987, a multi-patch (six patches) grid structure with spherical inner and outer boundaries. They have become more popular in computational astrophysics and were first successfully implemented for black holes evolutions only in 2004. In a nutshell, a multi patch system is a multi-block scheme[2] without coordinate singularities that allows spherical boundaries using several logically rectangular grids (a grid is a set of coordinates on which data values can be located, with points equally or non-equally spaced).

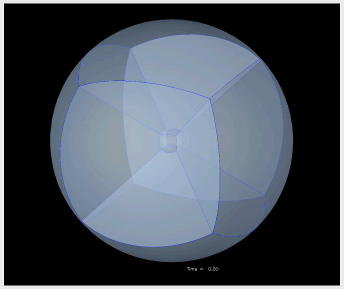

Figure 1: *The spherical inner and outer boundary can be observed in the above multi patch system*

The multiple grid patches provide a new level of flexibility and organization for implementing new coordinate systems. There are several advantages in using multiple grid patches: there are no coordinate singularities and the Cartesian grid is replaced by multiple angular grid patches so that there can be a constant angular resolution. A disadvantage is the complexity of the implementation. One particular challenge of the visualization of a multi patch system is that each patch has its own independent coordinate system.
The degree of success in visualizing multi patch simulations depends on the tools one uses. Some issues to consider when choosing these tools are the cost, the versatility and the program's ability to handle large amounts of data, hardware acceleration, disk space, etc. We solved this problem by running almost all of the simulations on high-performance computers or supercomputers.

3. OpenDX

OpenDX [3] is an interactive software used in research for visualization ofscientific 2D or 3D data based on experiments, numerical simulations, etc. These data are represented by numbers at each point on the regular or irregular grid.
Different tools -modules and macros- for transforming, rendering and animating data are used in complex networks to visualize and analyze the scientific information. A module is a part of a program that carries out a specific function; it can be used with other modules within a program. A macro is a sequence of modules collected together that can be used as a single tool.
OpenDX is a flexible and extensible software that allows the user to create his or her own features and to modify and combine the existing ones, making the visual program look simpler. In the current research we use OpenDX to visualize multi patch data.

4. Convergence Analysis

OpenDX is not only used for visualization, but also for analyzing the data. In order to test the accuracy of the multi patch code we used simulations of the scalar wave equation,

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0. \qquad (1)$$

Solving the wave equation (1), we obtained a numerical approximation to the analytic solution. The numerical solution obtained at a finite resolution has a certain error that comes from the approximation of the derivatives. By increasing the resolution of the multi patch grid structure, the numerical solution approaches the exact one and the error decreaseases. The smaller the space between two adjacent points on the grid, the better the approximation. The relationship between the numerical solution, analytical solution, and grid spacing resolution is given by the following formula:

$$f(\Delta) = f_0 + c\Delta^n, \qquad (2)$$

where c is the scale factor, Δ is the resolution, n is the convergence factor, f_0 is the exact solution to the wave equation and $f(\Delta)$ is the numerical solution at a certain resolution,

$$f(\Delta) = u(t, x, \Delta). \qquad (3)$$

Here, f_0, c and n are independent of the resolution. In order to estimate f_0, c and n we performed runs at three different resolutions and wrote a module in C that takes as an input these resolutions and gives the value of n as the output. This module can now be used as a research tool to check convergence of codes.

5. Visualization techniques

It is usually easier to visualize data, rather than to interpret different types of charts. For each value of the function we have a different color; for example the green color corresponds to zero in all the images. Using the generated images, we can analyze the unusual behaviour of the wave, observe the small and large scale features of the data, how much the field changes, and what is going on on the dynamics of the field. There are several techniques of visualization we were using in our research. The isosurface technique creates 2D contours in 3D space, mapping points in the domain that have the same value. The most important one is the slicing technique through the grid with one, two or more planes. This provides information about the data within the plane. Moving the slicing along the z axis, we actually sample at different steps in the domain. Three different results of this technique are shown and discussed in the next section.

6. Results

For each of the images generated at the time step equal to 110, we used data from simulations of the wave equation on a rotating black hole background. The time symmetric initial data are shells at $r = 20M$ and angular structure of spherical harmonics with $l = 2$, $m = 2$. The same data is visualized in different ways showing different aspects of the simulation and offering more information. For example, the first image below shows the evolution of a wave equation, taking a slice in a six multi patch grid structure (like the one showed in Figure 1). The second image is an unfolded cube. This is actually the same multi patch structure, but for a constant radius we took a 2D slice that intersects the six different patches. The last image also contains 2D slices through the same grid structure during a simulation of a wave in the same background of a rapidly rotating black hole. These planes are perpendicular.

a)
b)
c)

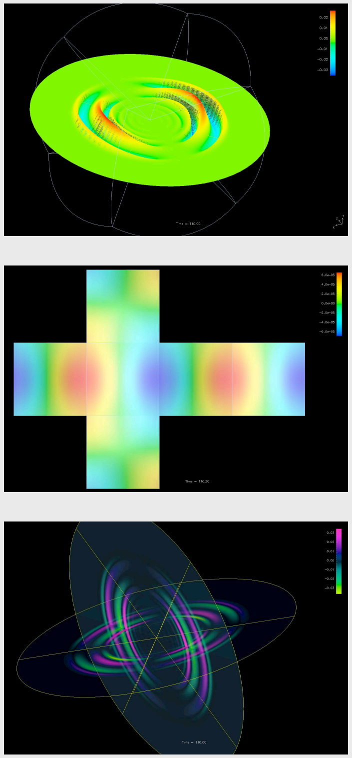

Figure 2: *a) Evolution of a scalar wave equation*
b) An unfolded multi patch grid structure
c) Two slicing planes for a simulation of the wave in a rapidly rotating black hole background

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References

[1] J. Thornburg, *Black hole excision with multiple grid patches*, Class. Quantum Grav. 21 (2004) 3665-3691

[2] Erik Schnetter, Peter Diener, Ernst Nils Dorband, and Manuel Tiglio, *A multi − block infrastructure for three − dimensional time − dependent numerical relativity*, 2006, gr-qc/0602104

[3] http://www.opendx.org